THE PENTAGON

Volume XXXVI  Fall, 1976  Number 1

CONTENTS

National Officers .................................. 2
Editorial Notice .................................. 3
Introduction to Basic Concepts of Graph Theory ........ 4
Fermat Numbers and Polygonal Numbers ............... 22
The Mathematical Scrapbook ....................... 25
The Problem Corner ................................ 31
The Book Shelf .................................... 36
Kappa Mu Epsilon News ............................. 40
Installation of New Chapters ........................ 61
Kappa Mu Epsilon, mathematics honor society, was founded in 1931. The object of the fraternity is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics, due, mainly, to its demands for logical and rigorous modes of thought; to provide a society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; to disseminate the knowledge of mathematics and to familiarize the members with the advances being made in mathematics. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.
Editorial Note

The first article in this issue is the first attempt at presenting longer expository articles on mathematics not covered in the customary undergraduate program. Professor Nance was requested to write this article so that it could be studied by individual students, or by KME chapters and other small groups in a seminar fashion. Problems are spaced through the exposition and are designed to enhance and extend the content. Further reading is suggested for deeper study of graph theory.

The editor would be delighted to receive comments on the usefulness of this article. If it is well received by the readers of THE PENTAGON, other expository articles on topics not normally taught in the undergraduate program will be solicited and printed. Anyone desiring to write such an article should contact the editor.
Graph theory is an exciting, challenging branch of mathematics rapidly gaining popularity at the college and university level. One reason for this increasing popularity is that basic concepts can be readily understood with a minimum mathematical background. Consequently, a first course in graph theory is appropriate at the undergraduate level. The author's experience in offering such a course has been that students invariably get "caught up" in the material. They are enthusiastic in class and become excited when they discover basic relationships.

A second reason for graph theory's popularity is the manner in which it lends itself to application. According to Ore, "... recent developments in mathematics and particularly in its applications have given a strong impetus to graph theory. Already in the nineteenth century graphs were used in such fields as electrical circuitry and molecular diagrams. A present there are topics in pure mathematics, for instance, the theory of mathematical relations, where graph theory is a natural tool, but there are also numerous other uses in connection with highly practical questions: matchings, transportation problems, the flow in pipe line networks, and so-called 'programming' in general. Graph theory now makes its appearance in such diverse fields as economics, psychology, and biology."

[4,p.3]

The definitions in this article are widely, but not universally, used. When reading other graph theory articles, the reader should always check the author's notation and terminology.

A graph $G$ is a finite, nonempty set $V$ together with a set $E$ of two-element subsets of distinct elements of $V$. Elements of $V$ are called vertices and elements of $E$ are called edges. An edge $e = \{u,v\}$ is usually denoted $e = uv$. This definition lends itself to an immediate geometrical interpretation. Vertices can be represented by points and edges by arcs or lines. For example, let $G$ be the graph where $V = \{v_1, v_2, v_3, v_4\}$ and $E = \{v_1v_3, v_3v_4, v_4v_2, v_2v_1\}$, then $G$ can be depicted as in Figure 1. (Be careful not to confuse intersection
of arcs with vertices.) If \( G \) has been specified, the vertex set and edge set are frequently denoted by \( V(G) \) and \( E(G) \) respectively. When \( e = v_1v_2 \) is in \( E(G) \), we say \( v_1 \) and \( v_2 \) are adjacent and \( e \) is said to be incident with \( v_1 \) and \( v_2 \).

**Problem 1.** Examine variations produced by changing the definition of a graph \( G \). For example, (1) suppose \( V \) is not finite, or (2) suppose the two element subsets are not distinct, or (3) suppose the two element subsets are ordered pairs of elements of \( V \).

![Figure 1. Geometrical representation of a graph.](image1)

Graphs \( G_1 \) and \( G_2 \) are isomorphic if there exists a one-to-one mapping \( \alpha \) from \( V(G_1) \) onto \( V(G_2) \) such that \( v_1v_2 \in E(G_1) \) if and

![Figure 2. Which graph is not isomorphic to the others?](image2)
only if $a(v_1) a(v_2) \in E(G_2)$. If $G_1$ and $G_2$ are isomorphic, we write $G_1 = G_2$.

In Figure 2, $G_2$ is isomorphic to $G_3$ but $G_1$ is not isomorphic to either $G_2$ or $G_3$.

**Problem 2.** a) Explain why $G_1$ of Figure 2 is not isomorphic to $G_3$. b) For Figure 2, find a one-to-one mapping from $V(G_2)$ onto $V(G_3)$ that preserves adjacency.

Let $G$ be a graph with vertex set $V$ and edge set $E$. If the number of elements in $V$, $|V| = p$ and $|E| = q$, then $G$ is called a $(p,q)$ graph and the order of $G$ is $p$.

**Problem 3.** If $G$ is a $(p,q)$ graph, what can be said about upper and lower bounds for $p$ and $q$?

It is often necessary to talk about subgraphs and supergraphs in the same sense that one talks about subsets and supersets. Accordingly, we say a graph $H$ is a subgraph of a graph $G$ if $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$; $G$ is called the supergraph of $H$. Elementary subgraphs of $G$ can be obtained by deleting either a vertex or an edge. If $G$ is a graph with vertex set $V$, then $G - v_1$,

![Subgraphs](image3.png)

Figure 3. Subgraphs
The Pentagon

is the graph with vertex set \( V - \{v_i\} \) and edge set containing all the edges of \( G \) that were not incident with \( v_i \). \( G - e \) is the graph with vertex set \( V(G) \) and edge set \( E(G) - \{e\} \). Figure 3 illustrates such subgraphs.

The degree of a vertex \( v \) is the number of vertices adjacent to \( v \). In Figure 3, the degree of \( v_1 \) in \( G \) is 3 and we write \( \text{deg} \ v_1 = 3 \). It is relatively easy to show there is a relationship between the degrees of the vertices and the number of edges in a graph. If \( G \) is a \((p,q)\) graph with \( V(G) = \{v_1,v_2,\ldots,v_p\} \), then \( \sum \text{deg} \ v_i = 2q \). This immediately implies there is an even number of odd vertices in a graph.

Certain types of graphs are used frequently enough to warrant special mention. Several of these will now be discussed.

A graph \( G \) is regular of degree \( d \) if \( \text{deg} \ v = d \) for all \( v \in V \). In this case, \( G \) is called \( d \)-regular. In particular, if \( d = 3 \), \( G \) is 3-regular or cubic (cubic graphs play a prominent role in a later development). \( G \) is called a complete graph if every pair of vertices are adjacent. This may also be defined by saying \( |E| = \binom{p}{2} \).

If \( G \) is a complete graph of order \( p \), we write \( G = K_p \). \( K_1,K_2,K_3,K_4,K_5 \)

\[
K_1: \quad \bullet \ v_1 \\
K_2: \quad v_1 \rightarrow \bullet \ v_2 \\
K_3: \quad \bullet \ v_1 \rightarrow \bullet \ v_2 \rightarrow \bullet \ v_3 \\
K_4: \quad v_1 \rightarrow v_2 \rightarrow v_3 \\
K_5: \quad \\
\]

Figure 4. Complete graphs

and \( K_5 \) are given in Figure 4. Notice that \( K_5 \) cannot be drawn in the plane without having at least one edge crossing some other edge.
Problem 4. Let $G$ be a cubic $(p,q)$ graph with $q = 2p - 3$. What can be said about $G$?

Problem 5. Try to draw $K_3$ in the plane without having any edges crossing. Why can't this be done?

A graph $G$ is $n$-partite, $n \geq 2$, if it is possible to partition the vertex set $V$ into subsets $V_1, V_2, \ldots, V_n$ such that each edge $e = uv$ has the property that if $e \in V_i$, then $v \in V_j$ where $i \neq j$. The class of graphs for $n = 2$ are called bipartite graphs. Figure 5 illustrates such a graph. In this case, $V_1 = \{u_1, u_2, u_3\}$, and $V_2 = \{u_4, u_5, u_6\}$. Actually, this bipartite graph has an additional property. Note that each vertex of $V_1$ is adjacent to each vertex of $V_2$. When this occurs, we say $G$ is a complete bipartite graph and denote it $K(3,3)$. In general, complete $n$-partite graphs are denoted $K(p_1, p_2, \ldots, p_n)$ where $p_i = |V_i|$, $1 \leq i \leq n$.

Problem 6. Sketch $K(2, 3, 4)$.

Problem 7. a) How many edges are in $K(p_1, p_2)$?

b) How many edges are in a complete $n$-partite graph?

A very important class of graphs is the class of connected graphs. Before defining this concept, however, one needs some additional definitions. Let $G$ be a graph. A $u-v$ walk of $G$ is an alternating sequence of vertices and edges of $G$ such that each edge is incident with the vertices preceding and following it in the sequence. Figure 6 contains the $u_1-u_2$ walk $W$: $u_1, u_1u_5, u_5, u_5u_3, u_3, u_3u_2, u_2, u_2u_5, u_5, u_5u_1, u_1, u_1u_6, u_6, u_6u_3, u_3, u_3u_2, u_2$. In future work, this will be denoted by $u_1,u_2,u_3,u_4,u_5,u_6,u_1,u_2,u_3,u_4$. If no edge is repeated in the walk, the walk is called a trail. The trail is open if the first and
last vertices are not equal; otherwise, the trail is closed. If no vertex is repeated, it is called a path. A closed walk is called a circuit. If a circuit has distinct vertices (except for the first and last), then the circuit is a cycle. In Figure 6, \( C: u_1, u_2, u_3, u_5, u_2, u_6, u_1 \) is a circuit while \( C: u_1, u_2, u_3, u_5, u_1 \) is a cycle. If \( G \) contains no cycles, then \( G \) is called acyclic. An interesting relationship exists between cycles of graphs and bipartite graphs. This is stated in Theorem 1 and the proof is left to the reader.

**Theorem 1.** Let \( G \) be \((p,q)\) graph with \( p > 2 \). Then \( G \) is bipartite if and only if \( G \) contains no odd cycles.

![Figure 6. Paths and circuits](image)

We are now ready to define connected graphs. A graph \( G \) is connected if for every pair of vertices \( u,v \) in \( V(G) \) there is a \( u-v \) path in \( G \). If \( G \) is not connected, then the connected subgraphs of \( G \) are referred to as components of \( G \). If \( G \) has \( n \) components, we say \( c(G) = n \). Figure 7 is a graph where \( c(G) = 3 \).

**Problem 8.** Prove that “is connected to” is an equivalence relation on the vertex set of a graph.

**Problem 9.** Let \( G \) be a graph of order \( p \) such that the minimum degree of \( G \) is greater than or equal to \((p-1)/2\). Prove that \( G \) is connected.

Closely related to the concepts of connected graphs are the concepts of cut-vertices and cut-edges. A vertex \( v \) is a cut-vertex of a graph \( G \) if \( c(G) < c(G - v) \). A cut-edge (bridge) of a graph \( G \) is an edge \( e \) such that \( c(G) < c(G - e) \). In Figure 8, \( u,v \) and \( w \) are cut-vertices while \( e = vw \) is a bridge. Characteriza-
The Pentagon

Figure 7. A graph with three components.

Theorems for both cut-vertices and bridges exist. In particular, a characterization for bridges that allows the number of bridges to be counted by inspection is:

**Theorem 2.** Let $G$ be a graph. An edge $e$ is a bridge of $G$ if and only if $e$ is on no cycle of $G$.

**Problem 10.** Determine the maximum number of bridges possible in a graph of order $p > 2$.

Figure 8. Cut vertices and edges.

One of the special graphs to be considered has an application to a problem that is about two hundred years old—the Königsberg Bridge problem. The people of Königsberg lived by a river which contained two islands. These islands were connected to the river banks by bridges as shown in Figure 9. Local residents would try to take a walk in which they crossed each bridge one time without recrossing any bridge. However, none were able to do this. The problem can be stated as a graph theory problem if the land masses are represented by vertices of a graph $G$ and the bridges by edges of $G$. We would then obtain the “multigraph”* of Figure 10. The

---

* A change in definition would need to be made to allow for multiple edges.
The problem is then simplified to finding a trail containing all the edges of G. This problem leads to the following definitions: (1) an eulerian trail of a connected graph G is an open trail of G containing all the edges of G; (2) an eulerian circuit of G is a circuit containing all the edges of G; and (3) a graph is eulerian if it contains an eulerian circuit.

Figure 9. Königsberg Bridge Problem.

Figure 10. Multigraph corresponding to Königsberg Bridge Problem.

Characterizations of eulerian graphs and graphs with eulerian trails have been established [2,p.36]. They are as follows:

**Theorem 3.** Let G be a connected graph of order \( n > 2 \). Then (1), G is eulerian if and only if every vertex of G is even; and (2), G contains an eulerian trail if and only if G has exactly two odd vertices.

Similar results can be proven for multigraphs after the concepts of degree, trail, eulerian circuit, etc., have been defined. Hence the Königsberg bridge problem can be restated as: "Does the multigraph G of Figure 10 contain an eulerian trail?" According to Theorem 3, no such trail exists.
A concept similar to eulerian graphs is that of hamiltonian graphs. A graph $G$ is hamiltonian if there exists a cycle which contains all vertices of $G$. Such a cycle is called a hamiltonian cycle. In Figure 11, $G_1$ is hamiltonian and $G_2$ is not hamiltonian.

![Figure 11. Hamiltonian and nonhamiltonian graphs](image)

The similarity between eulerian and hamiltonian graphs would lead one to expect a nice characterization of hamiltonian graphs. However, this is not so. At this time, there is no known characterization of hamiltonian graphs. This is one of the major unsolved problems of graph theory. Several sufficient conditions exist for a graph to be hamiltonian. An interesting one is the result of Posa's work [5, pp. 355-361]. (This particular result was proved when he was in his early teens!)

**Theorem 4.** Let $G$ be a graph of order $p \geq 3$ such that for every integer $j$ with $1 \leq j < p/2$, the number of vertices of degree not exceeding $j$ is less than $j$. Then $G$ is hamiltonian.

A geometrically related problem can be stated in terms of hamiltonian graphs. Let $P$ be a polyhedron with faces $F_1, F_2, \ldots, F_m$. We can form an associated graph $G(P)$ of order $m$ by letting $v_i$ represent $F_i$ and letting $v_i$ be adjacent to $v_j$ if and only if $F_i$ and $F_j$ ($i \neq j$) share a common edge. Figure 12 shows a cube and its associated graph. Note that $G(P)$ is a hamiltonian graph. An interesting exercise is to show that graphs of the tetrahedron, octahedron, icosahedron, and dodecahedron are also hamiltonian.

Another type of problem involving hamiltonian graphs is the Traveling Salesman Problem. Suppose a traveling salesman wishes to visit $p$ cities that are connected by a network of highways. Suppose also he has good reason for not wanting to return to any city
other than the one he started from (perhaps this salesman was the cause of all those traveling salesman stories). The salesman could then decide whether or not such a trip was possible by examining the associated graph where vertices corresponded to cities and edges to roads connecting the towns. If the graph is hamiltonian, such a trip would be possible. Of a more practical nature, expenses associated with traveling could be assigned to edges (highways) joining the cities. Then the most economical way of traveling would be found by the "minimal" hamiltonian cycle.

Perhaps the most important class of graphs for applications are trees. A tree is a connected acyclic graph. If $G$ is acyclic but not connected, $G$ is called a forest. (What else would you expect a collection of trees to be called?) Trees were used by A. Cayley in the study of organic chemistry and by G. Kirchoff when working with electric networks [4,p.52]. Figure 13 shows all nonisomorphic trees of order five.

Several characterizations exist for trees. In fact, the reader may find some of these used as definitions in other texts.
Theorem 5. Let $G$ be a $(p,q)$ graph. Then (1), $G$ is a tree if and only if $G$ is acyclic and $p = q + 1$; (2), $G$ is a tree if and only if $G$ is connected and $p = q + 1$; and (3), $G$ is a tree if and only if every two distinct vertices of $G$ are joined by a unique path of $G$.

Using the results of this theorem, one can solve an agriculture problem. Suppose a rice paddy is in the shape depicted by Figure 14 where edges represent earthen dams between fields. If the source of water is exterior to the paddy, how many dams must be broken in order to flood all the fields? In graph theoretic terms, how many edges must be removed to make an acyclic subgraph (spanning tree)? For the graph $G$ of Figure 14, $p = 8$ and $q = 12$. According to Theorem 5, for a tree, $p = q + 1$; therefore, $q$ must be 7 and 5 edges (dams) will need to be removed.

![Figure 14. The rice paddy problem.](image)

Problem 11. Misters A, B, C, D, E, F, G, and H are political conspirators in what has become known as the "Blottergate Affair." In order to coordinate their cover-up efforts, it is vital that each of them be able to communicate directly or indirectly with every other conspirator. Such communications involve a certain amount of risk to everyone. Below is a list of "risk factors" associated with direct communications between the indicated parties. All other direct communications

<table>
<thead>
<tr>
<th>Pairs</th>
<th>A</th>
<th>A</th>
<th>A</th>
<th>A</th>
<th>B</th>
<th>B</th>
<th>B</th>
<th>B</th>
<th>C</th>
<th>C</th>
<th>D</th>
<th>D</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B</td>
<td>C</td>
<td>E</td>
<td>F</td>
<td>G</td>
<td>C</td>
<td>F</td>
<td>D</td>
<td>F</td>
<td>G</td>
<td>H</td>
<td>E</td>
<td>H</td>
<td>H</td>
</tr>
<tr>
<td>Risk Factor</td>
<td>9</td>
<td>3</td>
<td>8</td>
<td>3</td>
<td>4</td>
<td>10</td>
<td>6</td>
<td>6</td>
<td>4</td>
<td>5</td>
<td>7</td>
<td>6</td>
<td>3</td>
<td>5</td>
</tr>
</tbody>
</table>
are too likely to expose the scheme. What is the least total risk involved in a connecting system? [1]

Another class of graphs that have important geometric interpretations are planar graphs. A graph $G$ is planar if it can be drawn in the plane so that no edges cross. If $G$ is drawn in the plane in such a fashion, $G$ is called a plane graph. To avoid confusion in terminology, consider the graph $G$ in Figure 15. If $G$ is depicted as on the left, $G$ is not a plane graph. However, if $G$ is drawn as on the right, $G$ is a plane graph. In any case, $G$ is a planar graph. In Figure 16, $K_{(2,3)}$ is shown to be planar.

![Figure 15.](image)

The famous Euler Formula can be used to relate the order, number of edges, and number of regions of a planar graph. Recall for a polyhedron with $V$ vertices, $E$ edges, and $F$ faces that $V - E + F = 2$. If we then associate the polyhedra $P$ with a graph $G(P)$ as before (Figure 12), we obtain the graph theory version of Euler's Formula:
**Theorem 6.** If \( G \) is a connected plane graph with \( p \) vertices, \( q \) edges, and \( r \) regions, then \( p - q + r = 2 \).

Using Euler's Formula and graphs related to regular polyhedra, it can be shown that there are exactly five regular polyhedra [2,p.89]. Graphs of these polyhedra are given in Figure 17.

![Graphs of regular polyhedra](image)

**Figure 17.** Graphs of regular polyhedra.

A natural question that arises when studying planar graphs is, "How many edges can there be for a graph \( G \) of order \( p \) such that \( G \) is planar?" A planar graph with the maximum number of edges is called *maximal planar*. It is relatively easy to see that in a

\[
G: \quad p = 6 \quad q = 12
\]

**Figure 18.** A maximal graph.
maximal planar graph the boundary of every region will consist of three edges (otherwise nonadjacent vertices could be joined by an edge in the interior of a region). Using Euler's Formula, one can then show that if \( G \) is a maximal planar \((p,q)\) graph with \( p \geq 3 \), then \( q = 3p - 6 \). Figure 18 shows a maximal planar graph.

What are some of the nonplanar graphs? Recall that in Figure 5, \( K_{(3,3)} \) appeared to be nonplanar. \( K_{(3,3)} \) is nonplanar as is the complete graph \( K_5 \). To convince yourself, try to draw them so that no edges cross. \( K_5 \) and \( K_{(3,3)} \) play a significant role in the study of nonplanar graphs. However, some additional definitions need to be given before this can be shown.

An elementary subdivision of a nonempty graph \( G \) is a graph \( H \) obtained by removing an edge \( e = uv \) from \( G \) and adding a vertex \( w \) and the edges \( uw \) and \( vw \). A subdivision of \( G \) is a graph obtained from \( G \) by a succession of elementary subdivisions. A graph \( H \) is homeomorphic from \( G \) if \( H = G \) or \( H \) is a subdivision of \( G \). \( G_1 \) is homeomorphic with \( G_2 \) if there exists a graph \( G_3 \) such that both \( G_1 \) and \( G_2 \) are homeomorphic from \( G_3 \). In Figure 19, \( G_1 \) and \( G_2 \) are homeomorphic with each other since they are both homeomorphic from \( G_3 \). (Note also that \( G_2 \) is homeomorphic from \( G_1 \).)

![Figure 19. Homeomorphic graphs.](image)

**Problem 12.** Find examples of graphs \( G_1, G_2, \) and \( G_3 \) such that \( G_1 \) is homeomorphic from \( G_3 \) and \( G_2 \) is homeomorphic from \( G_3 \) but neither \( G_1 \) nor \( G_2 \) is homeomorphic from each other.

A concept related to homeomorphism is that of contraction. An elementary contraction of a graph \( G \) is obtained by replacing two adjacent vertices \( u,v \) by a new vertex \( w \) in such a fashion that all adjacencies of \( G \) are preserved. Vertices adjacent to \( u \) or \( v \) in \( G \) will then be adjacent to \( w \). A contraction of \( G \) is a composition of finitely many elementary contractions. Figure 20 illustrates this
with $G_2$ being a contraction of $G_1$. The contraction is obtained by the mapping indicated. A subcontraction of a graph is an edge-induced subgraph (a subgraph obtained by starting with a subset of $E(G)$ and including only these edges and the vertices incident with one of these edges) of a contraction of $G$. A relation between contraction and homeomorphism is given by the following theorem:

**Theorem 7.** If a graph $H$ is homeomorphic from a graph $G$, then $G$ is a contraction of $H$.

![Figure 20. Contraction of a graph.](image)

We are now in a position to show the role played by $K_5$ and $K_{(3,3)}$ in the study of planar graphs.

**Theorem 8.** (Kuratowski). A graph is planar if and only if it contains no subgraph homeomorphic with $K_5$ or $K_{(3,3)}$.

**Theorem 9.** A graph is planar if and only if it contains neither $K_5$ nor $K_{(3,3)}$ as a subcontraction.
A relatively famous graph, the Petersen graph, has resulted from the study of nonplanar graphs. It is illustrated in Figure 21. This graph is nonplanar since it has a subgraph homeomorphic with $K_{(3,3)}$ and it contains $K_5$ as a contraction. However, it does not have subgraph homeomorphic with $K_5$.

**Problem 13.** Find a contraction of the Petersen graph to $K_3$.

![Figure 21. The Petersen graph.](image)

The last graph theory concept to be discussed here is that of map coloring. One of the classic problems of mathematics is the Four Color Conjecture. Basically, this claims that a map can be colored by at most four colors in such a fashion that regions sharing a common boundary (edge, not point) will be different colors. A guided discovery approach to this problem is contained in [3,p.215]. Map coloring has a graph theory interpretation in that regions of a map can be associated with vertices of a graph $G$ and vertices of $G$ are adjacent if and only if the associated regions

![Figure 22. A map and its associated graph.](image)
The Pentagon

share a common boundary. Figure 22 shows a map and the associated graph.

Analogous to map coloring, we now define a coloring of a graph $G$ to be an assignment of colors to vertices of $G$ so that adjacent vertices are assigned different colors. If $n$ colors are used, we say $G$ is $n$-colored. The Four Color Conjecture is then stated as: every planar graph is 4-colorable. (It has been proven that every planar graph is 5-colorable. In particular, an article by K. Shockley [6,pp.3-9] contained such a proof (attributed to P. J. Heawood, 1890)).

There are several equivalent graph theory statements to the Four Color Problem. Before stating these, two more definitions are needed: (1) a block is a graph ($p \geq 2$) with no cut-vertices; and (2) a block of a graph $G$ is a subgraph of $G$ which is itself a block and which is maximal in this respect. We can now state the following equivalents to the Four Color Conjecture.

**Theorem 10.** The Four Color Conjecture is true if and only if every planar block is 4-colorable.

**Theorem 11.** The Four Color Conjecture is true if and only if every cubic plane block is 3-“edge” colorable.

**Theorem 12.** The Four Color Conjecture is true if and only if every bridgeless plane map is 4-colorable.

![Figure 23. A nonhamiltonian, cubic, planar, bridgeless graph.](image-url)
At one point, Tait \[7, p. 729\] proposed a "proof" of an equivalent to the Four Color Conjecture. However, he assumed that every cubic, planar, bridgeless graph is hamiltonian. Several years later, Tutte \[8, pp. 98-101\] illustrated the fallacy of Tait's assumption by producing a counterexample. This is illustrated in Figure 23.

Just a few of the basic concepts of graph theory have been discussed here. It is hoped the interested reader will pursue his study either independently or with some group of students and faculty. If the interested reader wishes to pursue his study, I would suggest *Introduction to the Theory of Graphs* by M. Behzad and G. Chartrand (Allyn & Bacon) or *Graph Theory* by F. Harary for the theoretically inclined student; and *Graphs and Their Uses* by O. Ore (Singer), *Puzzles & Graphs* by J. N. Fugii (NCTM), or *Graphs as Mathematical Models* by M. Behzad and G. Chartrand (Prindle, Weber, and Schmidt) for the application oriented student.

**REFERENCES**


Fermat Numbers and Polygonal Numbers

ROBERT W. PRIELIPP

Faculty, University of Wisconsin—Oshkosh

Fermat numbers are numbers of the form $F_k = 2^{2^k} + 1$, $k = 0, 1, 2, \cdots$. Thus $F_0 = 3$, $F_1 = 5$, $F_2 = 17$, $F_3 = 257$, $F_4 = 65,537$, $F_5 = 641 \cdot 6,700,417$, and $F_6 = 274,177 \cdot 67,280,421,310,721$. They are named after Pierre de Fermat (1601-1665) who is famous for his work in number theory. Fermat believed that each $F_k$ was a prime number. Although $F_0$, $F_1$, $F_2$, $F_3$, and $F_4$ are all primes, $F_5$ is composite since, as is indicated above, it is divisible by 641. Up to the present no prime Fermat numbers beyond the first five ($F_0$ through $F_4$) have been found.

In a rather unexpected way Fermat primes are related to regular polygons. Karl Friedrich Gauss (1777-1855), perhaps the greatest mathematician of all time, proved that the only regular polygons, which are constructible by ruler and compass according to the Euclidean restrictions, are those whose number of sides is $2^w$ times a product of distinct Fermat primes, where $w$ is a non-negative integer.

Polygonal numbers have intrigued mathematicians, particularly those interested in number theory, since at least the time of the ancient Greeks. Formulas for the six kinds of polygonal numbers that we will consider in this paper are given in Table 1.

<table>
<thead>
<tr>
<th>Formulas for Polygonal Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangular $\frac{1}{2}n(n + 1)$</td>
</tr>
<tr>
<td>Square $n^2$</td>
</tr>
<tr>
<td>Pentagonal $\frac{1}{2}n(3n - 1)$</td>
</tr>
<tr>
<td>Hexagonal $n(2n - 1)$</td>
</tr>
<tr>
<td>Heptagonal $\frac{1}{2}n(5n - 3)$</td>
</tr>
<tr>
<td>Octagonal $n(3n - 2)$</td>
</tr>
</tbody>
</table>

The general formula for an $r$-sided polygonal number is

$$\frac{1}{2}n[(r - 2)n - r + 4].$$

For more information on polygonal numbers (including a table of the first 120 $r$-sided polygonal numbers for $r = 3, 4, 5, \cdots, 10$), see pp. 126-129 of Number Theory Tables compiled by Brother
Our goal is to answer the question “Which r-sided polygonal numbers (r = 3, 4, 5, • • • , 8) are also Fermat numbers?” We shall do this in a series of six theorems.

Theorem 1. The only triangular number which is also a Fermat number is 3.

Proof: Let \( \frac{1}{2}n(n + 1) = 2^{2^k} + 1 \). Then \( n(n + 1) = 2^{2^k+1} + 2 \) so \((n + 2)(n - 1) = n^2 + n - 2 = 2^{2^k+1}\). But \( n + 2 \) and \( n - 1 \) are not both even. Thus \( n - 1 = 1 \). Therefore \( n = 2 \) and \( k = 0 \), which yields the stated result.

Theorem 2. There are no square numbers which are also Fermat numbers.

Proof: Suppose \( n^2 = 2^{2^k} + 1 \). Then \((n + 1)(n - 1) = n^2 - 1 = 2^k \). Hence \( n + 1 = 2^m \) and \( n - 1 = 2^t \). Thus \( n = 3 \) because the only powers of 2 which differ by 2 are 4 and 2. But then \( 2^k = 3 \), which is impossible.

Theorem 3. The only pentagonal number which is also a Fermat number is 5.
Proof: Let \( \frac{1}{2}n(3n - 1) = 2^k + 1 \). Then \( n(3n - 1) = 2^{k+1} + 2 \) so \((3n + 2)(n - 1) = 3n^2 - n - 2 = 2^{k+1} \). But \(3n + 2\) and \(n - 1\) are not both even. Thus \(n - 1 = 1\). Therefore \(n = 2\) and \(k = 1\), which yields the stated result.

Theorem 4. There are no hexagonal numbers which are also Fermat numbers.

Proof: Since \( n(2n - 1) = (2n - 1)(2n)/2 \) every hexagonal number is a triangular number. However, 3 is not a hexagonal number. Hence the stated result follows from Theorem 1.

Theorem 5. There are no heptagonal numbers which are also Fermat numbers.

Proof: Suppose \( \frac{1}{2}n(5n - 3) = 2^k + 1 \). Then \( n(5n - 3) = 2^{k+1} + 2 \) so \((5n + 2)(n - 1) = 5n^2 - 3n - 2 = 2^{k+1} \). But \(5n + 2\) and \(n - 1\) are not both even. Thus \(n - 1 = 1\) so \(n = 2\). But this makes \(2^k = 6\), which is impossible.

Theorem 6. There are no octagonal numbers which are also Fermat numbers.

Proof: Suppose \( n(3n - 2) = 2^k + 1 \). Then \((3n + 1)(n - 1) = 3n^2 - 2n - 1 = 2^k \). Clearly \(n - 1 \neq 1\) since 7 is not a power of 2. Hence \(3n + 1 = 2^m\) and \(n - 1 = 2^j\) where \(m\) and \(j\) are both positive integers. Multiplying both sides of the second equation by 3 and subtracting this new equation from the first equation, we have \(4 = 2^m - 3 \cdot 2^j = 2^j(2^m-j - 3)\) so \(2^j = 4\) and \(2^m-j - 3 = 1\). It follows that \(j = 2\) and \(m = 4\). Thus \(n = 5\). But this makes \(2^k = 6\), which is impossible.
Today the United States is the last major nation not using the Metric System as its system of measurement. On 23 December 1975, President Ford signed into law The Metric Conversion Act which provides for the voluntary metrification of the United States within the next few years. The United States, unlike most non-metric nations at the time of conversion, has not as yet set a mandatory metrification date. One reason for the "rough time" the metrification of the United States is currently having is that many believe that the Metric System is a fad and will eventually become unpopular like the "new math" of the last decade.

To better understand the history of the Metric System, one should be acquainted with the history of measurement.

Important dates are as follows:

Cave-man—Judged distances by eye or time; matched objects with trees, stones, or mountains; used body for more accurate measuring; and had no standard measurements.

6000 B.C.—Units of measurements were defined as the cubit (the prime measurement): the bent forearm from the point of the elbow to the tip of the middle finger of the outstretched hand (roughly 18 inches); the span: the length between the tips of the thumb and little finger of the outstretched hand (about 1/2 a cubit); the digit: the breadth at middle of middle finger (1/24 cubit); the foot: 2/3 cubit, 4 palms, or 15 digits.

4000 B.C.—Units of measurement were: the meridian mile: 4000 cubits or 1000 Egyptian fathoms; the fathom: length of the outstretched arms (about 6 feet); and the reed (forerunner to the rod): double fathom, 8 cubits, 12 feet, or 16 spans.
The Chaldeans—Established the circle as $360^\circ$, set the year as 6 months of 60 days each, plus 5 days at the end.

Alexander the Great—Carried ancient measurements to Greece and Rome.

The Greeks and Romans—Used as units of measurement, the stadia: 1/10 of a meridian mile; the schoinos: 1/10 of a degree; the parasang: 4 miles; the mille (mile): 1000 paces or double steps, 4/5 of a Meridian mile; and the inch: thumb-breath, 1/12 of a foot.

9th Century—The art of measuring the earth and its shape influenced measurements which were based on astrology and the meridian of the locale.

The Dark Ages—The return to natural and human measurements.

12th Century—The units of measurement were: the English foot: measure of a cubical vessel containing 1000 Roman ounces of water; and the yard: distance from the tip of King Henry I's nose to the end of his thumb.

1150 A.D.—The inch was defined as the mean measure of the thumbs of 3 men.

1324 A.D.—The inch was defined as 3 barleycorns taken from the middle of the ear and placed end to end.

1500 A.D.—Establishment of the English Mile. Units of measure were:

- 3 barleycorns = 1 inch
- 12 inches = 1 foot
- 3 feet = 1 yard
- 9 inches = 1 span
- 5 spans = 1 ell (original cloth measure)
- 5 feet = 1 pace
- 125 paces = 1 furlong
- 5 1/2 yards = 1 rod
- 40 rods = 1 furlong
- 8 furlongs = 1 English mile
- 12 furlongs = 1 league

1670 A.D.—Gabriel Mounton, the Vicar of St. Paul's Church in
Lyons, Franch, proposed a comprehensive decimal system of weights and measures that was based on a unit from the physical universe instead of the human body.

1700 A.D.—The poppyseed became a means of precise measure with the units:

\[
\begin{align*}
1 \text{ inch} & = 3 \text{ barleycorns} \\
1 \text{ barleycorn} & = 4 \text{ poppyseeds} \\
1 \text{ poppyseed} & = 4 \text{ human hairs} \\
\text{hand (used to measure horses)} & = 4 \text{ inches} \\
\text{shaftment: the fist with thumb protruding} & = 6 \text{ inches}
\end{align*}
\]

French Revolution—King Louis XVI was unifying the French measuring system when he lost his head.

1786 A.D.—The United States adopted a complete decimal system of coinage.

1790 A.D.—Thomas Jefferson submitted a report on weights and measures to Congress. A basic standard, derived from the motion of the earth on its axis was proposed to establish a decimal system of weights and measures. In France a decree led to the development of the metric system.

1793 A.D.—All scientific academies were closed and the principal member of the Metric Committee was sent to the guillotine.

1795 A.D.—In France a decree was issued officially adopting the metric system. Copies of the provisional standards were sent to several countries, including the United States.

1799 A.D.—The first federal weights and measures law was enacted in the United States.

1800 A.D.—Napoleon permitted the French to return to the old system of measurement which was standardized.

1812 A.D.—Napoleon Bonaparte, by decree, temporarily suspended the compulsory provisions of the 1795 metric system law.

1821 A.D.—Secretary of State John Quincy Adams submitted an exhaustive report on the subject of weights and measures to Congress in response to a resolution passed by the Senate in 1817. In this report Adams recommended retention of the
English customary system by the United States, but he proposed a program for achieving greater uniformity among the States.

1832 A.D.—By administrative action, the Secretary of the Treasury declared the yard, the avoirdupois pound, and the Winchester bushel to be the United States official system of weights and measures.

1837 A.D.—The metric system was made compulsory again in France after a 37 year absence.

1840 A.D.—The Netherlands, Italy, and Greece went metric.

1849 A.D.—Spain adopted the metric system.

1859 A.D.—Italy adopted the Metric System.

1866 A.D.—The use of the metric system was made legal in the United States by an Act of Congress.

1868 A.D.—Portugal adopted the Metric System. Bismark adopted the meter and made it compulsory in all of Germany in 1872.

1873 A.D.—The American Metrological Society was organized in New York for the purpose of improving existing systems of weights, measures, and moneys.

1875 A.D.—The Treaty of the Meter was signed in Paris by seventeen nations, one of which was the United States.

1880 A.D.—Most of Europe and South America converted to the Metric System.

1893 A.D.—The United States Superintendent of Weights and Measures issued a bulletin announcing that the United States prototype meter and kilogram would henceforth be considered the nation's "fundamental standards of length and mass".

1894 A.D.—The United States Congress passed a law defining and establishing units for electrical measurement. These units were based on the metric system.

1895 A.D.—A resolution establishing a commission to study and report on the feasibility of metric adoption was passed by
the United States House of Representatives. By mistake, the resolution was recorded as requiring the concurrence of the Senate in order to be put into effect. Consequently, the commission was never formally organized.

1897 A.D.—Legislation was enacted by Great Britain permitting full use of the metric system for measurement.

1907 A.D.—Following a refusal by the Committee on Coinage, Weights, and Measures to report favorably on the metric bill, intense promotional efforts died down until the outbreak of World War I.

1918 A.D.—General Order No. 1 was issued by the United States War Department provided for the usage of the metric system for wartime activities.

1937 A.D.—A bill to fix the standards according to the metric system was considered and recommended by the Committee on Coinage, Weights, and Measures. The bill was never enacted.

1948 A.D.—The “Twentieth Yearbook of the National Council of Teachers of Mathematics” was devoted solely to a discussion of the need for and advantages of using the metric system, especially for educational purposes.

1957 A.D.—The U.S. Army issued a regulation establishing metric linear units as the basis for weapons and related equipment.

1960 A.D.—At the 11th General Conference on Weights and Measures a new international standard of length, based on the wavelength of the element krypton was adopted in place of the original “metric bar.” At the same conference, the modernized metric system was officially renamed the Système International d’ Unités—The International System of Units.

1965 A.D.—Britain started its 10-year compulsory metrication program.

1968 A.D.—A study providing for a three-year program to determine the impact of increasing use of the metric system in the United States was passed by Congress and signed into law by President Lyndon B. Johnson.
The Pentagon

1970 A.D.—Canada issued a statement which said that metrication is a definite objective of Canadian policy.

1972 A.D.—The U.S. Senate unanimously passed the “Metric Conversion Act of 1972”.

1974 A.D.—Canada began compulsory metrication.

1974 A.D.—The U.S. House of Representatives in a 240 to 153 vote defeated a motion to suspend the rules to consider metric conversion legislation (H.R. 11035) without any amendments being attached.


With the above history of measurement at hand, is the metrication of the United States inevitable?

(Continued from page 60)

and “A Modification of Integration by Parts” by Bill McKee, MO Zeta, University of Missouri, Rolla. Two papers were listed as alternates: “Magic Squares” by Sam Miller, MO Iota, Missouri Southern State College, and “Are the Black Jack Games in Las Vegas Fair?” by Linda O’Neal and Karen Lefler, MO Alpha, Southwest Missouri State University.

The main address was delivered by Barbara Currier, KS Delta member during her undergraduate study and currently a Ph.D. candidate at the University of Missouri in Kansas City and member of the mathematics faculty at Rockhurst College in Kansas City, Missouri. Her illustrated lecture centered on “The Theory of Braids”.

There were 16 of the 24 chapters in Region IV represented at the meeting with the attendance totaling 122. Harold L. Thomas, Director of Region IV, and Eddie Robinson, National Treasurer, were present at the meeting with representatives from the following chapters: IL Eta, IA Alpha, IA Beta, IA Gamma, KS Alpha, KS Beta, KS Gamma, KS Delta, KS Epsilon, MO Alpha, MO Gamma, MO Zeta, MO Eta, MO Theta, MO Iota, and NE Gamma.
The Problem Corner

Edited by Kenneth M. Wilke

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before 1 September 1977. The best solutions submitted by students will be published in the Spring 1978 issue of The Pentagon, with credit being for other solutions received. To obtain credit, a solver should affirm that he is a student and give the name of his school. Address all communications to Kenneth M. Wilke, Department of Mathematics, 275 Morgan Hall, Washburn University, Topeka, Kansas 66621.

PROPOSED PROBLEMS

287. Proposed by Randall J. Covill, West Newbury, Massachusetts.

Solve the following alphametic in which each letter represents a unique digit in base 14 and $E \neq 0$.

$$
S U B T E N D
+ A D D E N D
\overline{A N S W E R S}
$$

288. Proposed by Charles Trigg, San Diego, California.

Factor $6x^5 - 15x^4 + 20x^3 - 15x^2 + 6x - 1$

289. Proposed by Charles Trigg, San Diego, California.

Each of the three consecutive integers 4, 5, and 6 terminates its own cube. That is $4^3 = 64$, $5^3 = 125$, and $6^3 = 216$. Find four pairs of larger consecutive integers in which each integer terminates its own cube.

289. Proposed by Charles Trigg, San Diego, California.

"Come on in, Bob," said Dan, "only small stakes tonight."
"That's good," replied Bob, "I haven't quite three dollars in nickels, dimes, and quarters." "I haven't any pennies either," said Dan, "but I have the same number of coins that you have. That includes twice as many dimes as you have." "Correct," replied Bob, "but my number of nickels is twice yours. It also equals the number of all our quarters combined. The
The Pentagon

total value of your change is the same as mine." "O.K., let's go," said Dan, "it appears that my lucky half-dollar is the largest coin on the table."

How many coins of each type did Bob and Dan have?

290. Proposed by Leigh James, Rocky Hill, Connecticut.

Prove that the quadrilateral having sides $a$, $b$, $c$, and $d$ has maximum area when the quadrilateral is cyclic; i.e., the sum of opposite interior angles is $180^\circ$.

SOLUTIONS

277. Proposed by the editor.

Consider the fraction $\frac{a^3 + 2a}{a^2 + 3a^2 + 1}$ where $a$ is a positive integer. Show that the fraction is reduced to lowest terms for any permissible choice of $a$.

Solution by John Kitchin, College of Science and Technology at the University of Southern Mississippi, Hattiesburg, Mississippi.

To prove that the above fraction is reduced to lowest terms it suffices to show that, for all prime numbers $p$, $p$ divides $a^3 + 2a$ implies $p$ does not divide $a^4 + 3a^2 + 1$.

Suppose $p$ divides $a^3 + 2a$. Then $p$ divides $a$ or $p$ divides $a^2 + 2$. Consider both cases:

1. If $p$ divides $a$, then $p$ divides $a^2(a^2 + 3)$. Consequently, $p$ does not divide $a^2(a^2 + 3) + 1 = a^4 + 3a^2 + 1$.

2. If $p$ divides $a^2 + 2$, then $p$ divides $a^2(a^2 + 2)$ and $p$ does not divide $a^2 + 1$. Consequently, $p$ does not divide $a^2(a^2 + 2) + (a^2 + 1) = a^4 + 3a^2 + 1$. It follows that for all prime numbers $p$, $p$ divides $a^3 + 2a$ implies $p$ does not divide $a^4 + 3a^2 + 1$. Therefore, if $a$ is an integer, the above fraction is reduced to lowest terms.

Also solved by Keith Hafen, University of New Mexico, Albuquerque, New Mexico; Fred L. Harrison Jr., Southwest Missouri State University, Springfield, Missouri; Charles W. Trigg, San Diego, California; and Tim Swindle, University of Evansville, Evansville, Indiana.
Editor's comment: Another solution can be based on the fact that if the fraction \( \frac{R}{S} \) is reduced to lowest terms, then \( \frac{S}{R} \) is also. Thus the given fraction is reduced to lowest terms when \( \frac{a^4 + 3a^2 + 1}{a^3 + 2a} = a + \frac{a^2 + 1}{a^3 + 2a} \); i.e., when \( \frac{a^2 + 1}{a^3 + 2a} \) is reduced to lowest terms. Repeating the process three more times the original fraction is reduced to lowest terms whenever \( \frac{1}{a} \) is reduced to lowest terms. This is always true. Note the similarity of this process to Euclid's Algorithm.

278. Proposed by the editor.

Dr. Knowitall noticed that his clock had stopped. So he wound it, noted the time to be 6:00 p.m. and went to a friend's house to play chess. He arrived at 8:30, according to the clock in the friend's house. Dr. Knowitall left at 11:00. When he arrived home, the time, according to his clock, was 12:30 a.m. He reset his clock to the correct time. Assuming that Dr. Knowitall walked at the same rate in both directions and assuming that his friend's clock kept perfect time, what was the correct time when Dr. Knowitall reset his clock?


6:00 p.m.—he left, his clock
8:30 p.m.—arrived, (friend's clock) perfect time
11:00 p.m.—he left, (friend's clock) perfect time
12:30 a.m.—arrived, his clock

\( x = \) lack of time

6:00 + \( x \) and 12:30 + \( x \) respectively represent the correct time when the Dr. left his home and returned.

8:30—(6:00 + \( x \)) = 12:30 + \( x \) - 11:00
2.5 hrs. - \( x \) = 1.5 hrs + \( x \)
1 hr. = 2\( x \)

\( x = .5 \) hrs. = 30 mins.
The Pentagon

Time when Dr. Knowitall reset his clock was:
12:30 a.m. + x
12:30 + 30 mins. = 1:00 a.m. QED

Also solved by Fred L. Harrison Jr., Southwest Missouri State University, Springfield, Missouri; Al Maurice, Eastern Illinois University, Chicago, Illinois; Charles W. Trigg, San Diego, California; and Tim Swindle, University of Evansville, Evansville, Indiana.

279. Proposed by the editor.

In Moldavia, people pay an income tax equal to a percentage of the weekly wage equal to the number of ducats earned each week; e.g., on a weekly wage of 10 ducats, the rate is 10 percent. Assuming the maximum salary is 100 ducats per week, what is the optimum salary in Moldavia?

Solved by Fred L. Harrison Jr., Southwest Missouri State University, Springfield, Missouri.

All salaries in Moldavia may be calculated by the formula:

\[ S = X - \frac{X^2}{100} \]

\( S = \) salary
\( X = \) number of ducats earned

By taking the first and second derivatives, we find:

\[ D(S) = -\frac{2X}{100} + 1 \]

\[ D^2(S) = -\frac{2}{100} \]

Since the second derivative of the equation is negative, it is known that the maximum value of \( X \) occurs where \( D(S) = 0 \). Therefore, if \( D(S) = 0 \), then \( X = 50 \). The optimum salary in Moldavia is 50 ducats.

Also solved by Randall J. Covill, West Newbury, Massachusetts; Al Maurice, Eastern Illinois University, Chicago, Illinois; Charles W. Trigg, San Diego, California; Daniel G. Selby, Indiana University of Pennsylvania, Indiana, Pennsylvania; and Tim Swindle, University of Evansville, Evansville, Indiana.
280. Proposed by the editor.

Solve the cryptarithmetic \( THAT = (AH) (HA) \) where each letter represents a unique digit in the decimal system.

Solution by Charles W. Trigg, San Diego, California.

\[
\begin{align*}
\frac{THAT}{HA} &= AH, \\
\frac{T(00)T}{HA} &= AH - 10. \\
T(713)/(HA) &= AH - 10
\end{align*}
\]

so \( HA = 78, T = 6, \) and \( 6786 = (78)(87). \)

Also solved by Al Maurice, Eastern Illinois University, Chicago, Illinois; and Tim Swindle, University of Evansville, Evansville, Indiana.

281. Proposed by the editor.

An algebra student encountered the following problem on an exam: Evaluate \( \frac{\log A}{\log B} \). Being pressed for time, he cancelled common factors from both numerator and denominator (including the common factor \( \log \)) to obtain the correct answer.

\[
\frac{\log A}{\log B} = \frac{A}{B} = \frac{3}{4} . \text{ What are } A \text{ and } B? 
\]

Solution by Terri O'Dell, Missouri Southern State College.

\[
\frac{\log A}{\log B} = \frac{3}{4} . \text{ Then } \frac{A}{B} = \frac{3}{4} \text{ or } B = \frac{4A}{3} \text{ or } 4 \log A = 3 \log B.
\]

Removing "logs" and substituting we have \( A^4 = B^3 = \left(\frac{4A}{3}\right)^3 = \frac{64A^3}{27} \).

Thus \( A = \frac{64}{27} \) and \( B = \frac{4A}{3} = \frac{256}{81} \).

Also solved by Fred L. Harrison Jr., Southwest Missouri State University, Springfield, Missouri; Al Maurice, Eastern Illinois University, Chicago, Illinois; Charles W. Trigg, San Diego, California; and Tim Swindle, University of Evansville, Evansville, Indiana.
The Book Shelf

Edited by O. Oscar Beck

This department of The Pentagon brings to the attention of its readers recently published books (textbooks and tradebooks) which are of interest to students and teachers of mathematics. Books to be reviewed should be sent to Dr. O. Oscar Beck, Department of Mathematics, University of North Alabama, Florence, Alabama 35630.


It should first be noted that these books are calculus texts and that almost all references to business, biology, social science, and economics are to be found in the exercises. Of the applied problems, more than ninety percent of those in the first book, Calculus, pertain to business. There are very few biology or other social science applications. In both texts, ninety-five percent of the problems are typical of those in traditional calculus texts.

In the authors' words, both these books present calculus using an "intuitive approach" as compared to the rigorous approach to be found in "engineering calculus" texts. "Intuitive" means that the various concepts are presented more by illustration than by theory. The theorem-proof style is replaced with the example-solution technique. There are a large number of examples worked in each section.

Calculus is meant for students with a precalculus background, as it has very little review of this material. Concepts begins as a more elementary level, designed to students having only two years of high school algebra. Calculus covers one variable differential and integral calculus, an introduction to functions of several variables, and series and sequences. Concepts goes through one variable differential and integral calculus, but has an optional computer feature containing a chapter introducing BASIC. Computer related calculus exercises are spread throughout this text. Both books present the
use of tables for evaluating integrals and have sections on differential equations. An interesting and novel feature of *Calculus* is a section on probability density functions as an application of integrals.

Those teachers who have been ingrained with the traditional calculus books will find these books represent a radical departure. For instance, continuity and the Mean Value Theorem are mentioned, but never used in *Calculus*; neither is mentioned in *Concepts*. Other theoretical aspects of calculus are treated more or less similarly. While this approach is fine for teaching the mechanics of calculus, it will probably handicap those students who wish to continue on to other math courses. For this reason, it seems that these books are most appropriate for terminal math courses.

J. Myron Hood
Occidental College


This refreshingly small book is stated by the authors to be "A Short Course with Applications to Business, Economics, and the Social Sciences". While such a description might imply a shortage of physical applications, such is not exactly the case. Actually, the text may well be used in almost any terminal course of three, four, or five semester hours where a study of trigonometric calculus is not desired. It is, of course, especially written for students majoring in the above mentioned areas, so its use in engineering and physical science classes would be, at best, provisionally sound.

The authors pre-suppose only high school algebra. This book is so well written that a mathematically able college freshman with algebra through quadratic equations and a fair amount of experience in graphing can easily follow the material as presented or as edited by particular class requirements. An excellent first chapter called *Preliminary Concepts* reviews early algebra and at the same time brings into focus the introductory material necessary for what follows.

The remaining chapters are entitled, in order, *Derivatives and the Concept of Limit*, *Applications of Differentiation*, *Integration*, *Calculus of Several Variables*, and *Exponentials, Logarithms, and*
Growth Problems. Several tables are included at the end of the book. Trigonometry is sparse, perhaps too much so.

The entire text is easily read, with pleasing format. Many excellent exercises are included, some of a different nature than those usually appearing in such a textbook. While this reviewer feels that it is almost impossible to correctly judge the merits of a textbook without actually teaching with it, one should feel safe in adopting this one on appearance alone. It could be used to advantage in advanced high school classes as well as in early college courses.

D. H. Erkiletian, Jr.
University of Missouri—Rolla

Basic Mathematics for Engineers and Technologists, Alan Jeffrey,

This text was written to satisfy the mathematical needs of first year engineering students. The content is mostly calculus and differential equations, but also included are complex numbers and vectors, matrices, and an introduction to probability and statistics. The book is well written and should be very useful to the students the author has in mind.

There are some features of the text which are not good in a textbook for a course, however. The concept of a derivative is introduced in Chapter 5 after a long discussion of functions, complex numbers, limits and continuity, and even vectors. An earlier introduction would be preferable. The level of rigor varies in an inconsistent fashion. Functions are not defined precisely, but limits are treated rigorously and completely including $\varepsilon$-$\delta$ proofs of theorems.

In defining definite integrals a very careful treatment of partitions and upper and lower sums is followed by defining the definite integral as a limit of sums. After all of the careful preparation one would expect the author to follow with the usual precise definition of a Riemann integral. There are many other similar unexpected variations in level. For engineering students much of the fine points of theory could be eliminated and should be.

Problem sets are good with most answers given; and over-all, the good features of this book outweigh the faults for student users.
I would recommend the text to engineering or other science students as a useful reference book at least.

Ben F. Plybon
Miami University (Ohio)


This is not a mathematics book at all, but it is a book which uses mathematics. Very often we teachers and students of mathematics lose sight of the very rich variety of applications of our subject. Even a casual reading of this book (if indeed it is possible to read casually such a thorough study of information processing as this), will reveal these topics of mathematics applied to solutions of real world problems: combinatorics, graph theory, finite geometries, finite fields, and statistics. This is not an easy book to read but is well worth the effort you expend on it.

The undergraduate student not acquainted with computer science will have rough going in this book. The teacher seeking a textbook for an undergraduate course on this topic should have an adequate supply of problems to give his class in conjunction with the text. As a reference text for self study by the professional in this area I believe the book would be eminently satisfactory. The references are quite extensive.

James E. McKenna
State University College, Fredonia

(Concluded from page 64)
Kappa Mu Epsilon News

EDITED BY SISTER JO ANN FELLIN. Historian

News of Chapter activities and other noteworthy KME events should be sent to Sister Jo Ann Fellin, Historian, Kappa Mu Epsilon, Benedictine College, North Campus Box 43, Atchison, Kansas 66002.

CHAPTER NEWS

Alabama Beta, University of North Alabama, Florence

38 actives, 19 pledges

Officers serving during 1976-77: Jim Diehl, president; Judy Thorne, vice-president; Patti Coggins, secretary and treasurer; Jean Parker, corresponding secretary; Oscar Beck, faculty sponsor.

Dr. Peter Casazza from the University of Alabama in Huntsville dealt with false reasoning and paradoxes of the infinite in his interesting talk entitled “Paradox Lost”. Initiation of 19 new members took place at the spring banquet. The speaker for the evening was a former University of North Alabama student, Harold Jeffreys. Mr. Jeffreys, a physicist from NASA, spoke on the Laser Doppler System with which he is quite experienced. Ten members were initiated into Phi Kappa Phi this year and eighteen members were honored at Recognition Day with special awards for scholarship and service. Outgoing chapter president, Anthony Eckl, was selected as “man of the year”.

Alabama Gamma, University of Montevallo, Montevallo

15 actives, 5 pledges

Officers serving during 1976-77: Sandra Kay Hayes, president; Allen Smith, vice-president; Connie Flouronoy, secretary; Linda Stevens, treasurer; Angela Hernandez, corresponding secretary; William Foreman, faculty sponsor.

The chapter provided a “problem of the month” bulletin board. The student (non-KME member) with the best solution was presented a medal. At the Honors' Day Program the chapter presented a mathematical handbook to the outstanding freshman mathematics student. The chapter sponsored its annual spring cookout for the
mathematics faculty and all students with majors or minors in mathematics. The senior mathematics majors were honored with a dinner in late spring as well.

**California Gamma, California Polytechnic State University, San Luis Obispo**

46 actives, 20 pledges

Officers serving during 1976-77: Robert Watanabe, president; Inge Liem, vice-president; Sharon Kuge, secretary; Jack Terra, treasurer; George R. Mach, corresponding secretary; Adelaide T. Harmon, faculty sponsor.

The chapter joined the school district in sponsoring a Junior High Math Field day for about 250 students. KME members made demonstrations and displays for the mathematics department portion of Poly Royal (annual open house) and assisted with the annual mathematics contest for about 500 high school students. The speaker for California Gamma's spring banquet was Dr. Morris Marden, visiting distinguished professor.

**California Delta, California State Polytechnic University, Pomona**

17 members

Officers serving during 1976-77: Robert Beauchamp, president; Robert Bates, vice-president; Jan Aldridge, secretary and treasurer; Samuel Gendelman, corresponding secretary; Joseph Kachun, faculty sponsor.

California Delta conducted free tutoring sessions for students enrolled in any mathematics course on campus. During the spring quarter the chapter assumed the responsibility of preparing and staffing the mathematics department exhibit for the annual open house of the University. This event, called Polyvue, ran for three days in April. On 27 March the chapter held its annual picnic.

**Colorado Alpha, Colorado State University, Fort Collins**

10 actives, 9 pledges

Officers serving during 1976-77: Jane Darling, president; Doris Hammer, vice-president; Geri Grange, secretary and treasurer; Duane Clow, corresponding secretary; Ben Manuel, faculty sponsor.
The chapter's annual snow-shoeing trip held at Bear Lake in Rocky Mountain National Park was again a fun day for members. To top it off the faculty sponsor provided a chili supper. Other events included the annual spring picnic and a pot luck supper for the new initiates of the chapter.

**Colorado Beta, Colorado School of Mines, Golden**

20 actives

Officers serving during 1976-77 are elected in the fall. Officers for 1975-76 were: John Turner, president; Kendrick Killian, vice-president; Mark McCuen, secretary; Leonard Witkowski, treasurer; Ardel J. Boes, corresponding secretary; Donald W. Fausett, faculty sponsor.

A graph theory solution to "instant insanity" was presented by Dr. Strawther. Dr. Woolscy spoke at one of the meetings on survival in the real world. Three problems were posed in the campus newspaper, *The Oredigger*. Students winning the prizes were Gary Harms and Joe Smith. For Engineers' Day, 3 April, a computer terminal was set up to play roulette in a historic west atmosphere. On 21 April initiation was held for 42 students.

**Illinois Alpha, Illinois State University, Normal**

35 actives, 10 pledges

Officers serving during 1976-77: Lynne McKinty, president; Colleen Kirby, vice-president; Sue Parus, secretary; Scott Erwin, treasurer; Robert Ritt, corresponding secretary; Orlyn Edge, faculty sponsor.

The tutoring program provided a total of 350 hours of individual tutoring for the past academic year. Some topics considered at the monthly meetings were curriculum development and college entrance exam scores. The chapter sponsored a bake sale for a money making project. About 50 members and faculty attended the annual picnic on 2 May.

**Illinois Zeta, Rosary College, River Forest**

19 actives

Officers serving during 1976-77: Ann Stangarone, president; Donna Pawel, vice-president; Mary Rose Grano, secretary;
Thomas LeKostaj, treasurer; Mordechai Goodman, corresponding secretary and faculty sponsor.

Tutoring services were available to Rosary students. Final approval of a revised, up-dated constitution passed. On 8 March the chapter subsidized Rosary students and faculty who attended a meeting of the Associated Colleges of the Chicagoland Area held at Wheaton College. On 14 March three new members were inducted. Guest speakers for the ceremony were Richard Helmke and Dennis Witte of Concordia Teachers College. A party followed the installation of new officers on 13 May.

Indiana Alpha, Manchester College, North Manchester

12 actives, 16 pledges

Officers serving during 1976-77: Stanley Willmer, president; Richard Herbst, vice-president; Sandy Mohn, secretary; Sally Gradeless, treasurer; Ralph McBride, corresponding secretary.

Initiation of new members took place at the annual banquet 27 April. David S. Moore from Purdue University spoke on "Models of Doom" for this occasion.

Indiana Delta, University of Evansville, Evansville

42 actives, 46 pledges

Officers serving during 1976-77: Tim Swindle, president; Cliff Renschler, vice-president; Damon Thompson, secretary; Gene Bennett, treasurer and corresponding secretary; Ken Stofflet, faculty sponsor.

At the regular January meeting William Kenney talked about "The Many Faces of Non-associativity". A valentine party highlighted February for the members. Bruce Nance presented the four color problem at the March meeting. 46 new initiates were received at the initiation banquet on 4 April. Ken Stofflet delivered the address on "It's a Most Non-Linear Day". The annual picnic was held 25 April at Audubon Park, Kentucky.

Iowa Alpha, University of Northern Iowa, Cedar Falls

35 actives

Officers serving during 1976-77: Richard Kroeger, president; J. Scott Daup, vice-president; Diana Dickinson, secretary; Robert
Lutteneger, treasurer; John S. Cross, corresponding secretary and faculty sponsor.

Dick Kroeger was awarded first place for his presentation on the calculus of variations at the Regional Convention at Benedictine College. For the first time in eight years the weather for the annual picnic was ideal. (Partial credit goes to Professor Carl Wehner who volunteered to be in charge of the weather.) The picnic was a great success with large attendance by both students and faculty. Dr. W. E. Hamilton, head of the department of mathematics at the University of Northern Iowa since 1963, retired from administrative work and returned to the classroom on a full time basis following the 1976 summer session. His personal and administrative support of the activities of the Iowa Alpha Chapter of KME are greatly appreciated.

Iowa Beta, Drake University, Des Moines

10 actives, 6 pledges

Officers serving during 1976-77: Mary Bauer, president; Philip Warrick, vice-president; Rick Lang, secretary; Alan Carpenter, treasurer; Joseph Hoffert, corresponding secretary; Alex Kleiner, faculty sponsor.

Chapter activities began with a party at the home of Dr. Alex Kleiner. Talks during the semester were presented by the following: Steven Crandall on prime divisibility, William Ortmeyer on Lagrange's attempt to rigorize calculus, Dr. Basil Gillam on reciprocals of integers and repeating blocks of digits, Ann Jenkins on drawing three circles in a triangle, Philip Warrick on recursive methods of problem solving, David Emmon on ideas of Newton and Leibniz, and Darien Hall on Fibonacci numbers. Rodney Hanze and Harry Jamison won first and second place, respectively, in the 1976 Freshman Mathematics Contest. John Wells won $100 for the chapter in a basketball shooting contest sponsored by the local McDonalds. Initiation of new members and election of officers followed the annual spring banquet.

Iowa Gamma, Morningside College, Sioux City

28 actives

Officers serving during 1976-77: Kim Helmbrecht, president; Jim Richardson, vice-president; Jan Smith, secretary; Bob Barke,
treasurer; Elsie Muller, corresponding secretary and faculty sponsor.

On 31 March, the chapter held a mathematics symposium for high school students and teachers in the area. Rooms featuring the following topics were available to the participants: career possibilities, games and puzzles, courses in mathematics at Morningside, admissions and financial aid, computer center, filmstrips from *The Paradox Box*, and refreshments. Dr. Henry M. Walker, assistant professor of mathematics at Grinnell College, was a guest lecturer on 4-5 December. His topics were: "Math for Spies—Cryptanalysis before 1970" and "Calculus in Probability". At the April meeting Sister Michelle Nemmers, chairperson of the department of mathematics at Briar Cliff College, spoke on "Famous Unsolved Problems of Mathematics". Five members attended the regional meeting at Benedictine College. David Hall presented a paper at the meeting.

**Iowa Delta, Wartburg College, Waverly**

28 actives, 2 pledges

Officers serving during 1976-77: David Zelle, president; Deborah Ehlers, vice-president; Kathryn Thompson, secretary; Paul Kock, treasurer; William L. Waltmann, corresponding secretary and faculty sponsor.

On 27 January the Iowa Delta Chapter welcomed Dr. Alexander F. Kleiner, Jr. of Drake University. Dr. Kleiner presented a lecture on "Mathematics of Apportionment". His visit to Wartburg was sponsored by the Iowa Section M.A.A. visiting lecture programs. Dr. Glenn C. Fenneman of the Wartburg Department of Mathematics presented a lecture demonstration on the topic "Hexes and Flexes" on 17 February. Initiation of five members and election of officers took place on 16 March at the banquet in the Castle Room. The annual picnic was held at Cedar Bend Conservation Park on 10 May.

**Kansas Alpha, Kansas State College of Pittsburg, Pittsburg**

65 actives

Officers serving during 1976-77: Polly Mertz, president; Theresa Audley, vice-president; Gale Russell, secretary; Sara Nelson, treasurer; Harold Thomas, corresponding secretary; J. Bryan Sperry, faculty sponsor.
The February meeting was preceded by a banquet held at the home of Professor Annabelle Loy. At the meeting Professor Nelson Dinerstein presented the program on "Topics in Computer Science" and nineteen members were initiated. The March program was given by Don Mertz on "Foundational Mathematics". Ten student members and five faculty members attended the Region IV Convention at Benedictine College on 3 April. The regular April meeting included a report from those at the convention as well as a presentation on "Geometric Art" by Gary Stice. The final meeting for the semester was highlighted by the election of officers. In addition, the annual Robert M. Mendenhall Award for scholastic achievement was presented to Roy Bryant. Roy, who received a KME pin in recognition of this achievement, gave the program on "Finite Difference Methods".

Kansas Beta, Emporia Kansas State College, Emporia
65 actives
Officers serving during 1976-77: Eugene Scheckel, president; Kyle Fanning, vice-president; Roberta Loeffler, secretary; Dennis Whaley, treasurer; Sherry Anderson, historian; Donald Bruyr, corresponding secretary; Tom Bonner, faculty sponsor.

Sixteen new members were initiated during the spring semester. Dr. J. W. Maucker, Vice-president for Academic Affairs, spoke at the banquet which followed initiation. Kyle Fanning and Eugene Scheckel presented papers at the regional meeting. Eugene was awarded third prize for his presentation. Kyle Fanning and Dennis Whaley presented programs at chapter meetings during the semester. The annual picnic was held in May.

Kansas Gamma, Benedictine College, Atchison
16 actives
Officers serving during 1976-77: Le Ann Fischer, president; Wayne Heidman, vice-president; Ann Bremehr, secretary; Mike Hannon, treasurer; Sister Jo Ann Fellin, corresponding secretary; Jim Ewbank, faculty sponsor.

Dr. J. C. Kelly of the University of Missouri at Columbia is a favorite speaker for the Kansas Gamma Chapter. Dr. Kelly, an alum of Benedictine, returned to campus on 17 February, to inspire
the group with his presentation of "Fixed Point Theorems". New members initiated on 9 March were: Martin Asher, Ann Bremehr, Len Champ, Joe Gress, and John Kohler. The seventh biennial mathematics tournament for area high school students was held at Benedictine on 20 March with Wayne Heideman as coordinator. At the closing event of the semester, a steak cookout honoring the graduates, Wayne was presented the Agnesi Award by the chapter for his contribution in organizing the tournament. Chapter members participated actively in the tournament activities as well as the Region IV Convention hosted by Kansas Gamma on 3 April. Le Ann Fischer was named as the 12th recipient of the Sister Helen Sullivan Scholarship.

**Kansas Delta, Washburn University, Topeka**

27 actives, 8 pledges

Officers serving during 1976-77: Ron Wasserstein, president; Eldon Specht, vice-president; Peggy Strunk, secretary; Jamie Rozich, treasurer; Robert H. Thompson; Billy E. Milner and Gary Schmidt, faculty sponsors.

Kansas Delta sponsored the annual Mathematics Day on 25 March with about 300 high school students in attendance. Members of KME prepared the tests, scored the tests, and awarded team prizes. Monthly meetings were held by the chapter.

**Kansas Epsilon, Fort Hays Kansas State College, Hays**

27 actives

Officers serving during 1976-77: Deana Bowman, president; Kent Huffman, vice-president; Bernice Ruda, secretary and treasurer; Eugene Etter, corresponding secretary; Charles Votaw, faculty sponsor.

Four new members were initiated on 7 April at the annual banquet. Three other students were initiated earlier in the semester on 16 February. Four meetings were held during the spring semester.

**Kentucky Alpha, Eastern Kentucky University, Richmond**

11 actives, 13 pledges

Officers serving during 1976-77: Lois Coulter, president; Bruce Campbell, vice-president; Martha Maggard, secretary; Tony Casamento, treasurer; Karen Burns, recording secretary; Bennie
R. Lane, corresponding secretary; Amy C. King and Glynn Creamer, faculty sponsors.

The chapter entertained new mathematics students with a tea in January. Thirteen students were initiated at the annual banquet in March. Guest speaker for the event was Dr. Lynn Osen from California who spoke on “Women in Mathematics”. At the April meeting Jeff Kuchek presented material on actuarial science. A picnic was held by the chapter with the physics club.

Maryland Alpha, College of Notre Dame of Maryland, Baltimore

14 actives, 2 pledges

Officers serving during 1976-77: Kate Wildberger, president; Rhette Murtha, vice-president and treasurer; Bonnie Stingl, secretary; Sister Marie Augustine Dowling, corresponding secretary and faculty sponsor.

February and March meetings were devoted to exploring the metric system. Eight members attended the regional meeting at Bloomsburg State College. Rhette Murtha, Kathy O'Grady, and Kate Wildberger contributed papers. On 6 April a joint meeting was held with Maryland Beta. Maryland Alpha members presented a paper on matrices and cryptography. At the initiation meeting on 4 May the following two papers were given: “1776-1976 Mathematical Activities” by Jane Marini and “Population Dynamics” by Karen Pichler.

Maryland Beta, Western Maryland College, Westminster

23 actives, 6 pledges

Officers serving during 1976-77: Deborah Simmons, president; Theresa Holland, vice-president; Chris Dryden, secretary; Susan Burgess, treasurer; Lida Karick, historian; James E. Lightner, corresponding secretary; Robert P. Boner, faculty sponsor.

Mathematics Careers Night on 25 February featured a career counselor and representatives from IBM and NSA. New members were inducted at the annual banquet on 5 March. Tony Sager won honorable mention for his paper on geometric transformations at the Region I Meeting at Bloomsburg. Theresa Koontz presented a paper at the joint meeting with Maryland Alpha. Chapter members participated in two money making activities—May Day Carnival
and a car wash. The spring picnic was at the home of Dr. Lowell Duren. At the spring Honors Convocation Theresa Koontz and Virginia Bevans won the Pyne Mathematics Award.

**Mississippi Alpha, Mississippi State College for Women, Columbus**

8 actives, 10 pledges

Officers serving during 1976-77: Ellen Kirby, president; Jill Johnson, vice-president; Mary Ann Wright, secretary and treasurer; Jean Ann Parra, corresponding secretary and faculty sponsor.

**Mississippi Gamma, University of Southern Mississippi, Hattiesburg**

39 actives

Officers serving during 1976-77: Van Andrew Fortenberry, president; Russell E. Freeman, vice-president; Marie Aileen Clement, secretary; Nolann Nelson, social chairman; Alice W. Essary, corresponding secretary and faculty sponsor.

An original skit was presented by Mississippi Gamma when the chapter was host for a joint meeting with Mississippi Delta on 5 April. Ten new members were initiated at the spring cookout on 30 April. A book of mathematical tables was awarded to Kent Keys, outstanding freshman mathematics student. Larry Lok, past president of Mississippi Gamma, received the Outstanding Undergraduate Student Award and the Graduating Senior Award by the mathematics faculty.

**Mississippi Delta, William Carey College, Hattiesburg**

12 actives, 6 pledges

Officers serving during 1976-77 are elected in the fall. Gaston Smith, corresponding secretary and faculty sponsor.

On 1 April David Gruchy, Janie Pittman, Patti Purdum, Nora Mae Kelly, and Paul Aderinto were initiated into Mississippi Delta. These students had roles in a comedy performed at a joint meeting with Mississippi Gamma. The play entitled “The Three Stoogii and the Dude with the Golden Thigh” was written in contemporary language giving the history of the school of Pythagoras in Crotona,
Italy and some of its accomplishments. The chapter worked on a bicentennial project obtaining up-to-date information on all Mississippi Delta members from its formation in 1968.

**Missouri Alpha, Southwest Missouri State University, Springfield**

45 actives  
Officers serving during 1976-77: Pam Grassle, president; Steve Hansen, vice-president; Carol Wickstrom, secretary; Fred Harrison, Jr., treasurer; Eddie W. Robinson, corresponding secretary; L. T. Shiflett, faculty sponsor.

Chapter members served as proctors for Math Relays in which 1300 high school students competed. Carol Wickstrom was elected Senator from Mathematics to the Student Government Association. Karen Lefler was awarded the KME merit award for service to the chapter and the department. Both attended the regional meeting in Atchison, Kansas. Diana King and Michael Yehle were chosen to be speakers for the senior class. Diana graduated summa cum laude and won the Harry Siceluff Memorial Award. Michael was admitted to law school at Stanford University. Regular monthly meetings were held.

**Missouri Beta, Central Missouri State University, Warrensburg**

33 actives, 12 pledges  
Officers serving during 1976-77: Beverly Parnell, president; Steve Lacey, vice-president; Mary Beth Snodgrass, secretary; Glenda Arwood, treasurer; Homer Hampton, corresponding secretary; Keith Stumpff, faculty sponsor.

The chapter participated in a "work day" which grossed $50. Two guest speakers presented programs. A joint honors banquet was held with Sigma Zeta.

**Missouri Gamma, William Jewell College, Liberty**

17 actives, 18 pledges  
Officers serving during 1976-77: Howard Hays, president; Karen McAninch, vice-president; Howard Brooks, secretary; Tuyen Nguyen, treasurer; Sherman Sherrick, corresponding secretary; Joseph T. Mathis, faculty sponsor.

Monthly meetings were held with students presenting talks. In
March the pledges were installed in a formal ceremony followed by a banquet where Dr. Glenn Bernt of Evangel College was the invited speaker. Four members represented the chapter at the Region IV Convention held at Benedictine College.

**Missouri Iota, Missouri Southern State College, Joplin**

22 actives

Officers serving during 1976-77: Robert Dampier, president; Terri O'Dell, vice-president; Sam Miller, secretary and treasurer; Mary Elick, corresponding secretary; Charles Allen, faculty sponsor.

Team members Terri O'Dell, Bob Dampier, Mary Baldwin, and Cynthia Carter took the first place trophy in the Phi Theta Kappa College Bowl on campus. Missouri Iota sponsored a math contest for students at Missouri Southern. Prizes were awarded in each of the three divisions: individual, organization, and mathematics puzzles. To enter this last division an individual student had to buy one of the puzzles made by chapter members. In this way the contest became also a money-making project. Chapter members donated over 200 hours in free tutoring service and participated as proctors and graders for the Math League held for area high school students. Sharon McBride and Mary Baldwin competed in the National Putnam Exam. Eight students and three faculty attended the regional meeting at Benedictine College. Sam Miller's paper on magic squares was listed as an alternate. The chapter won $100 in the Missouri Southern Student Involvement Awards Contest.

**Nebraska Beta, Kearney State College, Kearney**

19 actives, 1 pledge

Officers serving during 1976-77: Ken Hutton, president; Horld Trease, vice-president; Diane Meyer, secretary; Nancy Burr, treasurer; Charles Pickens, corresponding secretary; Randall Heckman, faculty sponsor.

**Nebraska Gamma, Chadron State College, Chadron**

22 actives, 6 pledges

Officers serving during 1976-77: Mark Weiting, president; Vila Fuavao, vice-president; Chris Mahr, secretary; Cyndi Mondle,
treasurer; James A. Kaus, corresponding secretary; Albert Fox, faculty sponsor.

Rod Cape presented a paper at the regional in Kansas. Other members attending were: Sheryl Meyer, Vila Fuavao, and Mr. Kaus. The chapter sponsored the pie eating contest at Spring Daze held on campus on 1 May. Cindy Roffers was presented a mathematical tables book on Ivy Day. Regular meetings were held twice monthly.

**New Mexico Alpha, University of New Mexico, Albuquerque**

55 actives

Officers serving during 1976-77: Turner Laquer, president; Tony Buchen, vice-president; Phyllis Metzler, secretary; Gerald McNerney, treasurer; Merle Mitchell, corresponding secretary and faculty sponsor.

**New York Epsilon, Ladycliff College, Highland Falls**

10 actives, 5 pledges

Officers serving during 1976-77: Loretta Haarmann, president; Susan Schaeffer, vice-president; Michael J. Cellini, secretary-treasurer, corresponding secretary and faculty sponsor.

Monthly meetings consisted of a lecture series conducted by M. J. Cellini. Some of the topics covered were: “Lattice Theory”, “Measure and Integration”, and “The Four Color Problem”. New members Loretta Haarmann, Susan Schaeffer, Donna Witko and Elaine Gonzales were inducted in March. Chapter members attended various lectures at neighboring institutions.

**New York Eta, Niagara University, Niagara University**

32 actives, 12 pledges

Officers serving during 1976-77: John Schaefer, president; Joanne Esposito, vice-president; Dianne Fraher, secretary; Mike Drennan, treasurer; Robert L. Bailey, corresponding secretary; John J. Moore, faculty sponsor.

New York Eta was host for the Region II Convention held 26-27 March.

**New York Kappa, Pace University, New York**

40 actives, 20 inductees

Officers serving during 1976-77: Sal Vittorio, president; Jennifer
Costas, vice-president; Chris Szalay, secretary; Carleen Boote, treasurer; Sandra Pulver, corresponding secretary and faculty sponsor.

This year marked the first for inductees into New York Kappa from the Pleasantville campus. The 20 new members consisting of both faculty and students, 11 from New York and 9 from Westchester, were installed by out-going president Lawrence Jermyn. Guest speaker Gabriel Rosenberg spoke on the use of "Statistics at the Station-House". Mr. Rosenberg, a quantitative analyst for the New York Police Department for the last year and a half, discussed a system of pattern recognition using computer services to suppress crime.

**Ohio Alpha, Bowling Green State University, Bowling Green**

39 actives, 38 pledges

Officers serving during 1976-77: Diane Sparks, president; Nancy Johnson, vice-president; Karen Thomas, secretary; Paula Harvan, treasurer; Waldemar C. Weber, corresponding secretary; Thomas A. Hern and W. Charles Holland, faculty sponsors.

Students and faculty joined in a volleyball game on 22 February. On 2 March Aggie Gorup, recent graduate of Bowling Green now with Marathon Oil, and Professor William Hudson of Bowling Green led the meeting on "Mathematics in Industry and Business". The 25th Annual KME Exam was held 13 April with top awards going to William Tothe and Sally Nordquist for the best freshmen exam papers. The Excellence in Teaching Mathematics Award was presented to Professor Fred McMorris at the initiation banquet on 27 April with 79 in attendance. Professor V. Frederick Rickey spoke on "History of Mathematics in America and Bowling Green". Last activity of the year was the annual picnic and student-faculty softball game.

**Ohio Gamma, Baldwin-Wallace College, Berea**

20 actives, 20 pledges

Officers serving during 1976-77: Susan Kish, president; Bonnie Sprankle, vice-president; Lisa Chojnacki, secretary; Scott Steward, treasurer; Robert Schlea, corresponding secretary and faculty sponsor.
Ohio Zeta, Muskingum College, New Concord
35 actives
Officers serving during 1976-77: Becky Tucker, president; Jim Wingerter, vice-president; Janet Danison, secretary; Debbie Gutridge, treasurer; James L. Smith, corresponding secretary and faculty sponsor.
At the February meeting Dr. George Nieman of the chemistry department at Muskingum illustrated his talk on "Computer Synthesis of Chemical Kinetics" with computer terminals. New members Janet Danison, Don Graham, John Rogers, and Dave Swindler were received at the initiation banquet in March. Three students and three faculty attended the regional meeting at Niagara University. Dr. Larry Zettel of the department of mathematics and computer science at Muskingum presented a talk on "My Problems" in April. Art Patterson has been appointed as an undergraduate research assistant for the second semester 1976-77 at Argonne National Laboratory. The Spring Awards Banquet was held in May.

Oklahoma Gamma, Southwestern Oklahoma State University, Weatherford
21 actives, 20 pledges
Officers serving during 1976-77: Robert Whitenberg, president; Richard Cowen, vice-president; Lina Ommen, secretary; Pablo Reyes, treasurer; Wayne Hayes, corresponding secretary; Roy Fleischmann, faculty sponsor.
The chapter held monthly meetings, had one fund raising project, and took a trip to the severe storm center at Norman, Oklahoma. Twenty students were initiated on 20 April.

Pennsylvania Alpha, Westminster College, New Wilmington
57 actives, 19 pledges
Officers serving during 1976-77: Tyrene Zywar, president; Pete Andino, vice-president; Holly Baer, secretary; Tim Housholder, treasurer; J. Miller Peck, corresponding secretary; Thomas R. Nealeigh, faculty sponsor.
Paul Brown was honored at the initiation banquet on his retirement from the department of mathematics at Westminster after 29 years of service. He organized the Pennsylvania Alpha Chapter at Westminster in 1950.
Pennsylvania Gamma, Waynesburg College, Waynesburg

16 actives, 10 pledges
Officers serving during 1976-77: Clifford Quinn, president; Lois Swestyn, vice-president; Douglas Gapen, treasurer; Lee O. Hagglund, corresponding secretary; David S. Tucker, faculty sponsor.

Pennsylvania Gamma amended its constitution to assign the specific responsibility for program planning to the senior members. Programs last spring were: "Mathematical Paradoxes" by Lorraine Brocco and Douglas Mikula, "Some Applications of Mathematics to Economics" by James Keller and Michael Novosel, and "Cryptanalysis" by Richard Bailey.

Pennsylvania Epsilon, Kutztown State College, Kutztown

20 actives, 5 pledges
Officers serving during 1976-77: Rebecca Lykens, president; John Bloszinski, vice-president; Lynne Ott, secretary; Anastasia Maliaros, treasurer; Irving Hollingshead, corresponding secretary; Edward Evans, faculty sponsor.

Chapter members attended the PCTM conference at Valley Forge and conferences at Shippensburg and Bloomsburg. In addition to regular meetings the chapter held a dinner dance following initiation.

Pennsylvania Zeta, Indiana University of Pennsylvania, Indiana

43 actives
Officers serving during 1976-77: Nancy Neral, president; Scott Fickes, vice-president; Vicki Hirschman, secretary; Jacquelyn Harrington, treasurer; Ida Z. Arms, corresponding secretary; William R. Smith, faculty sponsor.

Dr. Leslie Ball, visiting professor in the safety science department, spoke on "Systems Engineering and its Mathematical Requirement" at the February meeting. In March Charles Maderer of the mathematics department discussed the "Mystery of the Square Root of 2". At the May banquet Dr. Melvin Woodard, chairman of the mathematics department, delivered the address entitled "Historical Perspectives of IUP Math Department". A slide presentation, "Indiana University, An Overview", was given by Neal Griffith.
Pennsylvania Eta, Grove City College, Grove City

30 actives, 20 pledges

Officers serving during 1976-77: Judy Moser, president; Chris Young, vice-president; Laurie Auletta, secretary; Gary Watson, treasurer; Marvin C. Henry; corresponding secretary; John Ellison faculty sponsor.

John Thompson, a 1963 graduate from Grove City College and a Fellow in the Society of Actuaries, spoke to the group about the actuarial profession. Mile Dove, who gave excellent leadership as president of Pennsylvania Eta during 1975-76, talked on "Stability Theory"—the subject of a paper he wrote as an independent study project. In May the chapter had their annual spring picnic at the Recreation Center.

Pennsylvania Iota, Shippensburg State College, Shippensburg

45 actives, 12 pledges

Officers serving during 1976-77: Dale Myers, president; Carol Novitch, vice-president; Dianne Ruminski, secretary; Howard Bell, treasurer; John S. Mowbray, corresponding secretary; James Sieber, faculty sponsor.

At the Region I meeting in Bloomsburg, Frank Schmidt tied for first place, John Saylor placed third and Keith Livingston received honorable mention. Other Pennsylvania Iota members who presented papers there were Sharon Byers and Kevin Livingston. The annual initiation banquet was held on 22 April. Other activities included regular meetings, a picnic, and a volleyball game.

Pennsylvania Kappa, Holy Family College, Philadelphia

8 actives, 2 pledges

Officers serving during 1976-77: Louise Wallowicz, president; Susan Capozio, vice-president; Janice DiGirolamo, treasurer; Sister M. Grace, C.S.F.N., corresponding secretary and faculty sponsor.

Louis Hoelzle, mathematics faculty member, gave an introduction to linear programming with applications at the chapter meetings. Chapter members offered free tutoring services twice each week. In keeping with the spirit of the bicentennial, chapter members visited historic sites in Philadelphia, the zoo, the Museum of Art
and the Mint during the spring semester. The chapter also donated money from its treasury towards the purchase of calculators for the mathematics department.

Pennsylvania Lambda, Bloomsburg State College, Bloomsburg

30 actives, 13 pledges
Officers serving during 1976-77: Richard Styer, president; Harland Shoemaker, vice-president; Diane Gilroy, secretary; Phyllis Ashenfalder, treasurer; Beverly Marey, historian; James C. Pomfret, corresponding secretary; Joseph Mueller, faculty sponsor.

Pennsylvania Lambda was host for the Region I Convention. Three of the 12 papers were given by Pennsylvania Lambda members. Member Richard Styer tied for first. The chapter sponsored talks given by Dr. Paul Cochrane and Dr. Paul Hartung, both of Bloomsburg State College. Their topics were "Complex Variables" and "Hot News in Number Theory", respectively. The final meeting of the year was held at the Hartung Resort.

Tennessee Alpha, Tennessee Technological University, Cookeville

75 actives, 44 pledges
Officers serving during 1976-77: Mary Anne Koltowich, president; Kent Berry, vice-president; Gregory D. Carr, secretary; Barry Sparkman, treasurer; Evelyn Brown, corresponding secretary; Donald Ramsey, faculty sponsor.

Tennessee Gamma, Union University, Jackson

12 actives
Officers serving during 1976-77: Vickie De Priest, president; Dana Northcut, vice-president; Maury Carrico, secretary; Jerry McKnight, treasurer; Richard Dehn, corresponding secretary; Joseph Tucker, faculty sponsor.

Tennessee Gamma held its annual initiation banquet on 18 March. Beth Seabrook, outgoing president, presided over the initiation of six new members—Maury Carrico, Jerry McKnight, Dana Northcut, John Oakley, Pam Pratt, and Donna Taylor. Phil Scott presented the address.
Tennessee Delta, Carson-Newman College, Jefferson City

27 actives, 21 pledges
Officers serving during 1976-77: Calvin Joyner, president; Ann Boyd, vice-president; Tammy Miller, secretary; Paul Collier, treasurer; Denver R. Childress, corresponding secretary; Carey R. Herring, faculty sponsor.
Activities included a lecture by Billy F. Bryant of Vanderbilt University, a bike hike to Cherokee Lake followed by an ice cream supper, and the initiation of 21 new members at the spring banquet.

Texas Beta, Southern Methodist University; Dallas

30 actives, 24 pledges
Officers serving during 1976-77: June Files, president; John Dunlop and Kathleen Hannigan, vice-presidents; Janet Hobratsch, secretary and treasurer; C J. Pipes, corresponding secretary; Robert C. Davis, faculty sponsor.

Texas Eta, Hardin-Simmons University, Abilene

19 actives, 7 pledges
Officers serving during 1976-77: Tim Zuhas, president; Sharon Carver, vice-president; Susan Porter, secretary and treasurer; Anne B. Bentley, corresponding secretary; Charles Robinson, faculty sponsor.
Chapter pledges were initiated on 10 April. Dr. Landon Colquitt, chairman of the mathematics department at Texas Christian University in Ft. Worth, was the speaker.

Wisconsin Alpha, Mount Mary College, Milwaukee

9 actives, 4 pledges
Officers serving during 1976-77: Kathleen Tandetzke, president; Karen Gabos, vice-president; Jane Reinartz, secretary; Terrie Flesch, treasurer; Sister Mary Petronia Van Straten, corresponding secretary and faculty sponsor.
The following programs were presented: "Approximations of pi" by Kathleen Tandetzke, "Tricky Problems" by Judy Gesell, "Developing Flow Charts" by Terrie Flesch, "Everything You Always Wanted to Know about the Number Six" by Karen Gabos, "Naming Numbers with 1976" by Sister Donna Griesbach. All of the pre-
senters were initiated on 24 February. A discovery lesson by Jane Reinartz completed the programs presented during the semester. The chapter conducted a mathematics contest for high school students on 3 April and presented "Miss A-metric-A Pageant" for the all-school talent show.

Report on the 1976 Region I Convention

On Saturday, 3 April 1976, the Third Conference for Region I was held at Bloomsburg State College, Bloomsburg, Pennsylvania. Dr. James Pomfert, corresponding secretary, and David Espe, president, of the Pennsylvania Lambda Chapter arranged the meeting.

The main part of the program was devoted to twelve student papers. Frank Schmidt, PA Iota, Shippensburg State College, and Richard Styer, PA Lambda, Bloomsburg State College, tied for first place with their presentations of "Programming Hexapawn — A Game You Play to Lose" and "Decision Making in Agriculture", respectively. Third place went to John Saylor, PA Iota, Shippensburg State College, for "Godel's Theorem — Impact on Mathematical Systems". Receiving Honorable Mention were: Rhette Murtha, MD Alpha, College of Notre Dame of Maryland, "A Finite Geometry"; Tony Sagar, MD Beta, Western Maryland College, "Transformational Geometry"; and Keith Livingston, PA Iota, Shippensburg State College, "The Brachistochrone Problem as Solved by Johann Bernoulli".

Other presentations included: "A Closure Complement Problem" by Anna Bucklar, PA Lambda; "Hard for Fermat — Easy for Us — How Light Refracts" by Sharon Byers, PA Iota; "Solving Systems of Equations Using the Computer" by Kevin Livingston, PA Iota; "A Look at Operations Research" by Kathy O'Grady, MD Alpha; "Information Networks" by Jeanne Seaman, PA Lambda; and "The Midas Touch" by Kate Wildberger, MD Alpha.

Sister Marie Augustine Dowling, Director of Region I and corresponding secretary of MD Alpha, addressed the group in a special session on "Mathematics in Colonial America".

Seven of the eighteen chapters in Region I were represented. A total of 46 were in attendance with 33 students and 13 faculty from the following chapters: MD Alpha, MD Beta, PA Delta, PA Epsilon, PA Theta, PA Iota, and PA Lambda. James E. Lightner, National Vice-President, was present for the meeting.
Report on the 1976 Region II Convention

A two day regional meeting was held at Niagara University, New York on 26-27 March 1976 with New York Eta as host chapter. Dr. John Moore, faculty sponsor, coordinated the meeting.

The banquet speaker on Friday evening was Dr. Stephen Brown of the State University of New York. The banquet was followed by a social gathering. On Saturday morning the following presentations were made: “Message Coding” by John Schaefer, student from New York Eta, Niagara University; “Functional Geometry” by Dr. James L. Smith, Director of Region II and corresponding secretary of Ohio Zeta, Muskingum College; and “Trapezoidal Numbers” by Dr. John Moore, faculty sponsor of New York Eta, Niagara University. John Schaefer received an award for his paper.

About 40 delegates attended, with representatives from Michigan Beta at Central Michigan University in Mount Pleasant, New York Eta at Niagara University, and Ohio Zeta at Muskingum College, New Concord.

Report on the 1976 Region IV Convention

Benedictine College in Atchison, Kansas was the setting for the Region IV meeting on 3 April 1976. Sister Jo Ann Fellin, National Historian and corresponding secretary of the host chapter, Kansas Gamma, directed the arrangements.

The significant feature of the meeting was the presentation of eight student papers. Richard Kroeger, Iowa Alpha, University of Northern Iowa, took first place with his paper on “The Calculus of Variations”. Second place went to the president of the host chapter, Le Ann Fischer, for “The Golden Rectangle”. Gene Scheckel, Kansas Beta, Emporia Kansas State College, received the third place award for “On the Sums of Powers of Natural Numbers”.

The remaining student presentations were: “Primitive Pythagorean Triples” by Rodney Cape, NE Gamma, Chadron State College; “One-to-one Transformations of the xy-Plane” by Kyle Fanning, KS Beta, Emporia Kansas State College; “Challenge of the Transfinites” by David Hall, IA Gamma, Morningside College; “Angle Trisection” by Wayne Heideman, KS Gamma, Benedictine College;

(Continued on page 30)
Installation of New Chapters

Edited by Loretta K. Smith

Information for the department should be sent to Mrs. Loretta K. Smith, 829 Hillcrest Road, Orange, Connecticut 06477.

IOWA DELTA CHAPTER

Wartburg College, Waverly, Iowa

The Iowa Delta Chapter was installed at the nineteenth biennial convention of Kappa Mu Epsilon at Morningside College. The ceremony occurred at the banquet on the evening of 6 April 1973. The installing officer was George R. Mach, National President, who was assisted by Fred W. Lott, past president. The speaker was Robert V. Hogg of the University of Iowa, talking on "Adaptive Statistical Inference: An Introduction for Mathematicians."

Initiates were:

Glenn C. Fenneman, Faculty
William L. Waltmann, Faculty
Terry Ackman
Robert Basham
Patricia Bubke
Bruce Foster
Kenneth Harris
Loren Heckathorne
Sharon Leslein
David Riley
Sandra Roecker
Marsha Paepker Shaffer

PENNSYLVANIA LAMBDA CHAPTER

Bloomsburg State College, Bloomsburg, Pennsylvania

The Pennsylvania Lambda Chapter was installed at Bloomsburg State College on 17 October 1973 by the National Vice-President Jane E. Lightner, who spoke on the history of Kappa Mu Epsilon. The charter officers were Arthur Klemick, president; Mary Lepley, vice-president; Janet Fiora, recording secretary and historian; and Susan Meyer, treasurer. Mr. Joseph Mueller and Dr. James Pomfret were selected as advisor and corresponding secretary, respectively. The following is a list of the charter members:
The installation of the North Carolina Beta Chapter took place on 4 June 1974 and was presided over by the National Secretary, Mrs. Elizabeth T. Wooldridge. The topic of Mrs. Wooldridge's talk was "A Brief History of Kappa Mu Epsilon." The following list is that of the initiates.

Linda M. Agner
John M. Baker, Faculty
Beverly B. Banks
Edward T. Blair
Patrick D. Cheung
Knox B. Cobb
Drucilla D. Connor
Carol I. Dills
Starline A. Godfrey
R. Kac Henderson
Roger D. Hinson
Elizabeth B. Jones

Joseph B. Klerlein, Faculty
Connie Y. Lester
Mary Ann Miller
Lee H. Minor, Faculty
Brenda J. Patterson
Drusilla Scott
Cherie M. Stevens
Alan G. Story
Pamela B. Teague
J. Pelham Thomas, Faculty
Halford I. Willis
Ralph H. Willis, Faculty
TEXAS ETA CHAPTER

Hardin-Simmons University, Abilene, Texas

The installation of the Texas Eta Chapter was performed on 3 May 1975 at Moody Student Center by Professor Mike Reagan of Northeastern Oklahoma State University, Tahlequah, Oklahoma. Prof. Reagan talked about the history of Kappa Mu Epsilon and the role of the organization in the undergraduate program. The faculty and students initiated were:

Mrs. Anne Bentley, Faculty
Sharon Carver
Keith Clark
Dr. Richard Garner, Faculty
Ofelia Gonzales
Mrs. Susan Helms, Faculty
Dr. William Helms, Faculty
Dr. Edwin Hewett, Faculty
Debbie Irwin

Billy Keith
Jane Morgan
Susan Porter
Rod Rackley
Dr. Charles Robinson, Faculty
Krista Swilling
Perry Webb
Vernon Williams
Tim Zukas

MISSOURI IOTA CHAPTER

Missouri Southern State College, Joplin, Missouri

The installation of the Missouri Iota Chapter took place in the Ballroom of the Student Union of Missouri Southern State College, on 8 May 1975. Installing officers were Mr. Eddie Robinson, National Treasurer and Dr. Harold Thomas, Regional Director of Kappa Mu Epsilon, Kansas State College of Pittsburg, Pittsburg, Kansas. The Faculty initiates were Dr. Charles Allen and Mr. James Roubidoux and the student initiates were Julie Ann Atherton, Cynthia Carter, Mary Conrad, Robert Dampier, Monica Edwards, Gary Goodpaster, Stephen Grissom, Randall Haddock, Kenneth Jones, Michael Mayfield, Sharon McBride, Anna Howell Monteleone, Sam Miller, David Smith, and Mary Veith.

Members of Kansas Alpha Chapter and of Missouri Alpha Chapter were present for the ceremonies. Many parents and friends of the initiates were also present.

The program was given by Mr. Eddie Robinson. He presented some interesting highlights of the history of Kappa Mu Epsilon. A reception followed the installation.
GEORGIA ALPHA CHAPTER

West Georgia College, Carrollton, Georgia

On 21 May 1975, at 4:30 p.m. the Georgia Alpha Chapter was installed by Elizabeth T. Wooldridge, National Secretary, in the Math-Physics Building. Mrs. Wooldridge spoke on 'A Brief History of Kappa Mu Epsilon." A banquet followed at 8:00 p.m. in the College Student Center. The student initiates were:

Rebecca Allen  Billy Hattaway
George Bagwell  Brenda Hattaway
Bill Chestnut  Julia R. Heidt
Karen Furr  Wesley Stephens
Vivian G. Dunn  Lynn Torbert

and faculty initiates were:

Dean Richard Dangle  Dr. Earl Perry
Dr. Ben deMayo  Dr. Thomas J. Sharp

WEST VIRGINIA ALPHA CHAPTER

Bethany College, Bethany, West Virginia

The West Virginia Alpha Chapter held its formal initiation in Bethany, West Virginia on 21 May 1975. The installing officer was Professor James E. Lightner, Vice-President of Kappa Mu Epsilon, from Western Maryland College. In his installation address, Professor Lightner discussed the history of honor societies, the history and goals of Kappa Mu Epsilon, and the symbolism in the crest of Kappa Mu Epsilon.

The list of initiates and officers include:

Prof. James E. Allison, Chairman of Mathematics Department
Prof. David Brown, Corresponding Secretary
Prof. Ronald A. Ward

Leslie A. Burton  Nancy L. Heilman
Richard A. Davidson  Kerry L. Hovan
Rthe A. Dodson—President  Carla A. Howell
M. Jean Emeigh  Michael P. Kuniak
Carol S. Friday  John R. Manypenny
Rita M. Glaser  Thomas K. Miller
Raymond H. Hart  Cheryl R. Mitchell

(Continued on page 39)