The Problem Corner

Edited by Pat Costello

The Problem Corner invites questions of interest to undergraduate students. As a rule, the solution should not demand any tools beyond calculus and linear algebra. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following new problems should be submitted on separate sheets before January 31, 2025. Solutions received after this will be considered up to the time when copy is prepared for publication. The solutions received will be published in the Fall 2024 issue of *The Pentagon*. Preference will be given to correct student solutions. Affirmation of student status and school should be included with solutions. New problems and solutions to problems in this issue should be sent to Pat Costello, Department of Mathematics and Statistics, Eastern Kentucky University, 521 Lancaster Avenue, Richmond, KY 40475-3102 (e-mail: pat.costello@eku.edu, fax: (859) 622-3051)

NEW PROBLEMS 928-936.

Problem 928. *Proposed by José Luis Díaz-Barrero, School of Civil Engineering, Barcelona Tech - UPC, Barcelona, Spain.*

Solve in the set of positive integers the following equation: $x^2 + y^2 = 137(x - y)$.

Problem 929. *Proposed by José Luis Díaz-Barrero, School of Civil Engineering, Barcelona Tech - UPC, Barcelona, Spain.*

In how many ways can the rational 2025/2024 be written as the product of two rational numbers of the form $(n + 1)/n$, where n is a positive integer?

Problem 930. *Proposed by Mathew Cropper, Eastern Kentucky University, Richmond, KY.*

The Mycielski construction is done to a finite simple graph G producing a graph $M(G)$ as follows: set the vertex set of G to be $\{v_1, v_2, \dots, v_k\}$, then add a set of vertices $\{u_1, u_2, \dots, u_k\}$ and one more vertex w. Set u_i to be adjacent to every vertex in G to which v_i is adjacent and make w adjacent to every vertex in $\{u_1, u_2, \dots, u_k\}$. Note that the set of vertices $\{u_1, u_2, \dots, u_k\}$ is an independent set [Introduction to Graph Theory, West, pg. 205]. Let $M^n(G)$ denote the graph obtained from a given finite simple graph G by applying the Mycielski construction n times. Determine a formula for the number of edges in the graph $M^n(G)$.

Problem 931. *Proposed by Richard Hasenauer, Eastern Kentucky University, Richmond, KY.*

Prove that 7! divides $n^7 - 14n^5 + 49n^3 - 36n$ for all positive integers n.

Problem 932. *Proposed by Tom Richmond, Western Kentucky University, Bowling Green, KY.*

If a and b are distinct square-free natural numbers and c and d are nonzero rational numbers, find necessary and sufficient conditions for $c\sqrt{a} + d\sqrt{b}$ to be a nonzero rational number.

Problem 933. *Proposed by Tom Richmond, Western Kentucky University, Bowling Green, KY.*

For cube-free integers $a, b > 1$ and nonzero rationals c, d, show that $c\sqrt[3]{a} + d\sqrt[3]{b}$ is rational if and only if $a = b$ and $c = -d$.

Problem 934. *Proposed by John Wilson, recently retired from Centre College, Danville, KY.*

A Squarely puzzle is a logic puzzle played on a 5x5 grid. The solution requires that the digits 1 through 9 be placed in the grid with two rules: 1) No digit appears more than once in any row, column or diagonal; 2) the 25 cells must contain exactly 3 copies of 8 of the digits and one copy of the ninth digit. You are given the five digits of each row, column and diagonal.

Row	Column	Diagonal
13458	12459	
12369	34689	\12589
24569	35678	
24568	12569	/35689
13789	12348	

Get 10 free puzzles at squarelypuzzle.com.

Problem 935. *Proposed by the editor.*

The binomial transform of sequence $a_0, a_1, a_2, \ldots, a_n$ is sequence $b_0, b_1, b_2, \ldots, b_n$ where

$$b_k = \sum_{i=0}^{k} (-1)^i \binom{k}{i} a_i$$

Starting with sequence $a_0 = 1, a_1 = -2, a_2 = 4, a_3 = -8, a_4 = 16$, find b_4.

Problem 936. *Proposed by the editor.*

A pair of numbers (A,M) is called *amicable* when $\sigma(A)-A = M$ and $\sigma(M)-M = A$ where $\sigma(n)$ is the sum of all positive divisors of n. The smallest amicable pair is (220,284) which was known to Pythagoras. 4230338853909611459680566 1394194053 is one member of a previously-unknown amicable pair. Find the other member.