The Problem Corner

Edited by Pat Costello

The Problem Corner invites questions of interest to undergraduate students. As a rule, the solution should not demand any tools beyond calculus and linear algebra. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following new problems should be submitted on separate sheets before October 1, 2017. Solutions received after this will be considered up to the time when copy is prepared for publication. The solutions received will be published in the Fall 2017 issue of *The Pentagon*. Preference will be given to correct student solutions. Affirmation of student status and school should be included with solutions. New problems and solutions to problems in this issue should be sent to Pat Costello, Department of Mathematics and Statistics, Eastern Kentucky University, 521 Lancaster Avenue, Richmond, KY 40475-3102 (e-mail: pat.costello@eku.edu, fax: (859) 622-3051)

NEW PROBLEMS 789-797

Problem 789. *Proposed by Daniel Sitaru, "Thodor Costescu" National Economic College, Traian National College, Drobeta Turnu – Severin, Mehedinti, Romania.*

In triangle ABC, let I = the incenter, O = the circumcenter, G = the centroid, and a,b,c the lengths of the sides. Prove that $\left(\sum IA\right)\left(\sum OA\right)\left(\sum GA\right) < (a+b)(b+c)(c+a)$

Problem 790. *Proposed by Daniel Sitaru, "Thodor Costescu" National Economic College, Traian National College, Drobeta Turnu – Severin, Mehedinti, Romania.*

Prove that if $a, b \in \mathbb{R}$ with $a < b$, then

$$\left|\ln\left(\frac{2+\sin 2b}{2+\sin 2a}\right)\right| \le \frac{2\sqrt{3}}{3}(b-a)$$

Problem 791. *Proposed by Jose Luis Diaz-Barrero, School of Civil Engineering, Barcelona Tech - UPC, Barcelona, Spain.*

Determine whether the real number

$$\frac{\ln(11+5\sqrt{2})}{\ln(5+11\sqrt{2})}$$

is rational or not.

Problem 792. *Proposed by Jose Luis Diaz-Barrero, School of Civil Engineering, Barcelona Tech - UPC, Barcelona, Spain.*

Let x_1 , x_2, …, x_n be real numbers lying in the interval (0,π/2). Prove that

$$\left(\frac{1}{n}\sum_{k=1}^{n}\sin x_k\right)\left(\frac{1}{n}\sum_{k=1}^{n}\cos x_k\right) \le \frac{1}{2}$$

Problem 793. *Proposed by D.M. Batinetu-Giurgiu, "Matei Basarab" National College, Bucharest, Romania, Neculai Stanciu, "George Emil Palade", Buzau, Romania.*

If a $\in$[0,π/4], compute $\int_0^a (x^2 - ax + a^2)(\ln(1+\tan x\tan a)dx$

Problem 794. *Proposed by D.M. Batinetu-Giurgiu, "Matei Basarab" National College, Bucharest, Romania, Neculai Stanciu, "George Emil Palade", Buzau, Romania.*

Let *a, b, c* be positive real numbers. Prove that

$$(1+a)(1+b)(1+c) \ge \left(1+\frac{2ab}{a+b}\right)\left(1+\frac{2bc}{b+c}\right)\left(1+\frac{2ca}{c+a}\right)$$

Problem 795. *Proposed by Michal Kremzer, Glicice, Silesia, Poland.*

Let Q be the set of rational numbers. Does there exist a function f: (Q-{0}) -> (Q-{0}) so that f(x) < f(3x) < f(2x) for all x in the set (Q-{0})?

Problem 796. *Proposed by Kadir Altintas, Turkey and Leonard Giugiuc, Romania.*

If A, B, and C are the angles of a triangle, prove that

$$\sqrt{6(1+\cos A\cos B\cos C} \ge 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$$

Problem 797. *Proposed by the editor.*

Let integer *n* be called a *consecutives concatenated number* when *n* is formed by concatenating two consecutive integers. For example, 67 and 1314 are consecutives concatenated numbers. 67 is prime, but 1314 is composite. It turns out there are lots of consecutives concatenated primes. Find a consecutives concatenated prime of 20 digits where the first integer of the two concatenated is divisible by $2^3 {*} 3^3$.