The Problem Corner

Edited by Pat Costello

The Problem Corner invites questions of interest to undergraduate students. As a rule, the solution should not demand any tools beyond calculus and linear algebra. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following new problems should be submitted on separate sheets before March 15, 2016. Solutions received after this will be considered up to the time when copy is prepared for publication. The solutions received will be published in the Spring 2016 issue of *The Pentagon*. Preference will be given to correct student solutions. Affirmation of student status and school should be included with solutions. New problems and solutions to problems in this issue should be sent to Pat Costello, Department of Mathematics and Statistics, Eastern Kentucky University, 521 Lancaster Avenue, Richmond, KY 40475-3102 (e-mail: pat.costello@eku.edu, fax: (859) 622-3051)

NEW PROBLEMS 760-768

Problem 760. *Proposed by Mathew Cropper, Eastern Kentucky University, Richmond, KY.*

Consider a 5x5 grid. There are $\binom{25}{13}$ ways to fill the grid with exactly 13 X's and 12 O's. How many of these have at least one row, one column, or one diagonal with 5 X's?

Problem 761. *Proposed by Ovidiu Furdiu, Technical University of Cluj-Napoca, Cluj, Romania.*
Let $n \geq 0$ be an integer and let T_{2n} denote the $2n$th Taylor polynomial corresponding to the cosine function at 0.

$$T_{2n}(x) = \sum_{k=1}^{n+1} (-1)^{k-1} \frac{x^{2k-2}}{(2k-2)!}$$

Let $I_n = \int_0^\infty \frac{T_{2n}(x) - \cos x}{x^{2n+2}} dx$.

a) Prove that $I_n = -\frac{1}{(2n+1)(2n)} I_{n-1}$, $n \geq 1$.

b) Calculate I_n.

Problem 762. *Proposed by Mohammad Azarian, University of Evansville, Evansville, IN.*

If $x \neq 0$, $x \neq 1$, $y > 0$, and $y \neq 1$, then find y as a function of x provided

$$y' + y(\ln y)^2 - \frac{1}{x(x-1)} y \ln y = 0$$

Problem 763. *Proposed by D.M. Batinetu-Giurgiu, "Matei Basarab" National College, Bucharest, Romania, Neculai Stanciu, "George Emil Palade", Buzau, Romania.*

Let $x \in \square$ and $A(x) = \begin{pmatrix} x+1 & 1 & 1 & 1 \\ 1 & x+1 & 1 & 1 \\ 1 & 1 & x+1 & 1 \\ 1 & 1 & 1 & x+1 \end{pmatrix}$.

Compute the matrix product A(0)*A(1)*A(2)*A(3).

Problem 764. *Proposed by D.M. Batinetu-Giurgiu, "Matei Basarab" National College, Bucharest, Romania, Neculai Stanciu, "George Emil Palade", Buzau, Romania.*

Calculate $\lim\limits_{n\to\infty} \sqrt{n}\left(((n+1)!)^{1/2(n+1)} - (\mathrm{n}!)^{1/2n}\right)$

Problem 765. *Proposed by Marcel Chirita, Bucharest, Romania.*

Let a, b, c be the lengths of the sides of a triangle in which $b^2 + c^2 = a^2$. Prove that $b^3 + c^3 < 3abc + (2\sqrt{2} - 1)a^3$

Problem 766. *Proposed by Marcel Chirita, Bucharest, Romania.*

Let x be an integer. Prove that if $x^5 + 5x^3 + 15x^2 > 21x$, then $x^5 + 5x^3 + 15x^2 - 21x \geq 30$.

Problem 767. *Proposed by the editor.*

Prove that the number 13^{17} cannot be written as the sum of a square and a fifth power of another integer.

Problem 768. *Proposed by the editor*.

Calculate the value of the series $\displaystyle\sum_{n=0}^{\infty} \frac{(1/2)^n}{(n+2)(n+4)}$