The Problem Corner

Edited by Pat Costello

The Problem Corner invites questions of interest to undergraduate students. As a rule, the solution should not demand any tools beyond calculus and linear algebra. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following new problems should be submitted on separate sheets before March 15, 2017. Solutions received after this will be considered up to the time when copy is prepared for publication. The solutions received will be published in the Spring 2017 issue of *The Pentagon*. Preference will be given to correct student solutions. Affirmation of student status and school should be included with solutions. New problems and solutions to problems in this issue should be sent to Pat Costello, Department of Mathematics and Statistics, Eastern Kentucky University, 521 Lancaster Avenue, Richmond, KY 40475-3102 (e-mail: pat.costello@eku.edu, fax: (859) 622-3051)

NEW PROBLEMS 780-788

Problem 780. *Proposed by Daniel Sitaru, Colegiul National Economic Theodor Costescu, Drobeta Turnu – Severin, Mehedinti, Romania.*

Prove that if $a,b,c \in [1,\infty)$, then $ab + bc + ca \geq 3 + 2\ln(a^b b^c c^a)$.

Problem 781. *Proposed by Daniel Sitaru, Colegiul National Economic Theodor Costescu, Drobeta Turnu – Severin, Mehedinti, Romania.*

Prove that if $a,b,c \in (0,\infty)$, then

$$\sum a\sqrt{(b^4+c^4)/2} \leq a^2(b+c)+b^2(a+c)+c^2(a+b)-3abc\,.$$

Problem 782. *Proposed by Jose Luis Diaz-Barrero, Barcelona Tech-UPC, Barcelona, Spain.*

Let a, b, c be the lengths of the sides of triangle ABC and m_a, m_b, and m_c the lengths of its medians. Prove that

$$\frac{2^{m_a}+2^{m_b}+2^{m_c}}{2^a+2^b+2^c} < 1.$$

Problem 783. *Proposed by Jose Luis Diaz-Barrero, Barcelona Tech-UPC, Barcelona, Spain.*

Find all real solutions of the following system of equations

$$x^3 + 2x + y \quad = 9 + 3x^2$$
$$3y^2 + 6y + z \quad = 21 + 9y^2$$
$$5z^3 + 10z + x = 33 + 15z^2.$$

Problem 784. *Proposed by D.M. Batinetu-Giurgiu, "Matei Basarab" National College, Bucharest, Romania, Neculai Stanciu, "George Emil Palade", Buzau, Romania.*

Prove that in any triangle ABC with $BC = a$, $CA = b$, $AB = c$ and area F, the following inequalities are true.

$$(b^2+c^2)\sin\frac{A}{2}+(c^2+a^2)\sin\frac{B}{2}+(a^2+b^2)\sin\frac{C}{2}\geq 4\sqrt{3}F,$$

$$ab(1+\sin^2\frac{C}{2})+bc(1+\sin^2\frac{A}{2})+ca(1+\sin^2\frac{B}{2})\geq 4\sqrt{3}F.$$

Problem 785. *Proposed by Iuliana Trasca, Scornicesti, Romania*.

Show that x, y, $z > 0$, then

$$\frac{x^6z^3+y^6x^3+z^6y^3}{x^2y^2z^2}\geq\frac{x^3+y^3+z^3+3xyz}{2}\ .$$

Problem 786. *Proposed by Thomas Chu, Macomb, Illinois*.

Prove that if $x,y,z > 1$, then

$$(x^2+y^2+z^2)(x+y+z)+x^3+y^3+z^3 > 4xy+4xz+4yz\ .$$

Problem 787. *Proposed by the editor*.

Mike buys some pants and shorts at the Great Pants Store. Mike buys shorts that cost \$11 each and pants that cost \$14 each. His total before taxes is \$283. How many shorts and how many pants did Mike buy?

Problem 788. *Proposed by George Heineman, Worcester Polytechnic Institute, Worcester, MA.*

A Sujiken™ puzzle has a triangular grid of cells containing digits from 1 to 9. You must place a digit in each of the empty cells with the constraint that no digit can repeat in any row, column, or diagonal. Additionally, no digit can repeat in the 3x3 large squares with thick borders or the three triangular regions with thick borders. The puzzle below is of intermediate difficulty.

			8	5	9	4		
			6			3		
			3				1	9
			1	9				7
					6			1
					5	8	4	6