The Problem Corner

Edited by Pat Costello

The Problem Corner invites questions of interest to undergraduate students. As a rule, the solution should not demand any tools beyond calculus and linear algebra. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following new problems should be submitted on separate sheets before August 1, 2008. Solutions received after this will be considered up to the time when copy is prepared for publication. The solutions received will be published in the Fall 2008 issue of *The Pentagon*. Preference will be given to correct student solutions. Affirmation of student status and school should be included with solutions. New problems and solutions to problems in this issue should be sent to Pat Costello, Department of Mathematics and Statistics, Eastern Kentucky University, 521 Lancaster Avenue, Richmond, KY 40475-3102 (e-mail: pat.costello@eku.edu, fax: (859)622-3051)

NEW PROBLEMS 616-623

Problem 616. *Proposed by Melissa Erdmann, Nebraska Wesleyan University, Lincoln, Nebraska.*

The birthday paradox is that in a room with 23 people the probability that two or more of them will have the same birthday (month and day) is at least 50%. Find the number of people needed so that there is a 50% probability that at least three or more of them will have the same birthday. Find the formula that will represent the probability that at least k of n total people in a room share a birthday.

Problem 617. *Proposed by Jose Luis Diaz-Barrero, Universitat Politecnica de Catalunya, Barcelona, Spain.*

Find all triplets (x, y, z) of real numbers such that

$$\sqrt{3^x(4^y+5^z)} + \sqrt{4^y(3^x+5^z)} + \sqrt{5^z(3^x+4^y)} = \sqrt{2}(3^x+4^y+5^z)$$

Problem 618. *Proposed by Jose Luis Diaz-Barrero, Universitat Politecnica de Catalunya, Barcelona, Spain.*

Let a, b, c be real numbers such that $0 < a \le b \le c < \pi/2$. Prove that

$$\frac{\sin a+\sin b+\sin c}{\cos a(\tan b+\tan c)+\cos b(\tan c+\tan a)+\cos c(\tan a+\tan b)} \le \frac{1}{2}$$

Problem 619. *Proposed by Duane Broline and Gregory Galperin (jointly), Eastern Illinois University, Charleston, Illinois.*

Several identical square napkins are placed on a table. They are placed in such a way that any two of them have a common area which is greater than half of the area of one of them. Is it always possible to pierce all the napkins with a needle going perpendicular to the plane of the table? If yes, prove it. If not, provide a counterexample.

Problem 620. *Proposed by Duane Broline and Gregory Galperin (jointly), Eastern Illinois University, Charleston, Illinois.*

Nick chooses 81 consecutive integers, rearranges them, and concatenates them together to form one long, multi digit number N. Michael chooses 80 consecutive integers, rearranges them, and concatenates them together to obtain the number M. Is it possible that $M=N$? If yes, provide an example. If not, prove it.

Problem 621. *Proposed by Lisa Hernandez, Jim Buchholz, Doug Martin (jointly), California Baptist University, Riverside, California.*

A standard technique for showing that .9999... = 1 is to let x=.999... and then 10x = 9.999... and so 9x = 10x - x = 9 which gives x=1. Can you derive an alternative proof that .999... = 1, perhaps using proof by contradiction?

Problem 622. *Proposed by the editor.*

Prove that there are infinitely many positive, palindromic integers containing just the digits 2, 7, and 9 which are divisible by 2, 7, and 9.

Problem 623. *Proposed by the editor.*

Let f(n) be the number of rationals between 0 and 1 (noninclusive) that have denominator less than or equal to n. Prove that $f(n) > (2/3)\, n^{3/2}$ for all integers $n \ge 2$.