

THE PENTAGON

Volume XLII(42)

Fall, 1982

Number 1

CONTENTS

Catalan Numbers <i>Kimberly Yarnall</i>	1
A Metric Characterization of the Betweenness Relation <i>Joseph Siler</i>	13
Pascal's Tetrahedron and the Trinomial Coefficients <i>Janet Shorter and F. Max Stein</i>	19
The Problem Corner.....	34
The Mathematical Scrapbook.....	45
The Hexagon.....	50
Kappa Mu Epsilon News.....	61
Twenty-Fourth Biennial Convention.....	83
Subscription Form.....	83

NATIONAL OFFICERS

- IDA Z. ARMS.....President
Indiana University of Pennsylvania
Indiana, Pennsylvania 15705
- JAMES L. SMITH.....President-Elect
Muskingum College
New Concord, Ohio 43762
- GEORGE R. MACH.....Secretary
California Polytechnic State University
San Luis Obispo, California 93407
- WILBUR J. WAGGONER.....Treasurer
Central Michigan University
Mount Pleasant, Michigan 48859
- HAROLD L. THOMAS.....Historian
Pittsburg State University
Pittsburg, Kansas 66762
- JAMES E. LIGHTNER.....Past-President
Western Maryland College
Westminster, Maryland 21157

Kappa Mu Epsilon, mathematics honor society, was founded in 1931. The object of the society is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics, due, mainly, to its demands for logical and rigorous modes of thought; to provide a society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; to disseminate the knowledge of mathematics and to familiarize the members with the advances being made in mathematics. The official journal, THE PENTAGON, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the chapters.

CATALAN NUMBERS

KIMBERLY A. YARNALL

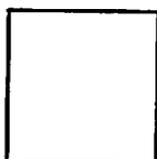
Student, Bloomsburg State College

There are literally thousands of interesting infinite sequences using positive integers. The Fibonacci numbers form a famous sequence (1,1,2,3,5,8,...) and a simple cubing of the integers forms another sequence (1,8,27,64,...). A less famous group of numbers which form a sequence are called the Catalan numbers. This sequence is of the form 1,1,2,5,14,42,132,429,1430, 4862,... and is of particular importance because of its knack in popping up to solve a variety of combinatorial problems.

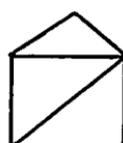
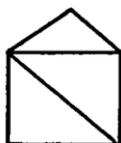
The first study of the Catalan numbers was by Leonhard Euler. He counted the number of ways to divide convex polygons into triangles by drawing in diagonals with no intersections. The number of ways to do this, starting with a triangle, then a square and so on, is actually the Catalan sequence.



1



2



5



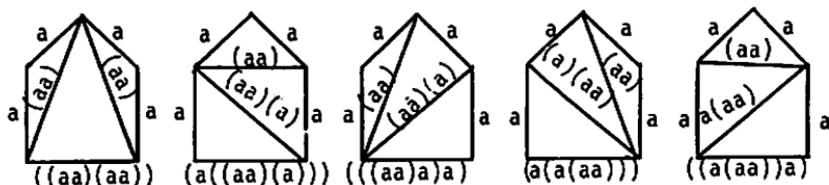
14

Another problem, which was solved by Eugene Charles Catalan, considers pairing a chain of n letters which are in a fixed order. The criteria for this pairing is

that $n-1$ sets of parentheses must be placed in the chain such that two terms are within them. The two terms could be simply two letters or a letter and a grouping of letters such as $(a(aa))$, or two groupings of letters such as $((aa)(aa))$. The problem asks, how many ways can a group of n letters be combined in this way. Taking two letters, there is only one way to combine them; (aa) . With three letters there are two ways to combine them; $((aa)a)$ and $(a(aa))$. Taking four letters they can be combined in five ways; $((aa)(aa))$, $(a((aa)a))$, $((aa)a)a$, $(a(a(aa)))$, $((a(aa))a)$. Again the Catalan numbers count the ways to pair n letters of a fixed order.

To show a correspondence between dividing polygons into triangles and pairing groups of letters, take a divided polygon and establish a base. Label each outer edge with a single letter. Now, for each line connecting two points label that with the labels from the two other sides. Continue this process until the base is labeled. For each of the other divided polygons repeat the process. These base labels are the different ways to combine the letters of an n -gon.

For example, consider the pentagons divided into triangles.



After performing the described process, you end up with five different ways of combining four $(n-1)$ letters.

A theorem to determine the number of ways to divide a convex polygon into triangles is as follows:

Let $h(n)$ be the number of ways of dividing a convex polygonal region with $n+1$ sides into triangular regions by inserting non-intersecting diagonals. Define $h(1)=1$. Then $h(n)$ is a solution of the recurrence relation.

$$H(n) = \sum_{k=1}^{n-1} H(k)H(n-k) \quad \text{for } n = 2, 3, 4, \dots$$

The solution of this recurrence relation with the initial value $H(1)=1$ is given by

$$h(n) = \frac{1}{n} \binom{2n-2}{n-1} \quad \text{for } n = 1, 2, 3, \dots$$

First I will prove the recurrence relation

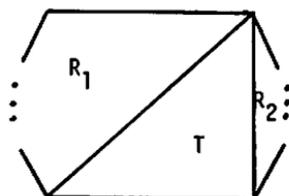
$$H(n) = \sum_{k=1}^{n-1} H(k)H(n-k)$$

For $n=1$, $H(1)$ is defined to be 1. For $n=2$, this would be a one-sided polygon, actually a line segment. So $H(2) = 1$.

Using the recurrence relation:

$$H(2) = \sum_{k=1}^1 H(k)H(2-k) = H(1)H(1) = 1 \cdot 1 = 1$$

For values of $n \geq 3$ consider a polygon R , with $n+1 \geq 4$ sides.



Take one of the sides, establish a base and form a triangle with it called T . Each of the other triangles in the polygon will have a base as one of the sides of this triangle T . The triangle T divides the polygon into two other polygons, say R_1 and R_2 . R_1 has $k+1$ sides while R_2 has $n-k+1$ sides where $k=1,2,\dots,n-1$.

So, the number of ways to divide R_1 into triangles by inserting non-intersecting diagonals is $h(k)$. Since

R_2 has $n-k+1$ sides it can be divided in $h(n-k)$ ways. So, the number of ways to divide the entire polygon, R , into triangles with a particular T chosen containing the base is $h(k)h(n-k)$ ways. But this must be done for all triangles T with the chosen base. Thus there are a total of

$$\sum_{k=1}^{n-1} h(k)h(n-k)$$

ways to divide R into triangles.

Therefore we have the recurrence relation as stated previously.

Now, to prove the non-recurrence formula; that is,

$$h(n) = \frac{1}{n} \binom{2n-2}{n-1}$$

Consider a grouping of letters say, a,b,c,d . If these are grouped in a fixed form, then $(ab)(cd)$ is the same as $(ba)(cd)$. But first allow them to be in any order so that $(ab)(cd)$ is different from $(ba)(cd)$.

Suppose we can make A_n different groupings of n letters. A_n can be formed in three ways from the grouping A_{n-1} . A new letter can be placed to the left of A_{n-1} , to the right of A_{n-1} , or it can be placed somewhere within the grouping A_{n-1} .

In the first two cases we would have (this is

assuming we picked the letter x as the new letter) $x(GN-1)$ or $(GN-1)x$ where $GN-1$ is any grouping of $n-1$ letters. So, this contributes $2A_{n-1}$ to the total A_n .

In the third case consider the building up of the grouping of $n-1$ letters. There will be a grouping, say $(P)(Q)$ where P and Q are letters or groupings of letters of some of the $n-1$ total letters. Now the new letter, x , can be inserted in four ways:

$$(((x(P))Q), (((P)x)Q), ((P)(x(Q))), ((P)((Q)x)).$$

But if there are $n-1$ letters there will be $n-2$ different ways of picking the P and Q groupings. So there is a total of $4(n-2)$ ways to insert a new letter within the grouping A_{n-1} .

Thus, the total number of groupings of n letters is:

$$\begin{aligned} A_n &= 2A_{n-1} + 4(n-2)A_{n-1} \\ &= (4n-6)A_{n-1} \\ &= (4n-6)(4(n-1)-6)A_{n-2} \\ &= (4n-6)(4n-10)A_{n-2} \\ &= (4n-6)(4n-10)(4n-14)A_{n-3} \\ &= \dots \\ &= (4n-6)(4n-10)(4n-14)\dots(A_r-2)A_r \end{aligned}$$

Since $H(1) = 1$

$$\begin{aligned} A_n &= 2 \cdot 6 \cdot 10 \cdot \dots \cdot (4n-6) \\ &= 2^{n-1} (1 \cdot 3 \cdot 5 \cdot \dots \cdot (2n-5)(2n-3)) \end{aligned}$$

But, we would like to know the number of ways to group letters when the order of the letters is fixed. So to obtain the solution, say B_n , we divide A_n by $n!$. This gives;

$$\begin{aligned} B_n &= \frac{2^{n-1} (2n-2)!}{n! (2 \cdot 4 \cdot \dots \cdot (2n-6)(2n-4)(2n-2))} \\ &= \frac{2^{n-1} (2n-2)!}{n! (2^{n-1}) (1 \cdot 2 \cdot \dots \cdot (n-3)(n-2)(n-1))} \\ &= \frac{(2n-2)!}{(n!) (n-1)!} \\ &= \frac{(2n-2)!}{n \cdot (n-1)! (n-1)!} \\ &= \frac{1}{n} \binom{2n-2}{n-1} \end{aligned}$$

Thus, we have the formula

$$h_n = \frac{1}{n} \binom{2n-2}{n-1} \quad \text{for } n=1,2,3,\dots$$

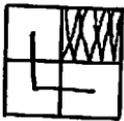
Another interesting recursive procedure for finding a Catalan number is to write the preceding Catalan number, then directly under it write that sequence backwards. For example:

$$\begin{array}{r} \\ x \\ 14 \\ \hline 14 + 5 + 4 + 5 + 14 = 42 \end{array}$$

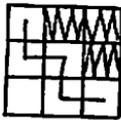
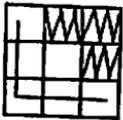
Now, multiply the top and bottom numbers and sum these products. This gives the next Catalan number in the sequence. This procedure was established by Johann Andreas Von Segner in the eighteenth century.

Now, I will give a few examples of combinatorial problems where the Catalan sequence provides the solution.

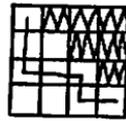
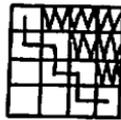
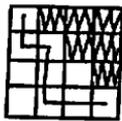
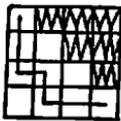
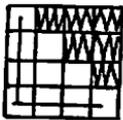
Take an $n \times n$ chessboard where $n=2,3,4,\dots$. The squares north and west of the main diagonal cannot be used. How many paths, starting at the top-left corner and ending at the bottom-right corner, are there? The allowable moves are down and to the right.



1



2



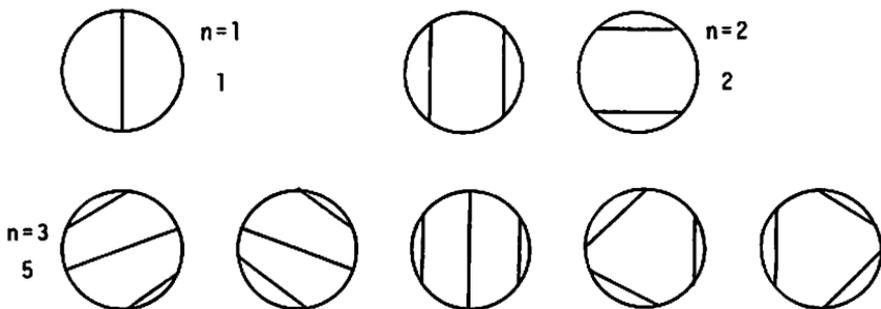
5

Notice that each square can be reached either from the one above it or the one to the left of it. So, a table can be made to determine the number of paths to any square below the main diagonal.

1					
1	1				
1	2	2			
1	3	5	5		
1	4	9	14	14	
1	5	14	28	42	42

There is another problem which could be called the "shaking hands across a table" problem.

Suppose there are $2n$ people seated at a round table. How many ways can these people shake hands in such a way that there are no intersections of arms?



The answer to the problem can be stated in a more geometric way as follows. The number of ways of joining $2n$ points on a circle joined by n non-intersecting chords is the $n+1$ Catalan number.

One final problem involves the number of ways to rhyme an n line poem. For one line there is only one way. With two lines there are two ways (both rhyming or both unrhymed). With three lines there are five ways. But with four lines there are fifteen ways. This sequence (1,2,5,15,...) is called the Bell sequence.

- | | | | | | | |
|----|------------------|------------------|------------------|------------------|------------------|----|
| 1. | a | | | | | 1 |
| 2. | \widehat{aa} | ab | | | | 2 |
| 3. | \widehat{aaa} | \widehat{aba} | \widehat{aab} | \widehat{abb} | abc | 5 |
| 4. | \widehat{aaaa} | \widehat{aaab} | \widehat{aaba} | \widehat{abaa} | \widehat{abbb} | |
| | \widehat{aabb} | \widehat{abba} | \widehat{abab} | \widehat{aabc} | \widehat{abac} | |
| | \widehat{abca} | \widehat{abbc} | \widehat{abcb} | \widehat{abcc} | abcd | 15 |

However, if you ask how many ways there are to rhyme an n line poem where the rhyming lines do not intersect, then we do have, once again, the Catalan sequence. This is called the "planar rhyme schemes" and was discovered by Joanne Crowley.

There are numerous problems for which the Catalan numbers supply the answer. Although they are not as well known as the Fibonacci numbers, they do pop up

in many combinatorial problems. I have shown various examples where the Catalan numbers provide the solution to different problems. They range from the first discovery of the sequence by Euler through the pairing of letters by Catalan to more practical problems such as chessboard problems, "shaking hands", and planar rhyme schemes. The list of problems does go on, but the basis for many of these combinatorial solutions is the set of Catalan numbers.

A METRIC CHARACTERIZATION OF THE BETWEENNESS RELATION

JOSEPH SILER

Student, Southwest Missouri State University

In many developments of geometry, "betweenness" is an undefined term. In this paper we show that by employing the concept of a metric space, betweenness may be defined in terms of a distance function.

Definition

Let M be a nonempty set and d be a mapping from the Cartesian product $M \times M$ into the set of non-negative real numbers which will satisfy the following three properties:

I) $d(x,y) = 0$ if and only if $x = y$

II) $d(x,y) = d(y,x)$

III) $d(x,y) + d(y,z) \geq d(x,z)$

for all x,y , and z , elements of M . Then M is called a *metric space*, and d is called a *metric*, or a distance function on M . Property I is known as the *positive definite* property, property II is called *symmetry*, and property III is known as the *triangle inequality*.

Definition

Let x,y , and z be elements of M , a metric space with metric d . ($d(x,y)$ will be abbreviated henceforth by xy .) y is said to be *between* x and z , abbreviated

xyz , if and only if $x \neq y \neq z$ and $xy + yz = xz$.

Note that it is not necessary to specify that $x \neq z$, since $x \neq y$ implies $xy > 0$, $y \neq z$ implies $yz > 0$, and so $xz > 0$, which establishes that $x \neq z$, by the positive definite property.

Theorem

In a metric space, distance is continuous.

Proof: It needs to be shown that for x and y elements of M , a metric space, if $x_n \rightarrow x$ and $y_n \rightarrow y$ then $x_n y_n \rightarrow xy$, where x_n and y_n are sequences of points in M and $x_n y_n$ is a corresponding sequence of non-negative real numbers. This is equivalent to the statement that if $x_n \rightarrow x$ and $y_n \rightarrow y$, then $|xy - x_n y_n| \rightarrow 0$.

$|xy - x_n y_n| = |xy - xy_n + xy_n - x_n y_n| \leq |xy - xy_n| + |xy_n - x_n y_n| \leq y y_n + x x_n \rightarrow 0 + 0 = 0$. This next-to-last step is from the fact that $|ab - bc| \leq ac$, which is easily shown from the triangle inequality for real numbers.

Theorem

In a metric space, betweenness satisfies the following properties:

- 1) *symmetry of outer points*: if pqr then rqp .
- 2) *special inner points*: if pqr then not prq and not qpr .

- 3) *transitivity*: if pqr and prs then pqs and qrs .
- 4) *closure*: $\bar{B}(p,q)$ which equals $\{p,q\} \cup B(p,q)$, (where $B(p,q)$ is the set of all x in M such that pxq is true) is a closed set, i.e., it contains all of its limit points.
- 5) *extension*: if M is a metric space consisting of the two distinct points p and q , then there exists a metric space M^* containing p,q , and another point r such that prq holds.

Proof of part 1: If pqr then $pq + qr = pr$. But $pq = qp$, $qr = rq$, and $pr = rp$ by symmetry of the metric. Since $qp + rq = rq + qp$ by commutativity of real number addition, we have $rq + qp = rp$, which implies rqp .

Proof of part 2: pqr implies $pq + qr = pr$. Assume qpr . Then $pq + pr = qr$, which implies that $pr = qr - pq = pq + qr$, which means that $2pq = 0$, so $pq = 0$ and $p=q$, a contradiction. Therefore qpr is false. Similarly, prq is false.

Proof of part 3: Let pqr and prs be given. Then $pq + qr = pr$ and $pr + rs = ps$. $pq + qr + rs = ps \leq pq + qs$ by the triangle inequality of the metric (property III). Subtraction gives $qr + rs \leq qs$, but again by the triangle inequality, $qr + rs \geq qs$, so that $qr + rs = qs$, which implies qrs . Substituting qs for

$qr + rs$ in $pq + qr + rs = ps$ above gives $pq + qs = ps$, and so qrs is shown.

Proof of part 4: Let a be an accumulation point (or limit point) of $\bar{B}(p,q)$, with $p \neq a \neq q$. Then $a = \lim_{n \rightarrow \infty} a_n$ for some sequence a_n , where a_n is an element of $\bar{B}(p,q)$ for every natural number n . To show that $\bar{B}(p,q)$ is closed, it must be established that a is an element of $B(p,q)$. Since a_n is an element of $\bar{B}(p,q)$ for all n , $pa_n + a_nq = pq$ for all n . By the continuity of the metric, we can take limits of both sides of the equation as $n \rightarrow \infty$, to give $\lim_{n \rightarrow \infty} (pa_n + a_nq) = pa + aq = \lim_{n \rightarrow \infty} pq = pq$. Therefore paq holds and so a is in $B(p,q)$. Hence $B(p,q)$ is a closed set.

The proof of part 5 is left to the reader.

For our purposes another definition is required.

Definition

If p and q are elements of M and p' and q' are elements of M' where M and M' are metric spaces, then the pair (p,q) is *congruent* to (p',q') (abbreviated $(p,q) \equiv (p',q')$) if and only if $pq = p'q'$.

Two subsets A and B of the same or of different metric spaces are *congruent* provided that there exists a one-to-one mapping f from A onto B such that each pair of points in A is mapped to a congruent pair of

points in B. This is denoted by $A \equiv_f B$. The mapping f is called an *isometry* of A onto B.

An isometry or congruence f of A onto B is biuniform, since if r and s are elements of A then $rs = f(r)f(s)$ and so $f(r) = f(s)$ if and only if $r = s$. If $A \equiv_f B$ then $B \equiv_{f^{-1}} A$, so $B \equiv_{f^{-1}} A$ congruence is *symmetric*. If $A \equiv_f B$ and $B \equiv_g C$, then $A \equiv_{f \circ g} C$, so congruence is *transitive*. *Reflexivity* is obvious, therefore congruence as defined above is an equivalence relation. Both f and f^{-1} can be shown to be continuous, so congruence is also a *homeomorphism*.

The notation $(p_1, p_2, p_3, \dots, p_n) \equiv (p'_1, p'_2, p'_3, \dots, p'_n)$ denotes that for each pair i and j elements of $\{1, 2, 3, \dots, n\}$, $(p_i, p_j) \equiv (p'_i, p'_j)$. Now we can state the sixth property of metric betweenness:

Theorem

In a metric space, betweenness satisfies

6) *congruence invariance*: if $(p, q, r) \equiv (p', q', r')$, then pqr implies $p'q'r'$.

Proof: $pq = p'q'$, $pr = p'r'$, $qr = q'r'$. Therefore, $p'r' = pr = pq + qr = p'q' + q'r'$, which implies $p'q'r'$.

It can be shown that these six properties are sufficient to 'characterize' betweenness. That is, for any

relation R defined on 3-tuples of a metric space which satisfies the properties 1-6 above, we have $R(p,q,r)$ iff pqr . This task is left to the interested reader.

BIBLIOGRAPHY

- Aronsjan, N. "Neuer Beweis der Streckenverbundenheit vollständiger konvexer Räume", Ergebnisse eines mathematischen Kolloquiums (Wien), 6 (1935), 45-56.
- Blumenthal, L. M. Theory and Applications of Distance Geometry, Oxford University Press, London, 1953.
- _____, "Distance Geometries:", University of Missouri Studies, 13 (1938), no. 2.
- _____, "A New Characterization of the Straight Line", Reports of a Mathematical Colloquium, Second Series, Issue 2 (1940), 25-27.
- _____, "New Characterizations of Segments and Arcs", Proceedings of the National Academy of Sciences, 29 (1943), 107-9.
- Menger, K. "On Linear Sets in Metric Spaces", Reports of a Mathematical Colloquium Second Series, Issue 1 (1939), 16-17.
- Wald, A. "Axiomatik des Zwischenbegriffes in metrischen Räumen", Mathematische Annalen, 104 (1931), 476-84.

PASCAL'S TETRAHEDRON AND THE TRINOMIAL COEFFICIENTS

Janet Shorter and F. Max Stein
Colorado State University

1. Introduction. The natural first step in trying to generalize the coefficients of the binomial expansion is to consider the coefficients which arise in the expansion of the trinomial. While in the case of the binomial expansion the coefficients can be arranged in a triangular array, Pascal's triangle, we shall show, among other things, that for the trinomial expansion the coefficients can be arranged in what we shall call Pascal's tetrahedron.

Rather than attempt to include the proofs of all of the relationships between the various coefficients that arise, we shall merely state these properties and relationships.

2. Coefficients in the binomial expansion. The coefficients in the expansion of $(a + b)^n$ can be arranged in a triangular array (Figure 1) called Pascal's triangle.

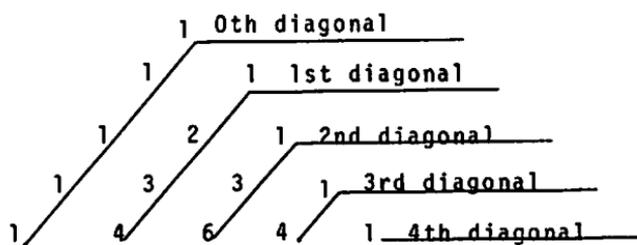
Prepared in an Undergraduate Research Participation Program at Colorado State University by Miss Shorter under the direction of Professor Stein.

0th row								$(a + b)^0$
1st row								$(a + b)^1$
2nd row								$(a + b)^2$
3rd row								$(a + b)^3$
4th row								$(a + b)^4$
5th row								$(a + b)^5$

Pascal's Triangle

Figure 1

In Pascal's triangle any given entry is obtained by adding the two entries which are above it in the preceding row. Except for entries above the vertex entry, all entries outside the triangle are considered to be zero. Note in Figure 2 that



Diagonals of Pascal's Triangle

Figure 2

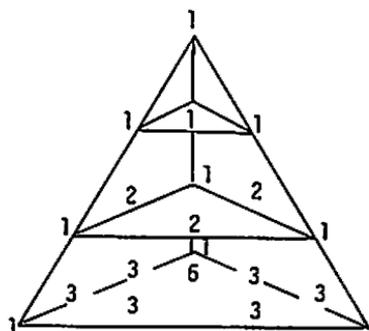
the 0th diagonal consists entirely of 1's. The 1st diagonal consists of the integers from 1 to m ; the m th entry on this diagonal is the sum of the first m entries on the 0th diagonal.

The elements on the 2nd diagonal are the triangular numbers $1, 3(=1+2), 6(=1+2+3), \dots, T_m = \frac{m(m+1)}{2}$. The m -th entry on this diagonal can be found by summing the first m entries on the preceding diagonal.

The 3rd diagonal is made up of the tetrahedral, or pyramidal, numbers $1, 4(=1+3), 10(=1+3+6)$, etc. The m th entry on the 3rd diagonal is the sum of the m triangular numbers on the 2nd diagonal. This procedure can be extended indefinitely to each new diagonal, and consequently we define generalized triangular numbers, see [1], by T_{mj} in which m is the number of the entry in the j th diagonal. Using this notation, the diagonal numbers of Pascal's triangle become:

$$\begin{aligned}
 T_{m0} &= 1 \quad \text{for all } m, \\
 T_{m1} &= \sum_{k=1}^m T_{k0} = m, \\
 T_{m2} &= \sum_{k=1}^m T_{k1}, \\
 &\dots \\
 T_{mj} &= \sum_{k=1}^m T_{k,j-1}.
 \end{aligned}
 \tag{1}$$

3. A model for the trinomial expansion. We now wish to consider the coefficients in the trinomial expansion, $(a + b + c)^n$, to see if we can find relationships similar to those for the binomial expansion. In the case of a trinomial we shall introduce the coefficients in a trigonal planar arrangement and stack them in layers to form a tetrahedron (Figure 3). Figure 4 shows the position of the coefficients in each layer.



Pascal's Tetrahedron

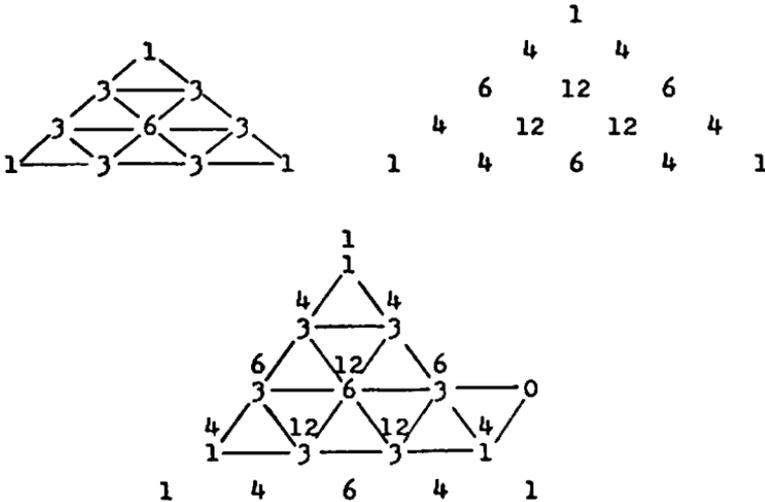
Figure 3

$$\begin{array}{rcccc}
 & & & & 1 \\
 & & & & (a + b + c)^0 = 1 \\
 & & & 1 & \\
 & & & 1 & (a + b + c)^1 = a + b + c \\
 1 & & & 1 & \\
 & & & 1 & a^2 \\
 & & & 2 & 2 & (a + b + c)^2 = & + 2ab + 2ac \\
 & & & 1 & 2 & 1 & + b^2 + 2bc + c^2 \\
 & & & 1 & & & a^3 \\
 & & & 3 & 3 & (a + b + c)^3 = & + 3a^2b + 3a^2c \\
 & & & 3 & 6 & 3 & + 3ab^2 + 6abc + 3ac^2 \\
 1 & & & 3 & 3 & 1 & + b^3 + 3b^2c + 3bc^2 + c^3 \\
 & & & 1 & & & a^4 \\
 & & & 4 & 4 & (a + b + c)^4 = & + 4a^3b + 4a^3c \\
 & & & 6 & 12 & 6 & + 6a^2b^2 + 12a^2bc + 6a^2c^2 \\
 & & & 4 & 12 & 12 & 4 & + 4ab^3 + 12ab^2c + 12abc^2 + 4ac^3 \\
 1 & & & 4 & 6 & 4 & 1 & + b^4 + 4b^3c + 6b^2c^2 + 4bc^3 + c^4
 \end{array}$$

Layers of Pascal's Tetrahedron

Figure 4

The entries in any given layer in Figure 4 can be obtained from those in the preceding layer in a manner similar to that used in obtaining entries in a Pascal's triangle, see Figure 1. Note also that we have a Pascal's triangle on each of the three faces; the edges are simply the binomial coefficients. Any entry in the interior of our tetrahedron, however, is obtained by adding the three entries at the vertices of the equilateral triangle directly above it in the preceding layer. For example (Figure 5) when layer 3 is superimposed on layer 4, each 12 in layer 4 appears in a triangle of entries in layer 3 with 3, 3, and 6 at the vertices. The sum of these three entries in layer 3 gives the entry in layer 4. Except for entries above the vertex entry, all entries outside the tetrahedron are considered to be zero. With this in mind, the entries on the edge of any layer can be obtained in a manner similar to that used to obtain interior entries; e.g., the edge entries in layer 4 can be obtained from the entries in layer 3 and the zeros outside the tetrahedron. In Figure 4 an entry 6 is obtained from $3+3+0$ in layer 3, and an entry 4 is obtained from $1+3+0$ in layer 3.



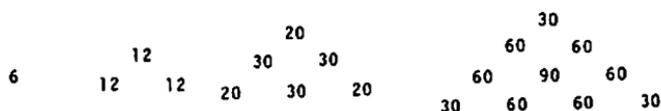
Layer 3 superimposed on layer 4

Figure 5

4. Nesting of tetrahedrons. If we look at the layers in Figure 4, we conjecture that only in every 3rd layer, starting with the 0th layer, do we get a number occurring directly in the center of the layer; i.e., at the centroid of the triangle of the layer. A number will occur in this position only when the powers of $a, b,$ and c in the expansion of $(a + b + c)^n$ are equal. For

example, we get $a^0b^0c^0$ in the 0th layer, $6abc$ in the 3rd layer, $90a^2b^2c^2$ in the 6th layer, etc. The proof of this conjecture depends on the fact that the number appearing in the center of a layer corresponds to the coefficient in the term in the expansion of $(a + b + c)^n$ in which the exponents are all equal. This implies that in the term $a^{n/3}b^{n/3}c^{n/3}$, n must be divisible by 3, and this will occur only for every third layer.

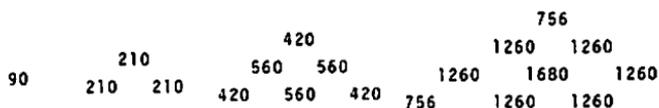
If we remove the Pascal's triangles (the faces) from our tetrahedron, we are left with the following interior numbers starting with the third layer; see Figure 6. These can be



First Tetrahedron

Figure 6

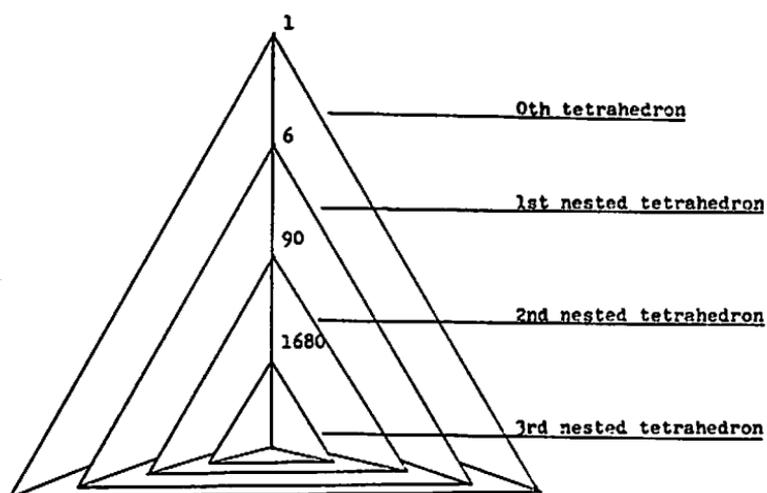
stacked to form another tetrahedron with 6 at its vertex. If we repeat the preceding process with this tetrahedron and remove the outer numbers from each layer, we are left with the following triangular arrays; see Figure 7. Here again,



Second Tetrahedron

Figure 7

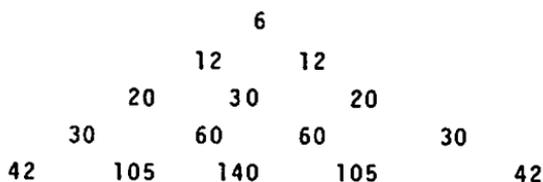
if we stack these layers, we find they will form a new tetrahedron with 90 at its vertex. We continue this process with a new interior tetrahedron being started every 3rd layer. We can, therefore, think of our model as a nesting of tetrahedrons. We will number these tetrahedrons as follows: let the 0th tetrahedron have 1 at its vertex, let the 1st tetrahedron have 6 at its vertex, etc.; see Figure 8. As we shall see later in (7), the nesting of tetrahedrons helps us to write a formula for finding any entry in our tetrahedral array—that is, any coefficient in the expansion of $(a+b+c)^n$.



Nested Tetrahedrons

Figure 8

5. Diagonals of the faces of the nested tetrahedrons. In Figure 9 we have a side view of one of the faces of the 1st nested tetrahedron in which the vertex element 6 is in the 3rd layer of the 0th tetrahedron, see Figure 3. In the left most



Face of First Tetrahedron

Figure 9

diagonal we note that 2 is a common factor. Removing the factor 2 we are left with 3,6,10,15,21,..., which we recognize as our triangular numbers $T_{m/2}$ for $m = 2,3,\dots$; see (1), where m is the diagonal number in Figure 2. That is, any entry on the outside diagonal of the 1st tetrahedron, the tetrahedron whose vertex entry, 6, is in the third layer, is of the form

$$(2) \quad H_{1m3} = 2 T_{m/2} = 2 \sum_{k=1}^{m/2} T_{k1} = T_{2,1} \sum_{k=1}^{m/2} T_{k1}, \quad m=2,3,\dots$$

In H_{1m3} the first subscript represents the number of the tetrahedron, the second subscript represents the number of the entry with reference to its position in the 0th tetrahedron, and the third indicates the number of the layer in the 0th tetrahedron.

The entries in the next diagonal (which start in the 4th layer of the 0th tetrahedron) have 3 as a common factor. Removing the factor 3 we are left with 4, 10, 20, 35, ..., the generalized triangular numbers

T_{m3} for $m = 2, 3, \dots$, where m is again the diagonal number in Figure 2. Therefore, any entry in this diagonal, which starts in the 4th layer of the 0th tetrahedron, is of the form

$$(3) \quad H_{1m4} = 3 T_{m3} = 3 \sum_{k=1}^m T_{k2} = T_{3,1} \sum_{k=1}^m T_{k2}, \quad m = 2, 3, \dots$$

The same procedure can be repeated for each diagonal with the m th entry on the diagonal which starts in the j th layer being given by

$$(4) \quad H_{1mj} = T_{j-1,1} T_{m,j-1} = T_{j-1,1} \sum_{k=1}^m T_{k,j-2}, \quad m = 2, 3, \dots$$

Similar patterns can also be found in the diagonals of the 2nd tetrahedron; see Figure 10. The entry 90 is in the

			90		
		210		210	
	420		560		420
756		1260		1260	756
1260	2520		3150		2520
					1260

Face of the Second Tetrahedron

Figure 10

6th layer of the 0th tetrahedron, and the entries in the left most diagonal of this face have 6 as a common factor. After factoring out the 6 we have remaining 15,35,70,126,210,..., the generalized triangular numbers T_{m4} for $m = 3,4,\dots$, where m is the diagonal number in Figure 2. Therefore, any entry on this diagonal can be represented as

$$(5) \quad H_{2m6} = 6 T_{m4} = 6 \sum_{k=1}^m T_{k3} = T_{3,2} \sum_{k=1}^m T_{k3}, \quad m = 3,4,\dots$$

The m th entry on subsequent diagonals of the second tetrahedron is given by

$$(6) \quad H_{2mj} = T_{j-3,2} T_{m,j-3} = T_{j-3,2} \sum_{k=1}^m T_{k,j-4}, \quad m = p+1, p+2, \dots,$$

where p is the number of the tetrahedron, m is the diagonal number in Figure 2, and j is the layer of the 0th tetrahedron.

In general for the p th tetrahedron, the m th entry on the diagonal starting at the j th level of the 0th tetrahedron can be represented by

$$(7) \quad H_{pmj} = T_{j-2p+1,p} \sum_{k=1}^m T_{k,j-p-1}, \quad m = p+1, p+2, \dots$$

Equation (7) is a formula for finding any coefficient in the expansion of $(a+b+c)^n$; see Figure 4.

8. Conclusion. In the preceding discussion we have attempted to indicate how the coefficients in the expansion of a trinomial can be arranged on a Pascal's tetrahedron. While the coefficients in Pascal's triangle lead to triangular and generalized triangular numbers, the coefficients in Pascal's tetrahedron can be arranged on nested tetrahedrons and lead to more generalized sets of numbers. Little attempt is made to prove the statements made; to do so would make the discussion quite lengthy and tedious. The reader is encouraged to verify the accuracy of the conjectures.

One might expect that extensions of the preceding discussion would lead to Pascal's pyramid with four or more faces, the entries on the faces being Pascal's triangles or Pascal's tetrahedrons. Also one might conjecture that there exist relationships involving generalized triangular numbers, nested pyramids, etc., similar to those for Pascal's tetrahedron. Although such conjectures are interesting to contemplate, one soon encounters difficulties when trying to establish such generalizations. One can get a hint of the complications by expanding $(a+b+c+d)^3$. The study of these relations is beyond the scope of this paper.

Additional information on Pascal's tetrahedron can be found in [3] and [4].

REFERENCES

- [1] J. Miller and F. Max Stein, A Generalization of Triangular Numbers, The Math. Teacher, Vol. LVI (1963), pp. 414-418.
- [2] E. B. Rosenthal, A Pascal Pyramid for Trinomial Coefficients, The Math. Teacher, Vol. LIV (1961), pp. 336-338.
- [3] N. Ya Vilenkin, Combinatorics, Academic Press, New York, 1971, pp. 97-98.
- [4] R. L. Keeney, On the Trinomial Coefficients, Math. Mag., Vol. 42 (1969), pp. 210-212.

THE PROBLEM CORNER

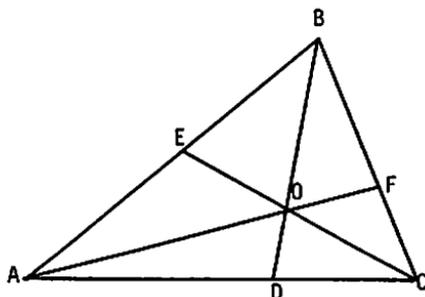
EDITED BY KENNETH M. WILKE

The Problem Corner invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before 1 August, 1983. The solutions will be published in the Fall 1983 issue of *The Pentagon*, with credit being given to student solutions. Affirmation of student status and school should be included with solutions. Address all communications to Kenneth M. Wilke, Department of Mathematics, 275 Morgan Hall, Washburn University, Topeka, Kansas 66621.

PROPOSED PROBLEMS

Problem 347: Proposed by Roger Izard, Dallas, Texas.

In the figure, line segments BD , AF and EC bisect the angles of triangle ABC . Also $BE^2 \cdot AO^2 = 3 \cdot BC^2 \cdot EO^2$. Prove that $EO = OD$.



Problem 348: Proposed by Charles W. Trigg, San Diego, California.

Martin Gardner ("Mathematical Games," Scientific

American, April 1964, page 135) showed that 999 is the minimum sum of three three-digit primes composed of the nine non-zero digits. With the nine non-zero digits form other trios of three-digit primes with sums that contain three like digits.

Problem 349: Proposed by Fred A. Miller, Elkins, West Virginia.

Prove that the area of a right triangle is equal to the product of the segments determined on the hypotenuse by the inscribed circle.

Problem 350: Proposed by Charles W. Trigg, San Diego, California.

A paper rectangle is five times as long as it is wide. With two cuts by scissors, dissect the rectangle into pieces that can be assembled into a square.

Problem 351: Proposed by Bruce Sommer, University of Wisconsin-Rock County Campus, Janesville, Wisconsin.

Consider a tangent to the curve $x^n + y^n = A^n$ where $n \neq 0$ or 1 . Suppose that this tangent intersects the x axis at P and the y axis at Q .

Show that $\frac{1}{p^{n/n-1}} + \frac{1}{q^{n/n-1}} = \frac{1}{A^{n/n-1}}$.

SOLUTIONS

337. Proposed by Dmitry P. Mavlo, Moscow, USSR.

(a) Find all solutions of the equation

$$\sqrt[3]{a_1 a_2 a_3 a_4} = a_1 + a_2 + a_3 + a_4$$

where a_1, a_2, a_3, a_4 are natural numbers or zero, $a_1 > 0$, and $a_1 a_2 a_3 a_4$ is a four digit number in the decimal system.

(b) Generalize the problem in part (a) to n variables.

Composite of solutions submitted by Joseph E. Bonin, Assumption College, Worcester, Massachusetts and Charles W. Trigg, San Diego, California.

(a) By checking a table of cubes, one finds that the only solutions are:

$$\sqrt[3]{4913} = 4 + 9 + 1 + 3 \text{ and } \sqrt[3]{5832} = 5 + 8 + 3 + 2.$$

(b) If the cube root of an integer equals the sum of its digits, then the number and the cube root have the same digital root; i.e. the cube root has one of the forms $9k-1$, $9k$ or $9k+1$. Then since the sum of the digits of the cube root cannot exceed 54, one needs only check the cubes 1, 8, 9, 10, 17, 18, 19, 26, 27, 28, 35, 36, 37, 44, 45, 46, 53 and 54. The only solutions not found in part (a) are:

$$\sqrt[3]{1} = 1, \sqrt[3]{512} = 5 + 1 + 2, \sqrt[3]{17576} = 1 + 7 + 5 + 7 + 6, \text{ and } \sqrt[3]{19683} = 1 + 9 + 6 + 8 + 3.$$

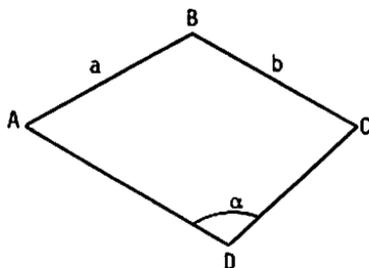
For $n \geq 7$, note that since $a_1 \neq 0$, the smallest possible value for $\sqrt[3]{a_1 a_2 \dots a_n}$ is $\sqrt[3]{10^{n-1}}$ while the largest possible value of $a_1 + a_2 + \dots + a_n$ is $9n$. An easy induction shows that $10^{n-1} \geq (9n)^3 \geq 9n$ for all integers $n \geq 7$. Hence the solutions given above are the only ones.

Also solved by Clayton W. Dodge, University of Maine at Orono, Orono, Maine and the proposer.

338. *Proposed by Dmitry P. Mavlo, Moscow, USSR.*

Given the angle $\angle ADC = \alpha$ and the two opposite sides $AB = a$ and $BC = b$ in quadrangle $ABCD$, as shown in the figure,

- find the quadrangle having the maximum area;
- express this maximum area in terms of a, b and α ; and
- give the Euclidean construction of this quadrangle of maximum area.



Solution by Charles W. Trigg, San Diego, California.

Let $\angle ABC = \theta$ and $AC = c$.

(a) Whatever the magnitude of θ , the area of triangle ACD will be a maximum when it is isosceles with altitude, $DE = \frac{c}{2} \cot \frac{\alpha}{2}$ and $AD = DC = \frac{c}{2} \csc \frac{\alpha}{2}$. In triangle ABC , the area is $\frac{1}{2} ab \sin \theta$ and $c^2 = a^2 + b^2 - 2ab \cos \theta$. Hence the area of quadrilateral $ABCD$ is

$$\begin{aligned} T &= \frac{1}{2} ab \sin \theta + \frac{1}{4} c^2 \cot \frac{\alpha}{2} \\ &= \frac{1}{2} [ab \sin \theta + \frac{1}{2} \cot \frac{\alpha}{2} (a^2 + b^2 - 2ab \cos \theta)] \\ &= \frac{1}{4} (a^2 + b^2) \cot \frac{\alpha}{2} + \frac{ab}{2} (\sin \theta - \cot \frac{\alpha}{2} \cos \theta). \end{aligned}$$

Now differentiating and equating $dT/d\theta$ to zero, we have

$$\frac{ab}{2} (\cos \theta + \cot \frac{\alpha}{2} \sin \theta) = 0.$$

Hence $\tan \theta = \frac{\sin \theta}{\cos \theta} = -\tan \frac{\alpha}{2}$, and $\theta = 180^\circ - \frac{\alpha}{2}$ which defines the quadrilateral with the maximum area.

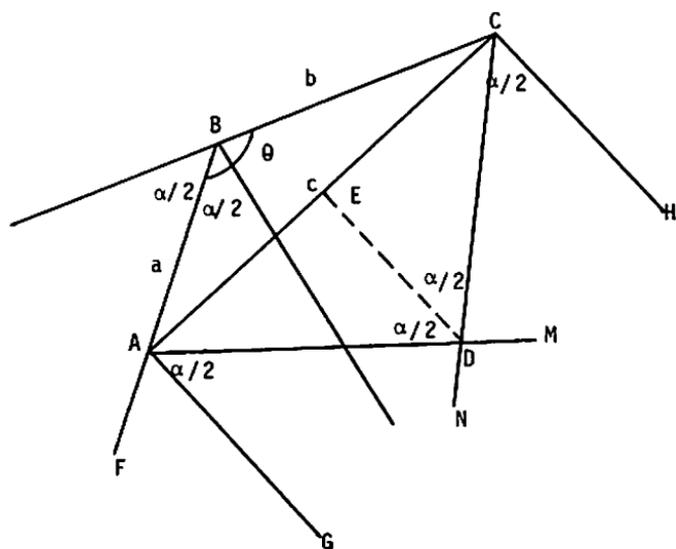
$$\begin{aligned} \text{(b) } \cos \theta &= 1/\sec \theta = 1/\sqrt{1 + \tan^2 \theta} = 1/\sqrt{1 + \tan^2 \frac{\alpha}{2}} \\ &= -\cos \frac{\alpha}{2}, \text{ whereupon } \sin \theta = \sin \frac{\alpha}{2}. \end{aligned}$$

It follows that the area of the quadrilateral is

$$T = \frac{1}{2}ab \sin \frac{\alpha}{2} + \frac{1}{4} (a^2 + b^2 + 2ab \cos \frac{\alpha}{2}) \cot \frac{\alpha}{2}$$

$$= ab \sin \frac{\alpha}{2} + \frac{1}{4} (a^2 + b^2) \cot \frac{\alpha}{2}.$$

(c) On a line, locate B and C so that $BC = b$. With CB extended as a side and using B as a vertex, construct an angle $= \alpha$. Bisect this angle with bisector BF. Lay off $BA = a$. Draw AC. On the side remote from B erect perpendiculars AG and CH to AC. With A and C as vertices and AG and CH, respectively, draw AM and CN, respectively, as sides, draw AM and CN between the parallels AG and CH so that $\angle GAM = \frac{\alpha}{2} = \angle HCN$. as in the figure.

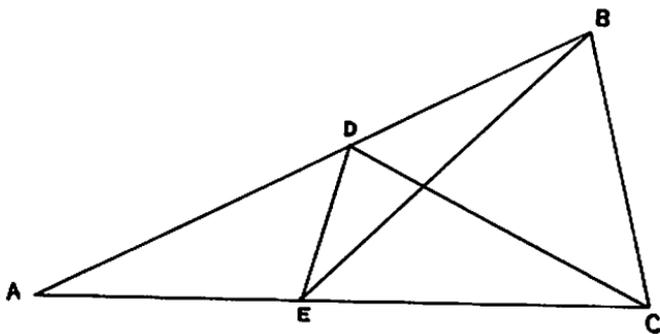


Denote the intersection of AM and CN as D . Then $ABCD$ is the desired quadrilateral with maximum area. The proof is obvious.

Also solved by the proposer.

339. *Proposed by Fred A. Miller, Elkins, West Virginia.*

In triangle ABC as shown in the figure, $AB = AC$, $\angle B = \angle C = 80^\circ$, $\angle BCD = 50^\circ$, and $\angle BEC = 40^\circ$. Find $\angle DEB$.



Solution by Clayton W. Dodge, University of Maine at Orono, Orono, Maine.

By subtraction we find that $\angle BDC = \angle BCD = 50^\circ$, so $BC = BD$. We apply the law of sines to triangles BCE and BDE to get

$$\frac{\sin 40^\circ}{BC} = \frac{\sin 80^\circ}{BE} \quad \text{and} \quad \frac{\sin 130^\circ}{BE} = \frac{\sin \angle DEB}{DB}$$

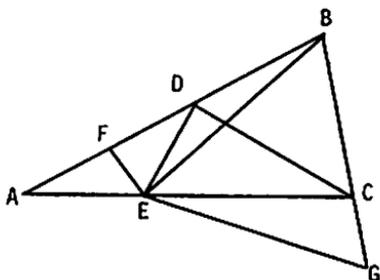
$$\text{so } \frac{BE}{BC} = \frac{BE}{BD} = \frac{\sin 80^\circ}{\sin 40^\circ} = \frac{\sin 130^\circ}{\sin \angle DEB}.$$

Then since $\sin 80^\circ = 2 \sin 40^\circ \cos 40^\circ$ and $\sin 130^\circ = \sin 50^\circ = \cos 40^\circ$, we obtain

$$\sin \angle DEB = \frac{\sin 40^\circ \sin 130^\circ}{\sin 80^\circ} = \frac{\sin 40^\circ \cos 40^\circ}{2 \sin 40^\circ \cos 40^\circ} = \frac{1}{2},$$

so $\angle DEB = 30^\circ$.

Solution by Oscar Castaneda, Edgewood High School, San Antonio, Texas.



On BA, extend BD to a point F such that $BF = BE$. Extend BC to a point G such that $BG = BF$. Draw EF and GE. From the angles whose sizes are given and since $AB = AC$ and $BE = BF$, one finds immediately that $\angle EBC = 60^\circ$,

$\angle DCE = 30^\circ$, $\angle EBF = 20^\circ$, $\angle BDC = 50^\circ$, and $BFE = BEF$. Hence triangles BCD and BEG are isosceles; in fact triangle BEG is equiangular since $\angle EBG = 60^\circ$. Hence $\angle GEC = \angle BAC = \angle EBA = 20^\circ$ and $AE = BE = EG$. Then triangles AEF and ECG are congruent, and $BF = BG$ and $BD = BC, GC = FD = FE$. Hence $\angle DEF = 50^\circ$ and $\angle DEB = 30^\circ$.

Also solved by Charles W. Trigg, San Diego, California (two solutions) and the proposer.

340. *Proposed by Charles W. Trigg, San Diego, California.*

Find all two-digit integers in all bases that are three times the number formed by reversing the digits.

Solution by Clayton W. Dodge, University of Maine at Orono, Orono, Maine.

Let b be the base and x and y the digits of the two-digit number. Then we seek integral solutions to the equation

$$xb + y = 3(yb + x) \text{ for } 0 < x, y < b. \text{ Hence } y = \frac{(b-3)x}{3b-1}.$$

There is a solution when $(b-3)/(3b-1)$ reduces by a factor of at least 4. In such cases, x is a multiple of $3b-1$ and y is the corresponding multiple of $b-3$. By the Euclidean Algorithm, $\gcd(b-3, 3b-1) = \gcd(b-3, 8)$ so the only possible common factors of $b-3$ and $3b-1$ are 4 and 8. Setting $b = 8k + r$ for some integers k and r and noting that $b-3$ is divisible by 4 for $r = 3$

or 7 we have: $\frac{b-3}{3b-1} = \frac{k}{3k+1}$ for $r = 3$ and $\frac{b-3}{3b+1} = \frac{2k+1}{6k+5}$ for $r = 7$. In the first case $y = 3k+1$ and $x = k$ or $y = 6k+2$ and $x = 2k$; in the second case $x = 2k+1$. Several solutions are listed below.

k	b=8k+3	y=3k+1	x=k	y=6k+2	x=2k	b=8k+7	y=6k+5	x=2k+1
0						7	5	1
1	11	4	1	8	2	15	11	3
2	19	7	2	14	2	23	17	5
3	27	10	3	20	6	31	23	7
4	35	13	4	26	8	39	29	9
5	43	16	5	32	10	47	35	11

Also solved by: Joseph E. Bonin, Assumption College, Worcester, Massachusetts; Fred A. Miller, Elkins, West Virginia; Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin and the proposer.

341. Proposed by Charles W. Trigg, San Diego, California.

Can each x in the diagram be replaced by an odd digit in such a fashion that the three digits on each side of the pentagon form a prime when read in either direction using ten distinct primes?

Solution by the proposer.

There are seven three-digit primes composed of odd digits whose reverses are also primes, namely: 113, 157, 179, 199, 337, 359 and 739. Among these there are three each of end 3's and 7's. Thus 337 is one side,

or there are the pairings 11359 and 15739 which would require the impossibility of a fifth side having like end digits. There are eight possible arrays; the four shown below and their reflections.

3 3 7	3 3 7	3 3 7	3 3 7
1 5	1 5	5 3	5 3
1 1 1	1 9 1	9 9 9	9 9 9
9 7	7 9 9	7 9 9	9 7
9	9	1	1

Also solved by: Joseph E. Bonin, Assumption College, Worcester, Massachusetts; Clayton W. Dodge, University of Maine at Orono, Orono, Maine; Fred A. Miller, Elkins, West Virginia; and Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin.

THE MATHEMATICAL SCRAPBOOK

EDITED BY RICHARD LEE BARLOW

Readers are encouraged to submit Scrapbook material directly to the Scrapbook editor. Material will be used where possible and acknowledgement will be made in THE PENTAGON. If your chapter of Kappa Mu Epsilon would like to contribute the entire Scrapbook section as a chapter project, please contact the Scrapbook editor: Dr. Richard L. Barlow, Department of Mathematics, Kearney State College, Kearney, NE 68849.

This edition of The Scrapbook contains numerous classical mathematical puzzles and games for your enjoyment. If you have any additional items of this type, please send them to the Scrapbook editor for inclusion in future issues of the Scrapbook section. Answers to selected problems will appear in the next edition of the Scrapbook.

A TRANSPORTATION PROBLEM

Jack brought a fox, a goose, and a sack of grain to the river. He wanted to take these three items to the other side, but the only available boat was just big enough so that he could take one item across at a time in addition to himself. If he left the fox and the goose alone, the fox would eat the goose. If he left the goose and the grain alone, the goose would eat the grain. How could he manage to take everything to the other side of the river so that the fox would not eat the goose and the goose would not eat the grain? We shall assume that neither the fox nor the goose will run away if left alone, and that the fox will not eat the grain. Jack may take more than 3 trips across the river if needed.

* * * * *

A BIRTHDAY PROBLEM

To guess somebody's birthday, tell that person to keep in mind two numbers: the number of the month and the number of the day of the month on which that person was born. The months are numbered in the usual way, from 1 to 12, beginning with January.

Now give the following directions: "Multiply the number of the month in which you were born by 5. Add 6. Multiply by 4. Add 9. Multiply by 5. Now add the number of the day on which you were born." When the calculations are finished, ask for the final result.

Now, silently (mentally) subtract 165 from the result. After subtracting, the last two digits tell you the days and the other digits tell you the month of the birthday. For example, if the result was 989, you subtract 165 and get 824. So you know he was born on the 24th day of the 8th month.

This trick is based on the fact that our numbers are written in the ten-scale. The directions you give to the person are a disguised way of adding 165, the day number, and 100 times the month number. When you subtract 165, the day number and 100 times the month number are left: Can you verify this algebraically?

* * * * *

A CHECKERBOARD PROBLEM

On a checkerboard, two diagonally opposite corner squares are removed. Can you cover the remaining 62 squares using domino pieces each covering two squares? Why?

* * * * *

A GRAPHING PROBLEM

Graph the following line segments, all on the same rectangular coordinate system.

$$\{(x,y) \mid x = 2, -2 < y < 2\}$$

$$\{(x,y) \mid y = 0, -5/2 < x < -3/2\}$$

$$\{(x,y) \mid x = -7, -2 < y < 2\}$$

$$\{(x,y) \mid y = 4x + 10, -3 < x < -2\}$$

$$\{(x,y) \mid x = 4, -2 < y < 2\}$$

$$\{(x,y) \mid y = 0, 4 < x < 6\}$$

$$\{(x,y) \mid y = -4x/3 - 22/3 \text{ and } -7 < x < -11/2\}$$

$$\{(x,y) \mid x = -4, -2 < y < 2\}$$

$$\{(x,y) \mid y = 2, 1 < x < 3\}$$

$$\{(x,y) \mid y = 4x/3 + 22/3 \text{ and } -11/2 < x < -4\}$$

$$\{(x,y) \mid y = -4x - 6, -2 < x < -1\}$$

$$\{(x,y) \mid x = 6, -2 < y < 2\}$$

* * * * *

VARIABLE VALUE PROBLEMS

If $\frac{\text{PORK}}{\text{CHOP}} = C$, and C is greater than 2, what different numbers do PORK and CHOP represent? None of the letters stands for zero, and the letters represent digits.

If NIPS and S=5, P=4, T=3, N≠1, evaluate QUICK. $\frac{+QUIT}{QUICK}$

If the number ABCDE x 4 = EDCBA, what different digits do these five letters represent?

* * * * *

HAT COLOR PROBLEM

Three men are instructed to stand in a straight line and to look straight ahead. (The man at the back of the line can see both other men, the center man can see only the front man, and the man in front can see neither of the others.) Three hats are then taken from a box containing three black and two red hats. A hat is placed on each man's head. The men are then asked to figure out what color of hat they are wearing. The man at the back says he does not know what color his hat is, and the man in the center then says he does not know what color his hat is either. From this, the man in the front can determine what color hat he is wearing. What color is it?

* * * * *

ELEVATOR PROBLEM

A man lived on the tenth floor of an apartment house. Each morning he rode the elevator down to the main floor and went to work. He returned again each evening, and walked up the last three flights. The man had no friends on the seventh floor so why didn't he ride all the way to the tenth?

* * * * *

ANOTHER BOAT PROBLEM

A boat was sitting in a bay. A 4 meter ladder hangs over its side and four of its rungs were under water. In another hour the tide will come in and the water will rise one meter. How many more rungs will now be submerged?

* * * * *

A SURPRISE RESULT PROBLEM

Multiply each of the following numbers by 7.

15,873	95,238
31,746	111,111
47,619	126,984
63,492	142,857
79,365	

The result...you'll be surprised!

* * * * *

A DIVERSION
MUSIC AND PYTHAGORAS

One of the most famous mathematicians of all times, Pythagoras, made several contributions to music. One of his discoveries proved that there was a connection between musical harmony and the whole numbers we count by -- 1, 2, 3, 4, 5, and so on. Pythagoras noticed that when you plucked a string, you obtained a note; but then if you would pluck an equally taut string twice as long as the first, you would hear a new note just one harmonic octave below the first. Continuing on with his new discovery, Pythagoras found that by starting with any string and the note sounded by it, you could go down the musical scale by increasing the length of the string according to simple fractions expressed as the ratios of whole numbers. For instance, $16/15$'s of a C string gives the next lower note B, $6/5$'s of it gives A, $4/3$'s of it gives G, $3/2$'s of it gives F, $8/5$'s of it gives E, $16/9$'s of it gives D, exactly 2 of it gives C again, but one octave lower. From these findings, Pythagoras was convinced that all harmony, all beauty, all nature could be expressed by whole-number relationships.

* * * * *

THE HEXAGON

EDITED BY IRAJ KALANTARI

This department of THE PENTAGON is intended to be a forum in which mathematical *issues* of interest to undergraduate students are discussed in length. Here by *issue* we mean the most general interpretation. Examination of books, puzzles, paradoxes and special problems, (all old or new) are examples. The plan is to examine only one issue each time. The hope is that the discussions would not be too technical and be entertaining. The readers are encouraged to write responses to the discussion and submit it to the editor of this department for inclusion in the next issue. The readers are also most encouraged to submit an essay on their own issue of interest for publication in THE HEXAGON department. Address all correspondence to *Iraj Kalantari, Mathematics Department, Western Illinois University, Macomb, Illinois 61455.*

Thoughts are expressible through language. Mathematical thoughts are studied within systems of formal languages. In a formal system, axioms are statements which are to reflect some indisputable aspect of the subject under study.

There are two other aspects which are most important about an axiom. Firstly, an axiom should be reasonably simple (according to some measure of complexity, for example the length of the statement). Secondly, an axiom should reflect, express or capture the intended meaning only. That is, no matter who interprets that axiom, the same basic idea emerges. It is difficult (at times even impossible) to find axioms satisfying all of these requirements.

The following article by Professor Ingrassia is about an intriguing case: axiom of infinity. A challenging problem is about searching for a simple expression which reflects the idea of infinite.

IK

CAN YOU CAPTURE INFINITY WITH A SHORT AXIOM?

Michael Ingrassia*

There's a sense in which it is unfortunate that Appel and Haken have proved the Four Color Theorem, that famous theorem which asserts that four colors suffice to color any map in the plane so that no two adjacent countries have the same color. The unfortunate aspect is that one can no longer point to the Four Color Theorem as an example of an easily-stated question which mathematicians don't know the answer to. Of course, there are many open problems in mathematics, but many of them require too much explanation before they become comprehensible or interesting. (For example, it is unknown if there are any non-trivial automorphisms of the upper semi-lattice of recursively enumerable degrees of unsolvability, but it would take many pages just to define all of the terms used, let alone to explain why the problem is so intriguing.) You know that no one has ever proved that the equation $x^n + y^n = z^n$ has no solutions in which $x, y,$ and z are positive and n is greater

*Professor Ingrassia received his Ph.D. in mathematics from University of Illinois at Urbana and is currently a member of the mathematics department of Western Illinois University. His interests include mathematical logic particularly recursion theory.

than 2; this is the problem known as Fermat's Last Theorem. You may have heard of Goldbach's conjecture, that every even number greater than 2 can be written as the sum of two primes. You may be able to think of a few other open problems; chances are they're questions of number theory.

In this article I'll show you an example of a fairly-easy-to-state open problem in logic, specifically, in quantification theory, which I think is pretty appealing. Let's start by investigating the truth of the statement

*There is an animal which has a horse's body
and has a horn on its head.*

Is that sentence true? (No, this is not the pressing open problem logicians would like to answer! We'll get to that later.) Let me make a seemingly outrageous claim. I claim the sentence is obviously true, because it's just the French for "Water boils at 212 degrees Fahrenheit." What's that? You object? Just my luck to be writing for someone who knows French. Okay, I lied. The sentence is actually Ugaritic for "Water boils at 212 degrees Fahrenheit." Ha! Got you there! You don't know Ugaritic, do you? Ah, but you still object that the sentence looks like English to you and the original

problem of ascertaining its truth still remains (assuming the words have their usual English meanings).

But it's not as simple as insisting that the sentence is in English. There's still room for argument. What do we mean by "animal"? What does "has a horse's body" mean? Must one have the whole body or is it enough to own a controlling share? Do we count a horse owner who wears a trumpet instead of a hat? Some words leave little scope for argument, however, such as 'and'. 'And' means 'and'. We don't want to allow 'and' to mean 'but it's not true that', because that would certainly change our investigation of whether the sentence is true. In a basic logic course one learns systematically to separate the *logical* parts of a sentence from the *non-logical* parts. In fact one would learn to write the sentence symbolically as something like

$$(\exists x)(Ax \wedge HBx \wedge HHx)$$

Here ' \exists ' and ' \wedge ' are logical symbols meaning 'there exists' and 'and' respectively. There's no arguing the meaning of these symbols. x is a variable, just as in algebra, and the parentheses are a sort of punctuation. The other symbols, 'A', 'HB' and 'HH' stand for predicates; they express properties which may or may not be true about x . We intend them to mean 'is an animal',

'has a horse's body', and 'has a horn on its head' respectively. But there's scope for argument here, as indicated above.

Our judgment of whether the sentence is true or not depends on just how we define the predicates. To give an extreme example, one might argue that the sentence is true because 'A' should mean 'is an animated cartoon character', 'HB' should mean 'appeared in a Hanna-Barbera cartoon, and 'HH' should mean 'costarred with Huckleberry Hound'. The sentence thus interpreted says

There is an animated cartoon character who appeared in a Hanna-Barbera cartoon and costarred with Huckleberry Hound.

This is of course true. Who could forget Yogi Bear?

There's a final hurdle even if we decide on the obvious intended interpretations of A, HH, and HB, for example insisting that 'HH' means 'has a horse's body' in the sense of 'is physiologically the same as a horse from the neck down'. For do we allow unicorns? Unicorns by definition have a horse's body and have a horn on their head. But unicorns are fictional. If they count, the sentence is true. If not, the sentence is false. So before we can tell whether a sentence is

true or false, we need to know the domain of discourse-- just what sorts of things we are talking about. As a numerical example, consider

There is a number whose square is -1

or, symbolically,

$$(\exists x)(\text{Sqmo } x).$$

Let the predicate statement $\text{Sqmo } x$ mean 'the square of x is minus one' as in ordinary mathematics. The sentence is true if our domain of discourse is the complex numbers, but false if our domain of discourse is the real numbers.

So, to determine the truth of a sentence written symbolically, we need to fix an interpretation.

An *interpretation* consists of the following:

- (1) Choice for the domain of discourse.
- (2) Decision for each predicate symbol when it is true about members of the domain of discourse and when it is false.
- (3) \exists means 'there is'.
- \forall means 'for all'.
- \neg means 'not'.
- \wedge means 'and'.
- \vee means 'and/or'.
- $=$ means 'equals'.

A sentence is *satisfiable* if there is some interpretation which makes it true.

For example, the sentence

$$(\forall x)(\exists y)((y > x) \wedge (\neg x=y))$$

is the symbolic transcription of 'for every x there is a y so that y is greater than x and x isn't equal to y '. Here ' $>$ ' is a *two-place predicate* (sometimes called a *relation*). This sentence is satisfiable. For let the domain of discourse be the natural numbers (1), let $>$ be the usual relation of greater than defined for natural numbers (2), and let $\forall, \exists, \wedge, \neg,$ and $=$ have their usual meanings (3). Under this interpretation, the sentence is true because it asserts that for every natural number there's a bigger natural number.

A curious thing is that this sentence also has an interpretation in which the domain of discourse is finite! This sounds odd because the sentence seems to say that for every x there's a bigger y , which would require infinitely many objects. But remember that we can give ' $>$ ' an unorthodox interpretation. Let's interpret ' $x > y$ ' as meaning ' x is bigger than y or x is equal to $y-2$ '. Take $\{0,1,2\}$ as our domain of discourse. The sentence is true under this interpretation--for every x there's a y so that $y > x$ but $y \neq x$. Namely,

for 0 there's 1 since $1 > 0$ and $0 \neq 1$; for 1 there's 2 since $2 > 1$ and $2 \neq 1$; and for 2 there's 0 since $0 > 2$ and $0 \neq 2$.

A sentence of this sort which is satisfiable over a finite domain of discourse is called (what else?) *finitely satisfiable*.

Is it possible to write a sentence which is satisfiable but not finitely satisfiable? Such a sentence might be called an *infinity axiom* since its truth entails that the domain of discourse is infinite. Our previous sentence fails to be an infinity axiom because of our unorthodox interpretation of '>'. But we can get an infinity axiom by trying a little harder. The example exhibited is taken from [1], slightly modified.

Consider the sentence

$$(\forall x)(\exists u)(\forall y)[((x \text{ loves } y) \vee (y \text{ loves } x)) \wedge (\neg(x \text{ loves } y) \vee \neg(y \text{ loves } u))]$$

The only predicate we must interpret is 'loves'. We must also choose a domain of discourse. Even before we do this we can see some of the implications of the sentence under discussion.

For example, the sentence implies that for any two

objects x and y in the domain of discourse, either x loves y or y loves x (possibly both). There's no reason why y and x can't be the same object, in which case we have x loves x or x loves x). So every object loves itself.

The sentence starts by saying 'for every x there is a u '. Let's call one of the u 's which corresponds to a given x (there may be several) x 's companion. Then for a given x and its companion u , the sentence says that for every y ... either x doesn't love y or y doesn't love u , maybe both. Let y be something x loves. Then y doesn't love u . That is, anything x loves will not love x 's companion.

In particular, for x itself, x doesn't love u . So x doesn't love its companion. But we know everything loves itself. Therefore x 's companion is not x itself. Also, since for any two objects, one of them loves the other, it must be that u loves x .

And by the same reasoning, anything x loves, u must also love (since u isn't loved by it).

But now suppose the domain of discourse is finite. Let A be the object which loves the most (in terms of numbers not passion!). Let C be A 's companion. C is not A . Anybody A loves, C also loves. But, in addition, C loves A ! So C , not A , is the most lover.

This is a contradiction! The sentence is not finitely satisfiable.

The sentence is satisfiable, however. Take as domain the natural numbers, and let 'A loves B' mean A is greater than B. The sentence is true under this interpretation. Check it out! As x's companion you may take x+1. Notice there is no number which *loves* the most.

Notice that the infinity axiom we just investigated starts with ' $\forall x \exists u \forall y$ '. We call it an $\forall\exists\forall$ -sentence. Now we're at the point where I can state the open logic problem I promised at the beginning of the article. The question is simply,

Is there an $\forall\forall\exists$ infinity axiom?

No one knows. Let's be clear about what we're looking for. The sentence should start with (say) $\forall x \forall y \exists z$. What comes after can involve the variables x, y , and z , any number of predicate symbols, and any of the logical symbols \neg, \wedge, \vee , or $=$ (\exists and \forall aren't allowed again or else the sentence wouldn't be an $\forall\forall\exists$ -sentence). The sentence must be satisfiable, but not over any finite domain of discourse.

To help you look, I'll mention a theorem that may be helpful.

Theorem. If a sentence is satisfiable, it is true under an interpretation in which the domain of discourse is a subset (or the entirety) of the set of natural numbers.

So it's not necessary to know anything about horses or unicorns; you can stick to interpretations in which you're talking about natural numbers.

An $\forall\exists$ sentence is a relatively simple type of sentence. But we don't know if one can write an infinity axiom of that type. Don't be surprised to find the problem is harder than it looks. If you can't solve it, don't forget there's always Fermat's Last Theorem.

BIBLIOGRAPHY

- [1] S. O. Aanderaa and H. R. Lewis, Prefix classes of Krom formulas, Journal of Symbolic Logic, v.38 (1973), pp. 628-642.
- [2] Burton Dreben and Warren D. Goldfarb. The Decision Problem: Solvable Classes of Quantificational Formulas. Addison-Wesley Pub. Co.: Reading, Mass. 1979.

KAPPA MU EPSILON NEWS

EDITED BY HAROLD L. THOMAS, HISTORIAN

News of chapter activities and other noteworthy KME events should be sent to Dr. Harold L. Thomas, Historian, Kappa Mu Epsilon, Mathematics Department, Pittsburg State University, Pittsburg, Kansas 66762.

REPORT ON THE 1982 REGION I CONVENTION

PA Lambda hosted the Region I meeting at Bloomsburg State College in Bloomsburg, PA on April 2 and 3, 1982. No other information was received about the convention.

REPORT ON THE 1982 REGION II CONVENTION

A large and enthusiastic group of 70 students and 19 faculty members enjoyed the Region II convention which was hosted by Ohio Alpha at Bowling Green State University in Bowling Green, Ohio on April 23 and 24, 1982.

Dr. Frederick Leetch and Ohio Alpha President, Mark Worline, together with their chapter members, planned a program which opened with a banquet in the Student Union. This was followed by initiation of new members, a talk by Dr. Yuri Gurevich, and a delightful barbershop quartet, the Logarythms, made up of faculty members who sang out in praise of Riemann sums, harmonized the decimal representation of π to thirty places, and entertained with many old favorites as well.

After coffee and donuts on Saturday morning, six students presented papers. These were, "Bell's Theorem" by Christopher McCord, Bowling Green State University; "Computer Generated Data Illustrating the Central Limit Theorem" by Sandra Dolde, Central Michigan University; "An Introduction to Homotopy" by Myron Dulkoski, Muskingum College; "Beauty in Mathematics, The Golden Proportion" by Marie Couture, Central Michigan University; "Graphical Solution to Partial Differential Equations" by Andrew Long, Bowling Green State University; and "An Investigation of the Negative Binomial Distribution" by

Beth Lapham, Central Michigan University. Professor Dorian Yeager and members of Kentucky Alpha, who acted as judges, awarded first prize, which was a KME pin, to Andrew Long of Ohio Alpha for his presentation. The judges also gave a special commendation to Marie Couture of Michigan Beta.

Sister Nona Mary Allard, Director of Region II, expressed appreciation to Ohio Alpha for the excellent job they did in preparing and hosting this convention.

REPORT ON THE 1982 REGION IV CONVENTION

Twelve Region IV chapters answered the roll call at the convention which was held at Kearney, NE on April 16 and 17, 1982. NE Beta hosted the convention which was attended by 83 students and faculty members. Professor Marilyn Jussel, faculty sponsor of NE Beta and Jeff Lodi, chapter president, were primarily responsible for the organization of the convention and the activities held.

A mixer was held Friday evening to give students the opportunity to get acquainted. Professor John S. Cross, Region IV director reports that the students actually did mix with many new friendships made. Many are looking forward now to the 1983 National Convention.

After registration on Saturday morning, the participants were welcomed by Dr. R. David Clark, Dean of the School of Natural and Social Sciences at Kearney State. The convention was then highlighted by the presentation of six student papers which were well prepared and well presented. These were, "Applications of Echelon Technique" by Martha Haeberle, Kearney State College; "Consequences of Rolle's Theorem" by Trent Eggleston, University of Missouri-Rolla; "The Classification of Chemical Crystals into the Seven Crystal Systems" by Dianne L. Hickert, Benedictine College; "Mathematics' Role in Telepathy -- the Ouija Board Curve" by Denise Rost, University of Missouri-Rolla; "Kant, Mathematics and Synthetic A Priori" by Sally Irvin, Fort Hays Kansas State University; and "The Fibonacci Numbers" by Annette Herz, Kearney State College.

Following a luncheon buffet, an address entitled "Life is for Lessons" was given by Dr. Marvin Knittel, Department of Educational Psychology at Kearney State. Entertainment was also provided by the Chamber Singers, directed by Annabell Zikmund.

The convention closed with awards presented to the top two student papers by Dr. John Cross, Region IV Director. The first place award was given to Denise Rost and second place went to Trent Eggleston.

Region IV chapters are now looking forward to the 1983 National Convention to be held at Eastern Kentucky University, Richmond, Kentucky on April 21-23, 1983.

CHAPTER NEWS

Alabama Zeta, Birmingham-Southern College, Birmingham
Chapter President - Alison Pool
21 actives, 25 initiates

At the spring initiation of new members, Professor Ouida Kinzey spoke on "Patterns in Nature -- A Mathematician's Viewpoint." Other activities included a talk by Dr. Don Riley from NASA, Marshall Space Flight Center in Huntsville, Alabama, on "The Space Shuttle," a discussion of careers in actuarial science, and a spring picnic with other science honoraries. Other 1982-83 officers: Thomas Donald Herring, vice president; Debra Sievers, secretary; Robert Sipe, treasurer; Lola F. Kiser, corresponding secretary; Sarah E. Mullins, faculty sponsor.

California Gamma, California Polytechnic State University,
San Luis Obispo
Chapter President - Dan Weeks
45 actives, 34 initiates

The chapter sponsored the annual Math Sciences Career Day for the entire campus as well as the annual county-wide Math Field Day for junior high school students. Assistance was also given to the Mathematics Department

with the annual Poly Royal Math Contest which attracted about 600 high school students to the campus. Chapter meetings featured alumni and industry speakers. The annual spring initiation banquet was attended by approximately 100 members and guests. Dennis Ikenoyama was the first recipient of the annual Arthur Andersen & Co. Professional Performance Award. Other 1982-83 officers: Mark Lucovsky and Dennis Ikenoyama, vice presidents; Nancy Lott, secretary; Fremont Bainbridge, treasurer; George R. Mach, corresponding secretary; Adelaide Harmon-Elliott, faculty sponsor.

Colorado Alpha, Colorado State University, Fort Collins
Chapter President - Michael Thomas
10 actives

Other 1982-83 officers: Mary Jo Black, vice president; Anne Murray, secretary; Paul Magnus, treasurer; Arne Magnus, corresponding secretary.

Connecticut Beta, Eastern Connecticut State College, Willimantic
Chapter President - Raymond Hill
26 actives, 11 initiates

Eleven new members were initiated this spring. One of the new members gave a talk at the initiation meeting. Colloquia were also held on Gödel's Theorem and on research using optimal control theory at NASA. Other 1982-83 officers: Karen Johnson, vice president; Kathleen Evans, secretary and treasurer; Ann M. Curran, corresponding secretary; Stephen Kenton, faculty sponsor.

Georgia Alpha, West Georgia College, Carrollton
Chapter President - Bob Ingle
24 actives, 6 initiates

The annual initiation of new members was held May 19, 1982. Election of officers followed the initiation ceremony as well as a reception in honor of the initiates.

Other 1982-83 officers: Darla House, vice president; Cindy Holladay, secretary; Steve Townsend, treasurer; Thomas J. Sharp, corresponding secretary and faculty sponsor.

Illinois Beta, Eastern Illinois University, Charleston
Chapter President - Michele Sinclair
33 actives, 25 initiates

Other 1982-83 officers: Kathy Graff and Nancy Marlin, vice presidents; Dawn Hoskins, secretary; Lisa Foltz, treasurer; Lloyd Koontz, corresponding secretary.

Illinois Zeta, Rosary College, River Forest
Chapter President - Brad Erickson
12 actives, 3 initiates

Three new members were initiated during the spring semester. Michael Renella and Sister Nona Mary Allard attended the Region II convention at Bowling Green State University in April. Ms. Marietta McPike, a manufacturer's representative and former programmer for IBM, spoke to the members about her career with IBM. Other 1982-83 officers: Joan Novak, vice president; Jean Rexroat, secretary; Laura Voss, treasurer; Sister Nona Mary Allard, corresponding secretary and faculty sponsor.

Illinois Eta, Western Illinois University, Macomb
Chapter President - Hans Hamilton
7 actives

Fall semester activities included: a lunch for mathematics department members to raise funds; a presentation by Dr. Gerald White of the WIU mathematics department on the TRS-80 microcomputer; a volleyball game with the physics and chemistry clubs (which KME won); a speaker from Bankers Life and Casualty Insurance Co. discussing actuarial careers. A pizza party was held at the end of the semester. Other 1981-82 officers: Karen Seehafer, vice president; Arthur Zenner, secretary and treasurer; Alan Bishop, corresponding secretary; Iraj Kalantari, faculty sponsor.

Illinois Theta, Illinois Benedictine College, Lisle
 Chapter President - Steve Becker
 30 actives, 12 initiates

The chapter held six meetings during the spring semester. At the Feb. 3 meeting, Dr. Phyllis Kitten spoke on "This is the Title of this Talk" (Geometry). A club social was held in late February. Seven alumni actuaries returned to campus in early April to discuss their profession with students. Other meetings included election of officers and initiation ceremony for new members. The chapter also provides a subscription to a recreational mathematics journal which is kept in the mathematical sciences department. Other 1982-83 officers: Ron Swanstrom, vice president; Mike Cooney, secretary; Joanne Connelly, treasurer; James M. Meehan, corresponding secretary and faculty sponsor.

Indiana Alpha, Manchester College, North Manchester
 Chapter President - James Brumbaugh
 26 actives, 8 initiates

The annual spring banquet was held April 28, 1982. Kevin Saylor of Miami University addressed the banquet on "Rubic's Magic Cube, Under One Minute." New members were also initiated and new officers elected. Other 1982-83 officers: Deb Hanson, vice president; Tammy Ulery, secretary; Kent Workman, treasurer; Ralph B. McBride, corresponding secretary; Tom S. Hudson, faculty sponsor.

Indiana Gamma, Anderson College, Anderson
 Chapter President - Lauri Van Norman
 12 actives, 10 initiates

Other 1982-83 officers: Douglas Skipper, vice president; Melissa Farlee, secretary and treasurer; Stanley L. Stephens, corresponding secretary and faculty sponsor.

Iowa Alpha, University of Northern Iowa, Cedar Falls
 Chapter President - Margaret Chizek
 34 actives, 5 initiates

The following students presented papers at local KME meetings -- Tina Walls on "Fermat's Last Theorem," Kirk Montgomery on "Pythagoras: Math and Music," and John Pestotnik on "Application of Computer Graphics." In March, Iowa Delta KME chapter from Wartburg College joined with Iowa Alpha for a pizza supper at Happy Joe's. In April three students, Margaret Chizek, Chuck Daws and Darla Dettman, along with Professors John Bruha and John Cross made the trip to Kearney, NE for the KME Region IV convention. These five adults and their luggage helped with energy conservation by traveling in a two door Dodge Colt. Professor Carl Wehner was in charge of weather for the picnic in May and he delivered a great day so attendance was very good. Iowa Alpha sold math T-shirts this spring. Other 1982-83 officers: Charles Daws, vice president; Kirk Montgomery, secretary; J'ne Day, treasurer; John S. Cross, corresponding secretary and faculty sponsor.

Iowa Beta, Drake University, Des Moines
Chapter President - Grant Izmirlian
13 actives, 5 initiates

The new initiates presented talks at the March and April meetings. They were then initiated at the annual banquet in May. In April, Tom Potempa was awarded first place in the annual KME freshmen mathematics contest. Other 1982-83 officers: Coni Johnson, vice president; Tom Potempa, secretary; Sheryl Shapiro, treasurer; Wayne Woodworth, corresponding secretary; Alexander Kleiner, faculty sponsor.

Iowa Delta, Wartburg College, Waverly
Chapter President - Edmund Bonjour
40 actives, 16 initiates

The chapter participated in 1982 Math Field Day when 150 high school students competed in math contests. Monthly presentations were given by Alan Guetzlaff on "Boolean Algebra" and Kathy Schulz on "Mathematical Games." A student-faculty department dinner was held at which time the film, "Sorting Out Sorting" was shown.

The annual banquet and initiation ceremony rounded out spring activities. Other 1982-83 officers: Tony Hogge, vice president; Brenda Augustine, secretary; Diane Smith, treasurer, Josef Breutzmann, corresponding secretary and faculty sponsor.

Kansas Alpha, Pittsburg State University, Pittsburg

Chapter President - Darren Smith

40 actives, 13 initiates

Kansas Alpha began the spring semester with a banquet celebrating the 50th anniversary of KME on the Pitt State campus. Two charter members attended the banquet. Former national historian, Dr. J. D. Haggard, welcomed the group and KME alum, Eddie Grigsby from Phillips Petroleum, spoke on "Mathematics in Industry." Thirteen new members were initiated following the banquet. The March program was given by Linda McCracken on "Organization Status." Hazel Kent presented the April program on "Euler's Trail." The chapter also assisted the Mathematics Department faculty in administering and grading tests at the annual Math Relays, April 27, 1982. The final meeting of the Spring Semester was held at Professor Sperry's home. It was highlighted by election of officers for the 1982-83 school year. In addition, the annual Robert M. Mendenhall awards for scholastic achievement were presented to Brenda Brinkmeyer, Paige Chilton, Mark Griffin and Hazel Kent. They received KME pins in recognition of this honor. Other 1982-83 officers: Wanda Lawson, vice president; Debbie Birney, secretary; Jeanine Carver, treasurer; Harold L. Thomas, corresponding secretary; J. Bryan Sperry, faculty sponsor.

Kansas Beta, Emporia Kansas State University, Emporia

Chapter President - Julie Romine

30 actives, 8 initiates

Monthly meetings were held during the spring semester at which time mathematical topics were discussed. The final meeting was a picnic when new officers were elected for the coming year. An initiation and banquet were also spring activities. The chapter attended the

Region IV convention held in Kearney, NE. Other 1982-83 officers: John Vogt, vice president; Phyllis Tidd, secretary; Patty Herrick, treasurer; John Gerriets, corresponding secretary; Tom Bonner, faculty sponsor.

Kansas Gamma, Benedictine College, Atchison

Chapter President - Kay Kreul

18 actives, 7 initiates

President Steve Pahls coordinated the Computer Dance held at the beginning of second semester. Steve revised the computer programs used for the matching process. This year's Math Tournament for high school students, held in February and coordinated by seniors Terri Beye and Tom Gallagher, had a special attraction -- a demonstration of dancing curves. Eric Heumann, a senior in the teacher education program built the model following the directions contained in the NCTM publication "Dancing Curves." KS Gamma initiated the following students on February 16: John Agnew, Ann Devoy, Jenny Farrell, Jane Feltmann, Karen Henneberry, Joe Schaefer and Eric Heumann. Richard Desko, data processing analyst for Hallmark Cards, gave a slide presentation on the Hallmark Distribution Center in Liberty, MO, at a chapter meeting on March 23. Students Dianne Hickert, Terri Beye, Kay Kreul, Therese Bendel and faculty moderator Sister Jo Ann Fellin attended the Regional meeting in Kearney, NE. Dianne presented her paper on crystal structures. KS Gamma seniors were surprised at the steak picnic on April 20 with gifts that will keep KME in the foreground for awhile -- T-shirts with an emblem containing the five leafed rose and the Greek words for Kappa Mu Epsilon. Kay Kreul, newly elected president of the chapter, was awarded the Sister Helen Sullivan Scholarship for 1982-83. Other 1982-83 officers: John Agnew, vice president; Ann Devoy, secretary; Jane Feltmann, treasurer; Jenny Farrell, historian and Exponent editor; Sister Jo Ann Fellin, corresponding secretary and faculty sponsor.

Kansas Delta, Washburn University, Topeka

Chapter President - Cindy Dietrich

20 actives, 3 initiates

The chapter helped sponsor Math Day for high school students on March 25, 1982. An initiation ceremony for new members was held in February. New officers for 1982-83 will be elected early in the fall semester.

Kansas Epsilon, Fort Hays Kansas State University, Fort Hays

Chapter President - Betty Burk
23 actives, 9 initiates

Monthly meetings were held with the Math Club featuring student presentations. The chapter has also been involved with publication of a departmental newsletter. Sally Irvin presented a paper at the Region IV convention. Other 1982-83 officers: Tim Seltmann, vice president; Arron Von Schrittz, secretary and treasurer; Charles Voltaw, corresponding secretary; Jeffrey Barnett, faculty sponsor.

Kentucky Alpha, Eastern Kentucky University, Richmond

Chapter President - Beth Stewart
39 actives, 20 initiates

Kentucky Alpha held bi-weekly meetings throughout the Spring 1982 semester, at which speakers presented talks on a variety of topics. A spring initiation banquet was held, at which Dr. Austin French of Georgetown College presented a talk entitled "Let's prove one for the Gipper." A spring picnic was held at the home of Dr. Dorian Yeager. Six students and one faculty member attended the Region II convention in Bowling Green, Ohio. Throughout the semester, members provided free weekly tutoring sessions for undergraduate mathematics and computer science students. Other 1982-83 officers: Monica Feltner, vice president; Carole Stagnolia, secretary; Karen Applegate, treasurer; Don Greenwell, corresponding secretary; Patrick Costello, faculty sponsor.

Maryland Alpha, College of Notre Dame of Maryland, Baltimore

Chapter President - Barbara Slezok
7 actives, 3 initiates

In March a Mathematics Olympiad was held for high school girls. Six high schools participated. Both the high school girls and the KME members were very enthusiastic about the Olympiad and are making plans for a repeat performance. The annual initiation dinner was held in May. At this time, Laura Nesbitt, '79 alum, spoke of her career at the National Security Agency. Officers for 1982-83 will be elected in the fall.

Maryland Beta, Western Maryland College, Westminster
Chapter President - Judy K. VanDuzer
22 actives, 2 initiates

The chapter saw the movie, Flatland, in February. The induction banquet was in March. A joint meeting was held in April with Maryland Alpha chapter of KME. Each chapter contributed a talk. Sally Carlson represented Maryland Beta. She spoke on Markov Chains as they relate to the book distribution field. The second career night of the year was also in April at which time three alumni members returned to tell members and majors about applications of mathematics in various professions. Three Maryland Beta members, Cynthia Bowden, John Wandishin, and Sally Townsend, gave talks at the Region I convention on April 2. Cynthia and Sally tied for first place. The chapter also sponsored a raffle and a weekly mathematics puzzle in the school newspaper. The year closed with a picnic at the home of one of the faculty members. Other 1982-83 officers: Cynthia B. Bowden, vice president; Amy S. Polashuk, secretary; Millard S. Mazer, treasurer; James E. Lightner, corresponding secretary; Robert P. Boner, faculty sponsor.

Maryland Delta, Frostburg State College, Frostburg
Chapter President - Vincent J. Costello
35 actives, 24 initiates

Programs during the semester included viewing of video-recordings concerning hypothesis testing and statistical estimation and a presentation of enrichment topics by the mathematics methods class. New members were initiated Feb. 7, 1982. Timothy Lambert gave a talk on the four-color problem following the initiation. Two members, David Zawodniak and George Wagner, received John Allison Outstanding Senior Awards. A picnic at Rock Gap State Park, with the chemistry and physics honor societies, concluded the semester. Other 1982-83 officers:

John Wagner, vice president; Margaret Neville, secretary; Lynn Harpold, treasurer; Agnes B. Yount, corresponding secretary; John P. Jones, faculty sponsor.

Massachusetts Alpha, Assumption College, Worcester
 Chapter President - Keith A. Brusio
 16 actives

Ten new members were initiated on April 30, 1982. Following a dinner in honor of the new members, John Heffernan spoke on "Goedel's Incompleteness Theorem." Other 1982-83 officers: Rosemary Kane, vice president; William A. Bilow, Jr., secretary; Charles Brusard, corresponding secretary and faculty sponsor.

Michigan Beta, Central Michigan University, Mount Pleasant
 Chapter President - Sandra Dolde
 40 actives, 24 initiates

Speakers during the spring semester were Dr. Joseph Aszenzo - statistician at Upjohn Pharmaceutical Company, Joann Ostrowski - 1976 CMU graduate and mathematician at General Motors Research Lab, William Lakey - CMU math professor, Fred Phelps - CMU physics professor. Fifteen students and two faculty attended the KME regional convention at Bowling Green, Ohio. Papers were given at the regional by Michigan Beta members Sandy Dolde, Beth Lapham, and Marie Couture. Michigan Beta held tutoring sessions for freshman-sophomore level mathematics classes. Other 1982-83 officers: Diane Francisco, vice president; Laurie Park, secretary; Dan Franck, treasurer; Arnold Hammel, faculty sponsor and corresponding secretary.

Mississippi Alpha, Mississippi University for Women,
 Columbus
 Chapter President - Julia Mann
 16 actives, 5 initiates

Other 1982-83 officers: Lesia Coleman, vice president; Merry L. Elliott, secretary and treasurer; Jean Ann Parra, corresponding secretary; Carol B. Ottinger, faculty sponsor.

Mississippi Gamma, University of Southern Mississippi,
Hattiesburg

Chapter President - Johnny Graves
30 actives, 9 initiates

The chapter sponsored weekly colloquia during the spring semester at which faculty members and students gave talks. The spring picnic and initiation was held April 17, 1982. Other 1982-83 officers: Charles Orr, vice president; Marie Turnage, secretary and treasurer; Liz O'Neal, public relations; Alice W. Essary, corresponding secretary.

Missouri Alpha, Southwest Missouri State University,
Springfield

Chapter President - Kathy Merlo
62 actives, 11 initiates

A banquet was held on April 29, 1982 in honor of the 50th anniversary of Missouri Alpha. Other 1982-83 officers: Belinda Butcher, vice president; Craig McCowan, secretary; Kim Tracy, treasurer; M. Michael Awad, corresponding secretary; L. T. Shiflett, faculty sponsor.

Missouri Beta, Central Missouri State University, Warrensburg

Chapter President - Jennifer Koch
35 actives, 21 initiates

Spring activities included two initiations, a Christmas party, a canoe float trip, an honors banquet, and five meetings with a speaker. Other 1982-83 officers: Vince Edmondson, vice president; Ruth Lichte, secretary; Randy Bush, treasurer; Homer Hampton, corresponding secretary; Larry Dilley, faculty sponsor.

Missouri Epsilon, Central Methodist College, Fayette

Chapter President - Michael Hanson
4 actives, 4 initiates

Other 1982-83 officers: Judy Frazer, vice president; Kirk Meyer, secretary and treasurer; William D. McIntosh, corresponding secretary and faculty sponsor.

Missouri Zeta, University of Missouri-Rolla, Rolla
 Chapter President - Eva Taylor
 22 actives, 9 initiates

Dr. Glen Haddock, Professor of Mathematics, was guest speaker at the first general spring meeting on Feb. 9, 1982. His topic entitled, "Pharmacokinetics" dealt with the manner in which various drug dosages are administered. The chapter hosted a pizza party at the Pizza Inn on February 21, 1982. Dr. David Goecke, Assistant Professor of Mathematics, gave a very intriguing talk on "The Earth is Neither Flat nor Round" at the March 9 meeting. Pictures were also taken for the yearbook. The program for the April 3 meeting was given by the two students who presented papers at the Region IV convention. Five students and one faculty member attended the convention on April 16, 17, 1982 at Kearney, NE. Denise Post received first place for her paper, "Mathematics' Role in Telepathy -- The Ouija Board Curve" and Trent Eggleston won second place with his presentation of "Consequences of Rolle's Theorem." Each received a cash award, certificate and pendant. Nine new members were initiated at the spring banquet on April 21. Dr. Charles Hatfield gave the banquet talk on "A Card-Shuffling Problem." New officers were elected at the final spring meeting on May 4. The chapter is looking forward to a very active fall semester. Other 1982-83 officers: Karen Anderson, vice president; Denise Rost, secretary; Trent Eggleston, treasurer; Melinda Arnold, historian; Johnny Henderson, corresponding secretary; James Joiner, faculty sponsor.

Nebraska Alpha, Wayne State College, Wayne
 Chapter President - Mike Ronspies
 24 actives, 15 initiates

Money making projects during the spring semester included monitoring the Math-Science building in the evenings and selling discs for the Apple II computers to students. Nine members and two faculty members attended the Region IV convention at Kearney State College on April 16 and 17. The Annual Wayne State College Bowl which was held April 20 & 21, 1982, was won by the KME team. Team members were Ken Hamsa, Sue Schrage, Kathy Hladky, Mike Ronspies, Marian Rhods and Ken Burns. Mike Ronspies was the winner of the club's \$25.00 scholarship which can be used to purchase text books or pay fees. This award is given each semester to one of the members. Club members select the recipient by secret ballot. The club also administered the annual test to identify the outstanding freshman studying mathematics. The award went to Debra Lofton of Omaha, Nebraska. The award includes the recipient's name being engraved on a permanent plaque, payment of KME National dues, one year honorary membership in the local KME chapter, and announcement of the honor at the annual spring banquet. Four chapter members, Ken Burns, Carol Francis, Brenda Mandel, and Kathy Hladky assisted the mathematics department in administering the Eighth Annual Wayne State College Mathematics Contest on May 10, 1982. Approximately 450 high school students from 60 high schools participated in the contest. Other 1982-83 officers: Kelli Goodner, vice president; Brenda Mandel, secretary and treasurer; Ken Burns, historian; Fred Webber, corresponding secretary; Jim Paige and Hilbert Johs, faculty sponsors.

Nebraska Beta, Kearney State College, Kearney
Chapter President - Martha Haeberle
35 actives, 8 initiates

The chapter hosted the Region IV convention April 16, 17, 1982. Over seventy-five members and faculty from eleven colleges attended. A field trip was taken to the SAC Air Force Base in Omaha where the group received the royal treatment. Members also hosted a tea for the seniors of the department. Other 1982-83 officers: Sharon Hostler, vice president; John Klimey, secretary; Melvin Joy, treasurer; William Meyer, pledge chairman; Dale Filsinger, historian; Charles Pickens, corresponding secretary; Marilyn Jussel, faculty sponsor.

Nebraska Gamma, Chadron State College, Chadron

Chapter President - Martin L. Egging

10 actives, 4 initiates

Other 1982-83 officers: Evonnda L. Sharp, secretary; Sterling S. Stumpf, treasurer; James A. Kaus, corresponding secretary; Monty G. Fickel, faculty sponsor.

New Jersey Beta, Montclair State College, Montclair

Chapter President - Vincent Corsaro

15 actives, 6 initiates

Spring semester activities included an attempted computer dating party, t-shirt sales, meetings and elections, and a picnic. The chapter also discussed possible field trips for next semester, fund raising activities, and a programming contest for local high schools. Other 1982-83 officers: Lisa Frobose, vice president; Linda Ruff, secretary; Johanna Serra, treasurer; William Parzynski, corresponding secretary; George Gugel, faculty sponsor.

New York Eta, Niagara University, Niagara

Chapter President - Cindy McDonald

18 actives, 6 initiates

Initiation ceremonies were held for five new initiates on March 28, 1982. Mr. John J. Moore, a former faculty member, spoke at the initiation on "Ravings of a Rambling Mathematician." The chapter sold flowers in March as a fund raising activity. A picnic was held in May in conjunction with the Math Club. Officers for 1982-83 will be elected in the fall.

New York Iota, Wagner College, Staten Island

Chapter President - Anthony Castellano

8 actives, 5 initiates

Other 1982-83 officers: Wanda Lalluse, vice president; Amy Wachs, secretary; Louis Capriotti, treasurer; William Horn, corresponding secretary.

New York Kappa, Pace University, New York City

Chapter President - Michael Klein

21 actives, 9 initiates

The chapter held their ninth annual dinner and initiation on April 26, 1982. The dinner was highlighted by a talk given by Mr. Nathan Morrison. He spoke on "The Role of the Actuary in Modern Society." Other 1982-83 officers: Judith Holden, vice president; Michael Scanna, secretary; Michael Kazlow, corresponding secretary; Martin Kotler, faculty sponsor.

Ohio Gamma, Baldwin-Wallace College, Berea
Chapter President - Larry Mills
38 actives, 6 initiates

The chapter held a spring picnic. Other 1982-83 officers: Marty Porter, vice president; Pam Botson, secretary; Rick Hughes, treasurer; Robert Schlea, corresponding secretary and faculty sponsor.

Ohio Zeta, Muskingum College, New Concord
Chapter President - Gail Yoder
39 actives, 8 initiates

Tina Oakley and Mark Killoran gave a program on micro-computers on Jan. 25, 1982. New members were initiated/ in February. Officers for 1982-83 were elected on March 22, 1982. Programs were also planned for next year. Bill Belknap of NCR gave a talk on computing in industry on March 24, 1982. Seven members attended the Region II convention at Bowling Green State University on April 23 and 24. Myron Dulkoski was one of the student paper presenters at the convention. A spring picnic was held at Dr. Smith's house on April 25, 1982. Dorothy Knight and L. Coleman Knight, retiring departmental members were honored. Other 1982-83 officers: Tom Bressoud, vice president, Shayne Fawcett, secretary; Paula Gomory, treasurer; James L. Smith, corresponding secretary and faculty sponsor.

Oklahoma Beta, University of Tulsa, Tulsa
Chapter President - Cathy DeHart
25 actives, 22 initiates

Other 1982-83 officers: Robert Duffy, vice president; Donna Hummel, secretary; Peggy Volz, treasurer; Richard Redner, corresponding secretary and faculty sponsor.

Pennsylvania Alpha, Westminster College, New Wilmington
Chapter President - Carl Schartner
37 actives, 16 initiates

Spring activities included a trip to the Youngstown State Planetarium, sponsoring campus chess tournament, and sponsoring a program on internships with two speakers and refreshments. The chapter also provided tutoring service for the Mathematics Department. A spring picnic was held in May. Other 1982-83 officers: Rob Streeter, vice president; Kirsten Pealstrom, secretary; Nick Kounavelis, treasurer; J. Miller Peck, corresponding secretary; Barbara Faires, faculty sponsor.

Pennsylvania Beta, LaSalle College, Philadelphia
Chapter President - Roseann Fisher
25 actives, 20 initiates

A panel discussion concerning cooperative education with Mr. Louis Lamorte and current students involved in the program was held March 17, 1982. New members were initiated on April 7. At that time, Mr. Raymond Kirsch gave a talk on hash tables. New officers were elected at the April 21st meeting. Other 1982-83 officers: Joanne Kelly, vice president; Kathryn Coccozza, secretary; Anne Galasso, treasurer; Hugh N. Albright, corresponding secretary; Carl McCarty, faculty sponsor.

Pennsylvania Epsilon, Kutztown State College, Kutztown
Chapter President - Melodie Schumaker
30 actives, 10 initiates

An initiation banquet was held on Feb. 27 and a spring picnic on May 2, 1982. Other 1982-83 officers: Stephen Caufield, vice president; Lori Klee, secretary; Jeff Herbein, treasurer; I. Hollingshead, corresponding secretary; Wm. Jones, faculty sponsor.

Pennsylvania Zeta, Indiana University of PA, Indiana
Chapter President - Rose Mary Zbiek
30 actives, 13 initiates

Thirteen new members were initiated in February. Following the initiation, Wm. Smith, faculty advisor, posed some mathematical problems for members to solve. In March, Dr. Donald McKelvey, member of the Chemistry Department faculty of IUP, gave a talk on "Continued Fractions." The annual banquet was held in April. It was prepared by student members under the guidance of Prof. Raymond Gibson, Mathematics Department faculty member, Dr. James Reber, chairman of the Mathematics Department, presented a talk on David Hilbert. Other 1982-83 officers: Mark Woodard, vice president; Edna Keba, secretary; Wendy Stilwell, treasurer; Ida Z. Arms, corresponding secretary; William R. Smith, faculty sponsor.

Pennsylvania Eta, Grove City College, Grove City
Chapter President - Mylene Klipa
30 actives, 11 initiates

The chapter held their annual Spring Picnic at the Grove City Country Club on May 3rd. Activities included Frisbee tossing and volleyball. Other 1982-83 officers: Leslie Demarest, vice president; Ron Ellenberger, secretary; Susan Kay, treasurer; Marvin C. Henry, corresponding secretary; Dan Dean, faculty sponsor.

Pennsylvania Kappa, Holy Family College, Philadelphia
Chapter President - Linda Czajka
5 actives, 4 initiates

Sister Clare Marie Butt, CSFN, and Sister Yoette Ortiz were both initiated on February 15, 1982. On March 3, 1982, the members attended a conference on the History of Mathematics held at St. Joseph's University. Group attendance for the viewing of Chariots of Fire was voted upon. Drawing parallels between math interest and the film portrayal of discipline, dedication, perseverance and faith were discussed after viewing the film. Other officers for 1982-83: Teresa McKeon, vice president and secretary; Patricia Kellner, treasurer; Sister M. Grace, corresponding secretary and faculty sponsor.

Pennsylvania Lambda, Bloomsburg State College, Bloomsburg

Chapter President - David Fox

20 actives, 8 initiates

Officers for 1982-83 will be elected later.

Pennsylvania Mu, St. Francis College, Loretto

Chapter President - Nancy Dudziec

12 actives, 2 initiates

Other 1982-83 officers: Penney Horner, vice president; Peter Laird, secretary and treasurer; Fr. John Kudrick, corresponding secretary; Adrian Baylock, faculty sponsor.

Tennessee Alpha, Tennessee Technological University, Cookeville

Chapter President - Sharon Lovett

40 actives, 20 initiates

A banquet honoring the new members was held on May 17, 1982 at the El Toro Steak House. New members were initiated at the banquet. Dr. Ed Dixon, chairman of the Mathematics and Computer Science Department, was the guest speaker. Other 1982-83 officers: Sherri Menees, vice president; Carolyn Talbert, secretary; Shelly Bynum, treasurer; Ed Dixon, corresponding secretary; Steve Khleif, faculty sponsor.

Tennessee Gamma, Union University, Jackson

Chapter President - Randal Brewer

22 actives, 10 initiates

The annual spring banquet and initiation of new members was held on April 15, 1982. The speaker was Mr. Carroll Griffin, a former student member. Other 1982-83 officers: Judy Escue, vice president; Donna Dixon, secretary; Bob Bond, treasurer; Richard Dehn, corresponding secretary; Joe Tucker, faculty sponsor.

Texas Eta, Hardin-Simmons University, Abilene
Chapter President - Marlies Johnston
50 actives, 11 initiates

The annual spring induction banquet was held April 8, 1982. Eleven new members were inducted including ten students and one faculty member. Professor Edwin J. Hewett of Hardin-Simmons University gave a talk entitled, "Mathematical Anecdotes." In addition, the chapter gave special recognition to Mr. and Mrs. B. C. Bentley. Mrs. Anne B. Bentley is retiring from the faculty of Hardin-Simmons University after twenty-one years of service. She has been the Corresponding Secretary of the Texas Eta Chapter of Kappa Mu Epsilon since the chapter's founding. The chapter presented Mrs. Bentley with an engraved silver platter and Mr. Bentley with a commemorative plaque recognizing their contribution to Kappa Mu Epsilon and to the Department of Mathematics at Hardin-Simmons University. Approximately twenty-five were in attendance. Other 1982-83 officers: Teresa Therwhanger, vice president; Debbie Smith Proctor, secretary and treasurer; Mary Wagner, corresponding secretary; Charles D. Robinson and Edwin Hewett, faculty sponsors.

Virginia Alpha, Virginia State University, Petersburg
Chapter President - David Battle
17 actives, 4 initiates

Papers were presented by pledges at the regular meetings. A plaque was given to Bridgette Evans as the graduating senior with outstanding performance in mathematics. The Louise S. Hunter scholarship award given to the outstanding junior member of KME was received by Jonathan Ransom. The chapter also held an initiation banquet. Other 1982-83 officers: Jonathan Ransom, vice president; Melissa Rainey, secretary; Martina Lewis, treasurer; LaVerne Goodridge, corresponding secretary; Emma Smith, faculty sponsor.

West Virginia Alpha, Bethany College, Bethany
Chapter President - James Kuzma
10 actives, 4 initiates

Other 1982-83 officers: Kathy Martin, vice president; Donna Gates, secretary and treasurer; David Brown, corresponding secretary; James Allison, faculty sponsor.

Wisconsin Alpha, Mount Mary College, Milwaukee
 Chapter President - Catherine Schueller
 7 actives, 2 initiates

Members were active participants in a careers conference for young women sponsored by the Mathematics and Science departments. The conference brought approximately 175 seventh through twelfth grade young women, their parents and teachers to the college campus to hear about mathematics and science related careers open to women. The two (2) initiates each gave a talk. Jane Grzechowiak spoke on computer programming while Bonnie Best's talk was concerned with topology. Initiation was held on April 28. After the initiation, the group went out to dinner. Some KME alumnae attended both the initiation and the dinner. Catherine Schueller, a prospective teacher, taught a discovery lesson involving polar coordinates to the KME members. Other 1982-83 officers: Bonnie Best, vice president; Jane Grzechowiak, secretary and treasurer; Sister Adrienne Eickman, corresponding secretary; Sister Petronia Van Straten, faculty sponsor.

Wisconsin Gamma, University of Wisconsin-Eau Claire, Eau Claire
 Chapter President - Candith Morzenti
 39 actives, 23 initiates

On April 20, 1982, the chapter sponsored a math bowl as part of honors week, a yearly event at UW-Eau Claire. There were six teams made up of math students from the local campus. Other 1982-83 officers: Linda Kelly, vice president; Jeanne Gandrud, secretary; Rosemary Slaby, treasurer; Tom Wineinger, corresponding secretary; Wilbur Hoppe and Robert Langer, faculty sponsors.

ANNOUNCEMENT OF TWENTY-FOURTH BIENNIAL CONVENTION

Eastern Kentucky University
Richmond, Kentucky

April 21-23, 1983

The 24th Biennial Convention of Kappa Mu Epsilon will be held on April 21-23, 1983 at Eastern Kentucky University; Richmond, Kentucky. Each chapter that sends a delegation will be allowed some travel expenses from National Kappa Mu Epsilon funds. Travel funds are disbursed in accordance with Article VI, Section 2 of the KME constitution.

IF YOUR SUBSCRIPTION HAS EXPIRED

We hope you have found THE PENTAGON both interesting and helpful. Your suggestions are always welcome and may be written on this form. They will be forwarded to the Editor.

If you wish to renew your subscription for two years, please send \$5* to THE PENTAGON, Department of Mathematics, Central Michigan University, Mt. Pleasant, Michigan 48859.

Please give your name below in the manner in which you wish your mail addressed (Miss, Mrs., Prof., etc.). If your name has changed, please indicated also the name to which THE PENTAGON was sent.

Name _____

Address _____

City _____ State _____ Zip _____

*Foreign subscriptions require an additional \$1 per year for postage.

