

# THE PENTAGON

*A Mathematics Magazine for Students*

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## General Convex Solid Modeling

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Presented at the 1993 National Convention and  
awarded FIRST PLACE by the Awards Committee.

### Introduction.

The problem of three dimensional modeling for the purpose of producing quality two dimensional images is a vast and complex one. The reason for this complexity stems from the virtually infinite number of types of three dimensional objects and the enormous number of visual models conceivable. While theoretically ray tracing is an all inclusive solution to this problem, it would be insane to neglect the various special properties of different kinds of three dimensional objects. In fact, ray tracing is often not possible because of the highly computational nature of the method. Therefore it is reasonable, indeed necessary in many cases, to use alternate techniques to solve this problem. In order to solve this problem we must analyze the kinds of objects to be modeled and then exploit their special characteristics to aid in the modeling process.

The kinds of objects that this paper deals with are convex solids and closed convex surfaces. We will use a method of three dimensional modeling, called polygonal, which requires that the surface of the object being modeled be broken up into polygons. It is then these individual polygons that are manipulated in the modeling process. This popular, widely used method is very flexible and can work on any three dimensional object — not just convex ones. Despite its enormous popularity and widespread use, I have never seen a general purpose algorithm to implement polygonal modeling on the types of objects we are interested in here. All of the published methods I have seen to solve

the problem set forth previously require one or more seemingly arbitrary restrictions on the data structures used to hold the points. One of the most common requirements placed upon the data structures used in the modeling process is that all of the points defining all of the polygons must be ordered consistently. The programmer, in complying with this, is forced to spend an enormous amount of wasted time producing and checking actual data. These kinds of arbitrary data restrictions should not be necessary to successfully model convex objects. Other restrictions are the actual data structures used to hold the points and polygons. Typically, multi-dimensional arrays are used. This leads to problems when the data must be treated as some other kind of structure elsewhere in the programs (as a b-tree, for example). Thus the problem to be solved within these pages arises: How to design and implement a general purpose polygonal modeling algorithm for convex solids which does not impose unnecessary restrictions upon the data or data-structures.

Before delving into the mathematics, we need to develop a feel for the general process of modeling three dimensional objects. The approach we will take is basically a three step one. The first step is to produce a model of the object (that is, obtain a group of polygons describing the surface of the object — the more polygons the smother the image). The second step is to transform the object using the standard linear transformations associated with rotation, scaling, partial scaling, etc., in order to orient the object appropriately for viewing. The choice of transformations used is irrelevant to the modeling process; in fact, even strange transformations may be used — as long as they are linear. Lastly, the third step is to project the appropriate parts of the object, shaded correctly of course, into a plane for viewing. The method of projection is, as was the choice of transformations, irrelevant. The two most common projection types are orthogonal and one or two point perspective. The part of this merry little process that this paper deals with is the most difficult part. That part is the determination of which polygons in the model are visible and how they should be shaded.

The cumulating event of the work done in this paper will be computer source code to implement the algorithms and techniques developed within the next few pages. The actual production of such source code to implement such general purpose algorithms may be the most difficult part of the problem solving process. The reason for this is not that the algorithms are particularly complex, but the data structures must be extremely non-specific in order to display the generality of the algorithms. Information structures which can adapt to any kind of storage scheme conceivable by a programmer must be developed (more on this latter in the paper). Then code which is as non-specific as these

data structures must be produced. Such highly dynamic code is normally very difficult to produce in actual practice. The hallmark of the algorithms produced is their isolation from the data and its storage structure; therefore, this feature should not be disguised by simple, limited data structures.

Overall, our work is made easier because most of the transformation and projection steps are standard and source code already exists. Making use of this previously existing source code and making an easy to use programmer interface for the new algorithms are thus goals for the programs which will be produced. Part of this interface will be to hide the underlying information structures so that the programmer does not know, or need to know, what they actually look like. The programmer can let the structure adapt to what the programmer wants, or needs, to use. In this way, the strange underlying data structures become an "ease of use" feature, as intended. Despite the practical nature of these programs, their primary use is for demonstration; therefore, they have been written with readability in mind — not speed. When the never ending struggle between speed and simplicity arose, simplicity and readability always won.

### The Directed Normal.

Now that we have a plan and feel for the process, it is time to proceed with the mathematics. First something I call a directed normal must be defined. A *directed normal* is a normal vector which, when based on the surface of a convex solid, is outwardly directed from the interior the object. By putting the vector on the surface of the object one may also think of a directed normal as a normal whose head is not within the object's interior. Convex solids have a very special property relating directed normals to visibility and shading. Any point on a convex object's surface that has a directed normal pointing away from the viewing plane is not visible. The term "pointing away from" will be defined precisely later in the paper. Now that the importance of directed normals has been established, it must be determined how to compute and use them efficiently.

The problem at this point is to find a way to compute directed normals. This problem may be approached in a general way as finding a normal to a plane which comes out of the plane on the same side as some arbitrarily chosen point in space (provided that the point is not in the plane of course). The point  $Q$  is the point in the plane from which we wish to find a directed normal. The points  $P$  and  $G$  are any two other points in the plane chosen such that the three points  $Q$ ,  $P$  and  $G$  are not

colinear. For our application, we may assume that two other suitable points are known. The reason for this is that any plane we consider in this manner will have a polygon defined in it and therefore we will know at least three non colinear points, the vertexes. By subtraction, the vectors  $\vec{QP} = \vec{P} - \vec{Q} = \vec{a}$ ,  $\vec{QR} = \vec{R} - \vec{Q} = \vec{b}$  and  $\vec{QG} = \vec{G} - \vec{Q} = \vec{g}$  are found. Now we name the two possible unit normal vectors to the plain at the point  $Q$ ,  $\vec{m}$  and  $\vec{w}$ . It is interesting to note that the normal vectors  $\vec{m}$ ,  $\vec{w}$  and the vector  $\vec{g}$  are coplanar. This is due to the fact that  $\vec{m}$  and  $\vec{w}$  are colinear by definition. Two approaches are obvious at this point to tell which vector  $\vec{m}$  or  $\vec{w}$  is closer to going in the direction of  $\vec{g}$ . This vector, either  $\vec{w}$  or  $\vec{m}$ , which actually is closer in direction to  $\vec{g}$  will be called  $\vec{n}$ . The first thing is to note the magnitude of  $d\vec{m}$  and  $d\vec{w}$ . If  $\|d\vec{m}\| < \|d\vec{w}\|$  (see Figure 1), then  $\vec{n} = \vec{m}$ , but if  $\|d\vec{w}\| < \|d\vec{m}\|$ , then  $\vec{n} = \vec{w}$ . If  $\|d\vec{m}\| = \|d\vec{w}\|$ , then the vector  $\vec{g}$  is in the plane and thus no directed normal with respect to the vector  $\vec{g}$  exists as defined. Now we need to find a way to compute  $d\vec{m}$  and  $d\vec{w}$  so we can use them. The following relationships come from the definition of vector arithmetic:  $d\vec{m} = \vec{g} - \vec{m}$  and  $d\vec{w} = \vec{g} - \vec{w}$ , but  $\vec{m} = -\vec{w}$  so  $d\vec{w} = \vec{g} + \vec{m}$ .

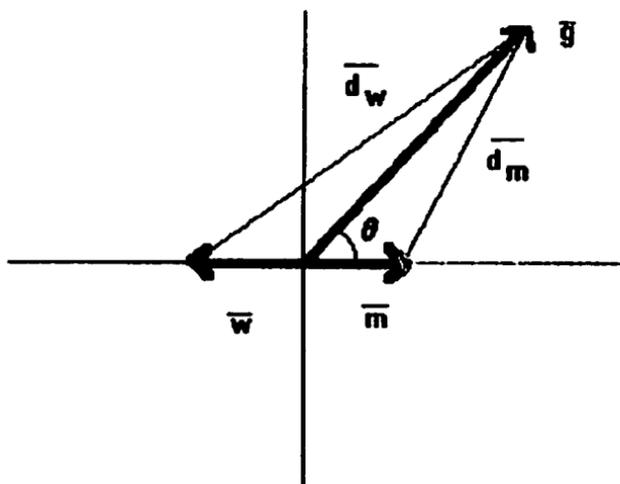


Figure 1.

We start with what we know:

$$\|d\vec{m}\| < \|d\vec{w}\| \Rightarrow \vec{n} = \vec{m} .$$

By substitution,

$$\|\vec{g} - \vec{m}\| < \|\vec{g} + \vec{w}\| \Rightarrow \vec{n} = \vec{m} .$$

By the definition of vector magnitude,

$$\left( \langle \vec{g} - \vec{m}, \vec{g} - \vec{m} \rangle \right)^{1/2} < \left( \langle \vec{g} + \vec{m}, \vec{g} + \vec{m} \rangle \right)^{1/2} \Rightarrow \vec{n} = \vec{m} .$$

By squaring both sides,

$$| \langle \vec{g} - \vec{m}, \vec{g} - \vec{m} \rangle | < | \langle \vec{g} + \vec{m}, \vec{g} + \vec{m} \rangle | \Rightarrow \vec{n} = \vec{m}$$

and, by a symmetric argument, we also have

$$| \langle \vec{g} - \vec{m}, \vec{g} - \vec{m} \rangle | > | \langle \vec{g} + \vec{m}, \vec{g} + \vec{m} \rangle | \Rightarrow \vec{n} = \vec{w}$$

This last expression is unnecessary as  $\vec{n} \neq \vec{m}$ , it must be that  $\vec{n} = \vec{w}$ .

Now we have a way to find the distances. Thus a concise way to determine directed normals is found. The other idea that comes to mind is to use the angle  $\theta$  and the properties of cosine and dot product. The relationship between the angle  $\theta$  and the vectors  $\vec{m}$  and  $\vec{g}$  is

$$\langle \vec{m}, \vec{g} \rangle = \|\vec{m}\| \cdot \|\vec{g}\| \cdot \cos(\theta) .$$

If the cosine of an angle is positive, then the angle is in the first or fourth quadrant; if it is negative, then the angle is in the second or third quadrant; and if it is zero, the angle is  $2k\pi - \pi/2$ , where  $k$  is a positive integer. Thus if the cosine is positive, the vector  $\vec{g}$  makes an acute angle with  $\vec{m}$  and is therefore closer to going in the direction of  $\vec{m}$  than  $\vec{w}$ . If the angle is zero, then the plane would be viewed as on its edge. Since  $\|\vec{m}\| \cdot \|\vec{g}\| > 0$ , by the definition of magnitude, this expression has no effect upon the sign of the cosine. Thus

$$\text{sign}(\langle \vec{m}, \vec{g} \rangle) = \text{sign}(\|\vec{m}\| \cdot \|\vec{g}\| \cdot \cos(\theta)) = \text{sign}(\cos(\theta)) .$$

This angular expression defines precisely the term "points toward" used previously. This expression is mathematically more interesting than the previous result and simpler to compute for a computer; however, the other result is useful in its own right. In fact, the distance concept will be used to find the correct normal, while this latter result will be used to determine the visibility. This choice makes the program simpler and easier to understand. This is also the obvious choice because of further

simplifying conditions specific to our application of primitive modeling. It is important to note at this point that sometimes one wants to pick normals that point in the wrong direction. This is to facilitate a technique known as far side modeling. In this modeling process we see the inside of the far side of the object. For many symmetric objects the images are exactly the same, to illustrate this fact some of the programs do far side modeling. Computer literate people might object to using different normals as only the if-then constructs need be changed to do far side modeling, but by using different normals both types of modeling can be done with the same code!

#### Application to the Problem.

Now we apply all this math to our current problem. To allow ourselves to concentrate on the concepts we simplify matters by modeling primitives. Primitives are the simple shapes used to form complex objects. These simple shapes include such things as spheres, cubes, cones, and cylinders. It is with these kinds of objects that complex graphics are produced in simulator games. These objects are all centered on or contain the origin. By using our previous analysis of directed normals we find an elegant relationship to test the cross products and find directed normals. Using the names in the figure, if  $\|\vec{Q} + \vec{m}\| > \|\vec{Q} - \vec{m}\|$  then  $\vec{n} = \vec{m}$  and if  $\|\vec{Q} + \vec{m}\| < \|\vec{Q} - \vec{m}\|$  then  $\vec{n} = \vec{w}$ . By computations similar to the general case, the above formulas are simplified to (Can you tell if this is for regular or for far side modeling?)

$$| \langle \vec{Q} + \vec{m}, \vec{Q} - \vec{m} \rangle | > | \langle \vec{Q} - \vec{m}, \vec{Q} - \vec{m} \rangle | \Rightarrow \vec{n} = \vec{m}$$

$$| \langle \vec{Q} + \vec{m}, \vec{Q} - \vec{m} \rangle | < | \langle \vec{Q} - \vec{m}, \vec{Q} - \vec{m} \rangle | \Rightarrow \vec{n} = \vec{w}$$

Visibility is simple to compute: if  $\langle \vec{n}, \vec{v} \rangle \geq 0$ , where  $\vec{v}$  is the view vector, then the polygon is visible. The view plane is  $yz$  which has normal  $\vec{i}$ ; this normal is the view vector. The intersection of the view plane and the  $x$ -axis is the source of light for shading; therefore, our light source vector is  $\vec{i}$  as well. This is particularly nice because dotting vectors with  $\vec{i}$  is simple, it is simply the first component of the vector. Let  $\vec{v} = [v_1, v_2, v_3]$  and  $\vec{i} = [1, 0, 0]$ . Then  $\langle \vec{v}, \vec{i} \rangle = v_1 \cdot 1 + v_2 \cdot 0 + v_3 \cdot 0 = v_1$ .

Now that we can tell whether a polygon is visible or not, we must compute a shading factor for those faces which are visible. The last part of our mathematical analyses of the situation will be just this problem of shading. While mathematics is the most beautiful art created by man,

the color and/or hashing patterns used are primarily a matter of aesthetics, and therefore are an issue to be addressed by another kind of artist. However, the color or hashing value is a simple matter of mathematics. By elementary optical laws and extremely simple lighting situations, we can realistically shade our three dimensional objects. We will consider only the incident light on an object; no transient, reflected or refracted light will be considered. The shading thus produced is quite realistic in spite of the simplicity of the laws and situation. It should be noted, however, that we have all the data necessary to completely describe each face and can therefore use the most sophisticated ray tracing models available. We have directed normals and the equations of the planes and this allows us to implement absolutely any optical model we could desire.

The amount of light reflected from an object to a plane is dependent upon the angle with which it meets the surface of the object. In particular, it is proportional to the angle between the normal of the surface at the point which the light hits and the light beam. Since our objects are made up of many polygons, which are coplanar figures, we can shade each polygon as a whole instead of shading each point. This is typical of any optically based shading scheme using polygonal modeling. The angle which our light source vector makes with the normal is the good old dot product again. If  $\vec{v}$  is the light source vector then  $\langle \vec{v}, \vec{n} \rangle / \|\vec{v}\| \cdot \|\vec{n}\| = \cos(\theta)$ . As mentioned before,  $\vec{v} = \vec{i}$  and  $\langle \vec{v}, \vec{n} \rangle = n_1$ , so  $\cos(\theta) = n_1 / \|\vec{n}\|$ . Since we will produce unit normal vectors, our shading will only need to look at  $\vec{n}$ . (Can you tell if this is for far side modeling or regular? Does it matter?)

We have now completed our theoretical work. The techniques developed thus far allow us to find directed normals and compute shading values. All of the necessary mathematics is now done and it is time to see if it all works on a computer.

### The Program.

The algorithms produced in this paper are embodied in a library (see Appendix A). The reasons for this are many. To mention a few, this kind of structure displays the compatibility, versatility, and transportability of the code. As mentioned previously, the most challenging part of the program was the necessity of an adaptable information structure capable of attaching and using many kinds of data structures without losing speed. The structure would have to be able to link to b-trees, arrays, linked lists, doubly linked lists, linked rings, linked matrices, globular structures and even polygons randomly scattered in memory. In short, if

the data was in main memory, the structure needed to be able to use it. This would display the data structure independent nature of the algorithms. The structure used is one step away from total point distribution. This level could be added in ten lines of code, but the language has facilities to help with continuous point schemes. If the programmer did not want to use the built in polygon drawing procedures this level of abstraction could be used with little code modification. In fact, this method might be preferable in a large computer environment, a Cray for example, where language facilities are far different. The data structure used can totally insulate the programmer from data requirements if this is desired.

The `Solid3dType` and `Polygon3dType` do not declare any memory for the actual data; in fact, they do not even make any assumptions about where or how that data is stored. They simply supply pointers to pointers and arrays of points for each polygon. The way memory is obtained for these arrays is unimportant. They could be declared as variables taking the correct amount of space or be allocated at run time or in some strange data structure. In the programs (see, for example, Appendix B), I have put all the points in huge arrays and then attached those arrays to the polygon arrays. This is simple, readable and very flexible. In order to use this method the programmer doesn't even need to know where his or her data is.

The data to fill the huge arrays used is produced in procedures. The procedures either contain an optimized database of points, have an algorithm to compute the points or both. These procedures are completely independent of the rest of the program. These points could come from anywhere. If the total point distribution extensions were added to the data structures the data in the optimized data bases would not even need to be copied. The information structure could simply attach itself to the data base and use the points. This would even let the same point be used multiple times in several polygons with out any data duplication. Again, the reason this total point distribution model was not used is that it would require that a procedures to draw polygons and to fill polygons be programmed. This kind of low level graphics programming is not what this paper is about and would just clutter things up. The clarity and cleanliness of using the built in functions seemed more important.

It must be noted that in a typical program the points would be computed as needed, not stored. The methods developed in this paper would work with computed values, but storing them in memory makes the program much more general and faster. As evidence to the power of

the data structures, the transformation and normal finding procedures work if the points are computed or stored with out any modification.

Due to space limitations the only program included models a cube (see Appendix B). I chose the cube from all of the other test programs because it's simple to understand and is thus a better teaching aid.

### Implementations.

Polygonal modeling is a very flexible method which can be applied to any three dimensional object. These techniques are particularly well adapted for the modeling of primitives and general convex solids. There are many ways to implement the techniques developed in this paper. They can be implemented directly as in the demo program or as a preprocessor for more conventional polygonal techniques. As mentioned previously, many data restrictions are encountered in polygonal modeling, like the ordering of points. This ordering could be determined by computing directed normals, thus the computer could do this data sorting rather than the programmer. In this way data could be preprocessed and used by conventional polygonal techniques. In effect turning the conventional techniques into a more general form. Due to the speed of the techniques developed here it is not necessary to use them as a preprocessor because of a slow speed. The advantage of using them as such would be the ability to use previously developed code and thus save much time in code modifications.

### Conclusion.

The methods and algorithms developed within these pages are significant, but required little sophisticated mathematics. In fact, the relationships turned out to be rather simple after finding the correct course of thought. Some knowledge about geometry, dot products, cross products and vectors was all that was really needed. From a computer science point of view the data structures used are exotic and could have be even more exotic. The end result of it all is a coherent method for modeling general convex solids and closed convex surfaces. A coherent method that is somewhat simple to boot.

The methods are probably not new, just well hidden in the literature. In fact, the reason I took up this project is that I became tired of looking in books. Many good books exist on the subject of modeling. One good book giving a good introduction, in a very nuts and bolts way, is *High-Performance CAD Graphics in C* by Lee Adams. Another good introduction is *Tutorial: Computer Graphics* by John C. Beatty and Kellogg S. Booth. A more general book is *Fundamental Algorithms for*

*Computer Graphics*, edited by R. A. Earnshaw. These books will give a good introduction to solid modeling.

*Acknowledgements.* In closing, I would like to thank Dr. Gary McGrath for his help in proofreading this paper. I would also like to thank Dr. Bobby Winters and Dr. Harold Thomas for their sponsorship and leadership of our local *KME* chapter, Kansas Alpha.

### Appendix A.

(Turbo Pascal, Version 5.0, Borland)

```

unit Lib3d;
interface
uses graph, crt;
{      constants for variable type definitions      }
const
    MaxNumPts = 4;           { This is the max index }
    MaxNumPoly = 4;         { This is the max index }
{      basic data type definitions      }
type
    rnum = real;
    Point3dType = record x: rnum; y: rnum; z: rnum; end;
    Point3daType = array[1..3] of rnum; { for flex only }
    ArrPoint3dType = array[1..MaxNumPts] of Point3dType;
    Polygon3dType = record
        Pts: ^ArrPoint3dType;
        Num: integer;
    end;
    ArrPolygon3dType = array[1..MaxNumPoly] of
        Polygon3dType;
    Solid3dType = record
        Poly: ^ArrPolygon3dType;
        Num: integer;
    end;
    ScrPointType = record x: integer; y: integer; end;
    ArrScrPointType = array[1..MaxNumPts + 1] of
        ScrPointType;
    ScrPolygonType = record
        Pts: ^ArrScrPointType;
        Num: integer;
    end;

```

```

ArrScrPolygonType = array[1..MaxNumPoly] of
                                ScrPolygonType;

TfrmDatType = record
  r1: rnum; { yaw   z }
  r2: rnum; { roll  y }
  r3: rnum; { pitch x }
  sr1: rnum; { sin r1 }
  sr2: rnum; { sin r1 }
  sr3: rnum; { sin r1 }
  cr1: rnum; { cos r1 }
  cr2: rnum; { cos r1 }
  cr3: rnum; { cos r1 }
  d  : rnum; { distance for perspective (not used)}
  sx : rnum; { x  scale }
  sy : rnum; { y  scale }
  sz : rnum; { z  scale }
end;
{          constants based on above data types          }
const
  Point3dSize      = sizeof(Point3dType);
  Polygon3dSize    = sizeof(Polygon3dType);
  ScrPointSize     = sizeof(ScrPointType);
  ScrPolygonSize   = sizeof(ScrPolygonType);
  i_vector: Point3dType = (x:1.0; y:0.0; z:0.0);
  j_vector: Point3dType = (x:0.0; y:1.0; z:0.0);
  k_vector: Point3dType = (x:0.0; y:0.0; z:1.0);
  0_vector: Point3dType = (x:0.0; y:0.0; z:0.0);
  i_ScrVector: ScrPointType = (x:1; y:0);
  j_ScrVector: ScrPointType = (x:0; y:1);
  0_ScrVector: ScrPointType = (x:0; y:0);
  StdTfrmData: TfrmDatType =
    (  r1 : 0.0;   r2 : 0.0;   r3 : 0.0;
      sr1: 0.0;   sr2: 0.0;   sr3: 0.0;
      cr1: 1.0;   cr2: 1.0;   cr3: 1.0;   d : 1.0;
      sx : 100.0; sy : 100.0; sz : 100.0          );
  StdTfrmData2: TfrmDatType =
    (  r1 : 0.2;   r2 : 0.2;   r3 : 0.2;
      sr1: 0.198669330795;
      sr2: 0.198669330795;
      sr3: 0.198669330795;
      cr1: 0.980066577841;
      cr2: 0.980066577841;
      cr3: 0.980066577841;
      d  : 1.0;
    );

```

```

    sx : 100.0; sy : 100.0; sz : 100.0           );
shd: array[1..14] of FillPatternType =
  ( ($00, $00, $00, $00, $00, $00, $00, $00),
    ($00, $00, $00, $20, $08, $00, $00, $00),
    ($00, $40, $20, $00, $00, $02, $04, $00),
    ($00, $40, $A8, $00, $00, $0A, $04, $00),
    ($00, $42, $A4, $00, $00, $1A, $44, $00),
    ($00, $4A, $A5, $00, $00, $9A, $54, $00),
    ($0A, $42, $A4, $A1, $10, $1A, $44, $01),
    ($1A, $42, $A4, $A1, $1A, $1A, $44, $01),
    ($BA, $42, $A4, $AB, $BA, $1A, $44, $01),
    ($AA, $55, $AA, $55, $AA, $55, $AA, $55),
    ($CC, $33, $CC, $33, $CC, $33, $CC, $33),
    ($OF, $F0, $OF, $F0, $OF, $F0, $OF, $F0),
    ($96, $69, $96, $69, $96, $69, $96, $69),
    ($FF, $FF, $FF, $FF, $FF, $FF, $FF, $FF) );
{
  extra data types for easy programing
}
type
TriangleType = array[1..3] of Point3dType;
QuadType     = array[1..4] of Point3dType;
PentagonType = array[1..5] of Point3dType;
HexagonType  = array[1..6] of Point3dType;

procedure AttachArrPolyToArrPts(var InPolys;
                                var InPts; NumPts, NumPoly: integer);
procedure CompTfrmTrig(var td: TfrmDatType);
procedure TfrmPtToAll(InPt: Point3dType;
                      var OutPt: Point3dType; var sp:
                      ScrPointType; td: TfrmDatType);
procedure FindDirNorm(Polygon: Polygon3dType;
                      var OutNormal: Point3dType;
                      var Hiden: boolean);
procedure DrawSolid(Solid: Solid3dType; IsFill:
                    boolean; td: TfrmDatType);
procedure InteractSolidDisplay(Solid:
                                Solid3dType; td: TfrmDatType);
procedure initGraphicsToStd;

```

implementation

```

{ This attaches the pointers in an array of
  Polygon3dtype to a block of memory. This memory is
  viewed as a two dim. array of arrays of
  Point3dtypes. The actual memory could have been

```

```

allocated as a linear one dim. array or a two dim.
array or just as a general block of memory. All
that matters is that the data in this block of
memory is correct. }
procedure AttachArrPolyToArrPts(var InPolys;
                                var InPts; NumPts, NumPoly: integer);
var
  i: integer;
  ArrPoly: ArrPolygon3dType absolute InPolys;
  ArrPts: ArrPoint3dType absolute InPts;
begin
  for i := 1 to NumPoly do
    begin
      ArrPoly[i].Pts := addr(ArrPts[(i - 1) *
                                NumPts + 1]);
      ArrPoly[i].Num := NumPts;
    end;
  end;
{ This is used to compute the sin and cos fields of
a TfrmDatType. The angles stored in the angle
fields are used for the computations. }
procedure CompTfrmTrig(var td: TfrmDatType);
begin
  with td do
    begin
      sr1 := sin(r1); sr2 := sin(r2); sr3 := sin(r3);
      cr1 := cos(r1); cr2 := cos(r2); cr3 := cos(r3);
    end;
  end;
{ This procedure transforms a point to all possible
output types. This would be memory wasteful if used
on an entire array. }
procedure TfrmPtToAll(InPt:Point3dType; var OutPt:
  Point3dType; var sp:ScrPointType; td:TfrmDatType);
var
  xa, ya, za: rnum;
begin
  with td, InPt do
    begin
      x := x * sx; y := y * sy; z := z * sz;
      xa := cr1 * x - sr1 * z;
      za := sr1 * x + cr1 * z;
      x := cr2 * xa + sr2 * y;
    end;
  end;
end;

```

```

    ya := cr2 * y - sr2 * xa;
    z  := cr3 * za - sr3 * ya;
    y  := sr3 * za + cr3 * ya;
end;
OutPt.x := InPt.x;
OutPt.y := InPt.y;
OutPt.z := InPt.z;
sp.x := round(InPt.y);
sp.y := -round(InPt.z);
end;
{ This procedure finds a unit directed normal to a
  face. It also returns a flag about the normal. }
procedure FindDirNorm(Polygon:Polygon3dType; var
  OutNormal:Point3dType; var Hiden:boolean);
var
  a, b, n, P, Q, R: Point3dType;
  mag1, mag2: rnum;
begin
  P := Polygon.Pts^[1];
  Q := Polygon.Pts^[2];
  R := Polygon.Pts^[3];
  a.x := P.x - Q.x;
  a.y := P.y - Q.y;
  a.z := P.z - Q.z;
  b.x := R.x - Q.x;
  b.y := R.y - Q.y;
  b.z := R.z - Q.z;
  n.x := a.y * b.z - a.z * b.y;
  n.y := a.z * b.x - a.x * b.z;
  n.z := a.x * b.y - a.y * b.x;
  mag1 := sqr(Q.x+n.x)+sqr(Q.y+n.y)+sqr(Q.z+n.z);
  mag2 := sqr(Q.x-n.x)+sqr(Q.y-n.y)+sqr(Q.z-n.z);
  if mag1 < mag2 then
    begin
      n.x := -n.x; n.y := -n.y; n.z := -n.z;
    end;
  mag1 := sqrt(sqr(n.x) + sqr(n.y) + sqr(n.z));
  if mag1 <> 0 then
    begin
      n.x:=n.x/mag1; n.y:=n.y/mag1; n.z:=n.z/mag1;
    end;
  if n.x = abs(n.x) then Hiden := false
  else Hiden := true;

```

```

    OutNormal := n;
end;
{ This is the workhorse procedure. It draws a general
  convex solid. If the thing will fit into memory,
  this procedure will draw it. This version is fully
  dynamic and written to be easy to read rather than
  very fast. }
procedure DrawSolid(Solid: Solid3dType;
                   IsFill: boolean; td: TfrmDatType);
var
  i, j: integer;
  aPt: Polygon3dType;
  asPt: ^ArrScrPointType;
  somePt: Point3dType;
  hid: boolean;
begin
  for i := 1 to Solid.Num do
    begin
      getmem(aPt.Pts, Solid.Poly^[i].Num * Point3dSize);
      getmem(asPt, (Solid.Poly^[i].Num+1)*ScrPointSize);
      for j := 1 to Solid.Poly^[i].Num do
        begin
          TfrmPtToAll(Solid.poly^[i].Pts^[j],
                     aPt.Pts^[j], asPt^[j], td);
        end;
      asPt^[Solid.Poly^[i].Num + 1] := asPt^[1];
      FindDirNorm(aPt, somePt, Hid);
      if not hid then
        begin
          if somePt.x <> 0
            then
              begin
                setfillstyle(round(SomePt.x * 12), blue); }
                SetFillpattern(shd[round(SomePt.x*14)],white);
                if IsFill
                  then fillpoly(Solid.Poly^[i].Num + 1, asPt^)
                  else drawPoly(Solid.Poly^[i].Num + 1, asPt^);
                end
              else
                drawpoly(Solid.Poly^[i].Num + 1, asPt^);
            end
          end
        end
      }
    { To use color, use the command in this comment.
      Remember to kill the next command if you do use the
      commented out one.
      setfillstyle(round(SomePt.x * 12), blue); }
      SetFillpattern(shd[round(SomePt.x*14)],white);
      if IsFill
        then fillpoly(Solid.Poly^[i].Num + 1, asPt^)
        else drawPoly(Solid.Poly^[i].Num + 1, asPt^);
      end
    else
      drawpoly(Solid.Poly^[i].Num + 1, asPt^);
    end
  end
end;

```

```

    end;
    freemem(aPt.Pts, Solid.Poly^[i].Num*Point3dSize);
    freemem(asPt, (Solid.Poly^[i].Num+1)*ScrPointSize);
end;
end;
{ This is the procedure that lets the user rotate,
  scale and shade the object in real time. It works
  well on a 486 and reasonably well on a 386. If you
  are using a slower computer, I would suggest
  increasing RotStp and SclStp to speed things up. }
procedure InteractSolidDisplay(Solid: Solid3dType;
                               td: TfrmDatType);
var
  IsFill: boolean;
  c: char;
  RotStp, SclStp: rnum;
begin
  RotStp := 0.1;
  SclStp := 10;
  IsFill := false;
  repeat
    CompTfrmTrig(td);
    cleardevice;
    DrawSolid(Solid, IsFill, td);
    c := readkey;
    case c of
      'x': td.r3 := td.r3 - RotStp;
      'y': td.r1 := td.r1 - RotStp;
      'z': td.r2 := td.r2 - RotStp;
      'X': td.r3 := td.r3 + RotStp;
      'Y': td.r1 := td.r1 + RotStp;
      'Z': td.r2 := td.r2 + RotStp;
      'f', 'F': IsFill := not IsFill;
      'i': RotStp := RotStp * 2;
      'I': RotStp := RotStp / 2;
      'o': SclStp := SclStp * 2;
      'O': SclStp := SclStp / 2;
      'a': td.sz := td.sz + SclStp;
      's': td.sx := td.sx + SclStp;
      't': td.sy := td.sy + SclStp;
      'A': td.sz := td.sz - SclStp;
      'S': td.sx := td.sx - SclStp;
      'T': td.sy := td.sy - SclStp;
    end;
  until c = #0;
end;

```

```

    end;
    until c = chr(27);
end;
{ This procedure detects the graphics driver and
  monitor hardware. It then sets the graphics mode to
  the highest resolution possible. This program has
  been tested on VGA, EGA and CGA. Other hardware
  should work. VGA or better should be used for best
  results. }
procedure initGraphicsToStd;
var
  grdriver, grmode: integer;
begin
  grdriver := detect;
  InitGraph(grdriver, grmode, '');
  setviewport(getmaxx div 2, getmaxy div 2,
             getmaxx, getmaxy, clipoff);
  setwritemode(copyput);
  setlinestyle(0, 0, 3);
  setbkcolor(black);
  setcolor(white);
  setfillStyle(1, white);
  clearviewport;
end;

end.

```

#### Appendix B.

```

{      This program models a cube with lib3d.      }

program Cube;

uses crt, graph, Lib3d;

procedure UnitCubeDataFill(var solid:Solid3dType);
const
  cords: array[1..8] of Point3dType =
    ( (x: 1.0; y: 1.0; z: 1.0),
      (x: 1.0; y: 1.0; z:-1.0),
      (x: 1.0; y:-1.0; z:-1.0),
      (x: 1.0; y:-1.0; z: 1.0),

```

```

                (x:-1.0; y: 1.0; z: 1.0),
                (x:-1.0; y: 1.0; z:-1.0),
                (x:-1.0; y:-1.0; z:-1.0),
                (x:-1.0; y:-1.0; z: 1.0)    );
ptPtr: array[1..6,1..4] of integer =
    (    (1, 2, 3, 4),    { front  }
      (1, 5, 6, 2),    { r side }
      (5, 8, 7, 6),    { back   }
      (4, 3, 7, 8),    { l side }
      (4, 8, 5, 1),    { top    }
      (3, 2, 6, 7)     { bottom }    );
var
    i, j: integer;
begin
    Solid.Num := 6;
    for j := 1 to 6 do
        begin
            Solid.Poly^[j].Num := 4;
            for i := 1 to 4 do
                Solid.Poly^[j].Pts^[i] := Cords[ptPtr[j, i]];
            end;
        end;
end;
{    variables and constants for the main program    }
const
    NumPts = 4;
    NumPoly = 300;
    NumC = 20;
var
    Solid: Solid3dType;
    ListOfPoints: Array[1..NumPoly] of
        Array[1..NumPts] of Point3dType;
    PolyData: Array[1..NumPoly] of Polygon3dType;
{
    main program
}
begin
    initGraphicsToStd;
    Solid.Poly := addr(PolyData);
    AttachArrPolyToArrPts(PolyData, ListOfPoints,
                          NumPts, NumPoly);

    UnitCubeDataFill(Solid);
    InteractSolidDisplay(Solid, StdTfrmData);
    restorecrtmode;
end.

```

## How the Scientific Calculator Calculates<sup>1</sup>

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In this paper I will attempt to answer the question, "How does your scientific calculator work?" To do so I will use a simple algorithmic language that is essentially identical to BASIC.<sup>2</sup> I have chosen this topic to suggest how thinking at the level of school mathematical problems — sophisticated thinking to be sure — is still viable in the 20th Century. Here is how engineers solved problems that look quite complex and yet involve only the concepts you learned in school.

Some comments at the outset. First, I have often asked students and colleagues how they think their calculator operates. Their answer usually starts out something like, "It converts numbers to binary and then ...," and at that point their voice trails off. Or they suggest that everything is done by logarithms. When I ask how the logs are calculated, some say that there is probably a built-in table. Now think about that for just a moment. We're talking here about hand-held calculators, little \$10 jobs that calculate to eight or ten digits very quickly. Or finally, some suggest

---

<sup>1</sup> I dedicate these remarks to the finest high school teacher I have ever known: Don Stover of Arlington, Virginia, from whom I have taken their central content.

<sup>2</sup> The algorithms that follow have been programmed on a Texas Instruments TI-74 Basicalc with minor modifications to fit that calculator's particular demands. This algorithmic language would require only similar changes to work on any programmable calculator or computer. Interestingly smaller calculators like this often provide ten digit accuracy while personal computers only give six.

that the calculations are done by Taylor series. That is closer to the truth and is in fact sometimes the case, but I avoid advanced techniques here.

Second, the algorithms I will offer are oversimplified and in fact, especially in the case of trig calculations, may be replaced by more efficient programs that utilize simple (but still high school) ideas about complex numbers. I do not address these CORDIC chip techniques because of space.<sup>3</sup> You may then, if you wish, retitle this paper to "Approximately How Your Scientific Calculator Could Calculate," but most of the ideas here are incorporated in that chip.

And finally, I assume the four basic operations in what follows. You are to see how to progress from addition, subtraction, multiplication and division to powers, roots, trig functions and logarithms. How the four basic operations are carried out is the subject of another paper, one of interest at least equal to what follows.

### The Square Root Key.

We start with the square root key. You type in a number, say 3, and then you press a key that has a square root symbol on it. Almost instantaneously up comes 1.732050808. What is going on here?

When some of your older teachers and I went to school, we calculated square roots by a complex algorithm. It was like long division, but not quite. You multiplied and brought down and multiplied by twenty, but nobody ever explained why it worked — I now know that was because very few knew. Then, still before your schooling, another algorithm was introduced. It was simpler, but it still raised problems in those pre-calculator times.<sup>4</sup>

This new square root method used the idea that the arithmetic mean (or average) approximated the geometric mean. That is, that

---

<sup>3</sup> I refer you to Stover's development of those techniques which I will provide to anyone who writes to me for them.

<sup>4</sup> Do you realize that the hand-held scientific calculator is scarcely as old as you are? I bought one of the first reverse Polish Hewlett Packard scientific calculators for \$400 just over twenty years ago. And the four bangers that preceded them and performed only addition, subtraction, multiplication and division came not much earlier.

$$\sqrt{xy} \doteq \frac{x+y}{2}.$$

To find the square root of a number like 3, you guess a starting number — our first  $x$ , perhaps 2. Since  $xy = 3$ , we can find  $y$  by dividing 3 by 2. It is 1.5. We have effectively bracketed  $\sqrt{3}$  between 2 and 1.5. To get a better  $x$ , we average these two numbers. Clearly the average of 2 and 1.5 is 1.75. And this becomes our next approximation of  $\sqrt{3}$ .

Now let us record that process of going from one estimate to the next. We seek  $\sqrt{N}$ .

$$x_1 = \text{initial guess}$$

$$x_{n+1} = \frac{x_n + N/x_n}{2}$$

Now if we continue our process of finding  $\sqrt{3}$  by this means we would have the sequence of  $x_n$ 's: 2, 1.75, 1.732142857, 1.73205081, 1.732050808; and that, in just four iterations or repetitions, is  $\sqrt{3}$  to ten digit accuracy, a remarkably fast convergence. In fact, it is easily established that, once you gain accuracy to some decimal digits, that accuracy at least doubles the number of digits with each step.

Of course, the problem with that simple algorithm — guess, divide, average — was that it involved long division. Without a calculator, long division is a tedious and error generating process for any of us; and, for you who were brought up in an age when such calculations are done by machine, I expect that it is still worse. In any case, for a calculator, dividing a ten digit dividend by a ten digit divisor is just like dividing 3 by 2.

To build a square root function behind that key, then, all we need to do is hard wire a little program to carry this out. Here is an over-simplified version.<sup>5</sup>

```

INPUT N
X ← N + .25
FOR K = 1 TO 10
  X ← (X + N / X) / 2
NEXT K
OUTPUT X : END

```

That little routine runs very fast on a calculator even when it is not hard

<sup>5</sup> The second line merely gives us an  $X$  from which to start. We cannot guess in an algorithm.

wired.<sup>6</sup>

Now we're off and running. We know how the square root key works. Watch carefully. We'll use that subroutine in what follows several times.

### The $x^y$ Key.

Next we turn to the  $x^y$  key. And first, let's see how it might work for positive integer powers. How about this algorithm to calculate  $B^E$ ,  $E$  a positive integer.

```

INPUT B, E
P ← 1
FOR K = 1 TO E
  P ← P × B
NEXT K
OUTPUT P : END

```

That algorithm conforms to the definition. It multiplies  $B$  by itself  $E$  times. The problem is that it is too slow for big powers. For example, consider  $1.04^{1000}$ . On my calculator, it takes that program about 18 seconds to come up with the answer. In other words, the program is running through that loop at the rate of about 55 times a second, but still not fast enough.

Here is a program that speeds up this calculation of  $B^E$ .

```

INPUT B, E
P ← 1 : S ← B : Q ← E
1 R ← Q - 2 × INT (Q / 2)
IF R = 1 THEN P ← P × S
Q ← INT (Q / 2)
IF Q = 0 THEN OUTPUT P : END
S ← S × S
GOTO 1

```

When I calculate  $1.04^{1000}$  by this algorithm, the answer comes up in about a second, quite a saving in time.

---

<sup>6</sup> It gives correct roots in the range .0001 to almost 10000. For  $\sqrt{10000}$  it gives 100.0000003. And this is what I meant when I said that I was oversimplifying. I chose to run the loop ten times, but a better program would use some simple means to determine how many times to repeat. I do not address such details.

We can see what is happening by tracing the calculation of  $3^{13}$ . Effectively we consider 13 in binary form, as  $1101_{\text{two}}$ . Reading from the right, we get 3 (no  $3^2$ ) times  $3^4$  times  $3^8$ . For  $1.04^{1000}$ , since  $1000 = 1111101000_{\text{two}}$ , we would have

$$1.04^{1000} = 1.04^8 \times 1.04^{32} \times 1.04^{64} \times 1.04^{128} \times 1.04^{256} \times 1.04^{512}.$$

Now there was no saving in our calculation of  $3^{13}$ , but there was a great deal of saving in calculating  $1.04^{1000}$ . We used ten loops instead of 1000, not a saving of the full 99%, because each loop contained more steps, but a substantial savings indeed; in terms of time, I estimate about 90%.

Now we come to the important step. We can raise to positive integer powers. What about roots, or equivalently rational powers? Recall in this regard that  $\sqrt[5]{7}$  is the same as  $7^{1/5}$  or  $7^{0.2}$ .<sup>7</sup>

You may be surprised to learn that the tool that we will use to solve this problem is our square root algorithm. We will call upon that subroutine as  $\text{SQR}(x)$  in the following algorithm.

To calculate  $B^E$ ,  $E$  rational and  $0 < E < 1$ .

```

INPUT B, E
P ← 1 : S ← B : X ← E
1 F ← INT (2 × X)
S ← SQR (S)
IF F = 1 THEN P ← P × S
X ← 2 × X - F
IF X = 0 OR ABS (S - 1) < .0000001 THEN OUTPUT P : END
GOTO 1

```

A trace will provide some insight into how this eight line algorithm works for the calculation of  $5^{0.6875}$ .

B	E	P	S	X	F
5	0.6875	1	5	0.6875	1
		$\sqrt{5}$	$\sqrt{5}$	0.375	0
		$\sqrt[4]{5}$	$\sqrt[4]{5}$	0.750	1
		$\sqrt[8]{5}$	$\sqrt[8]{5}$	0.5	1
		$\sqrt[16]{5}$	$\sqrt[16]{5}$	0	1

<sup>7</sup> We need only consider rational powers between 0 and 1, because we can combine the last routine to get other powers. For example,  $1.04^{8.35}$  would be  $1.04^8$  times  $1.04^{0.35}$ .

What is happening here is that we are splitting up our exponent into powers of  $1/2$  and those powers,  $1/2, 1/4, 1/8, \dots$ , correspond to  $\sqrt{5}, \sqrt[4]{5} = \sqrt{\sqrt{5}}, \sqrt[8]{5} = \sqrt{\sqrt{\sqrt{5}}}, \dots$ . In this case:

$$\begin{aligned} 5^{0.6875} &= 5^{0.5} \times 5^{0.1875} = 5^{0.5} \times 5^{0.125} \times 5^{0.0625} \\ &= \sqrt{5} \times \sqrt{\sqrt{5}} \times \sqrt{\sqrt{\sqrt{5}}}. \end{aligned}$$

Notice that the roots used correspond to the exponent in binary representation:

$$0.6875 = 0.1011_{\text{two}}.$$

In fact, the algorithm includes a means of converting a decimal to a binary fraction.<sup>8</sup> You simply double your decimal and pick off the whole number part for your binary representation. In this case,  $0.6875 \times 2 = 1.375$ ; take off the 1. Then  $0.375 \times 2 = 0.75$ ; take off the 0. Then  $0.75 \times 2 = 1.5$ ; take off the 1. Then  $0.5 \times 2 = 1.0$ ; take off the 1 and you're done. Recording those 1's and 0's you peeled off in the same order, you have our .1011.

### The Circular Functions.

Next we turn to the calculation of the circular functions and I offer a program that calculates cosine. You should recall that we can calculate all of the other five functions through use of the relationships among them. For example, the secant is merely the reciprocal of the cosine, the sine  $x = \pm \sqrt{1 - \cos^2 x}$  with the sign determined by the quadrant, and so on.

First, here is the program, another remarkably short one.

To calculate cosine  $X$ ,  $X$  in radians.<sup>9</sup>

```

INPUT X
X ← ABS (X)
X ← X - PI × INT (X / PI)
S ← X × X / 4294967296
FOR K = 1 TO 16
  S ← S × (4 - S)

```

<sup>8</sup> Is that a binimal?

<sup>9</sup> Clearly, calculating  $\cos X$  for  $X$  in degrees would only require the addition of a second line  $X \leftarrow X \times \text{PI}/180$ .

NEXT K  
 OUTPUT 1 - S / 2 : END

In this program the second line converts negative angles to positive and the third uses a standard reduction formula to change to an equivalent arc or angle between 0 and  $\pi$ . Thus the actual calculation takes only five lines.

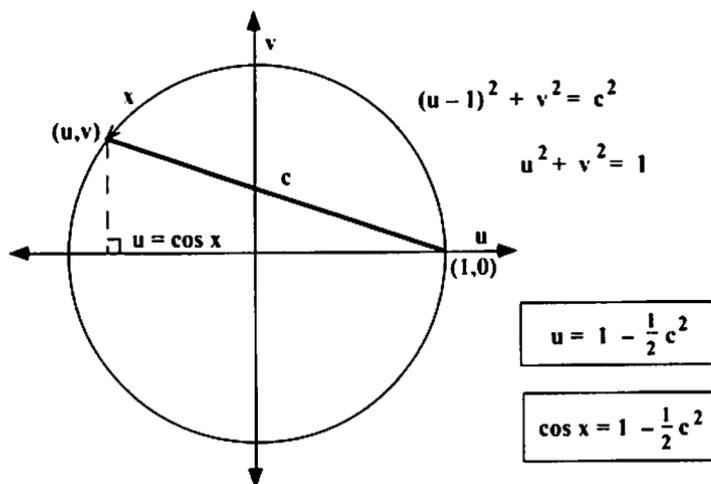


Figure 1.

To justify this simple program, we need two geometric theorems. You should be able to follow their derivation from the diagrams. The first relates cosine  $x$  to the length of a chord associated with the arc  $x$  (see Figure 1) and the second relates the length of a chord  $h$  to the length of the chord  $d$  of twice its arc (see Figure 2).

Now we see how those two theorems are applied to finding cosine  $x$  by the stated algorithm. Notice in Figure 3 how, each time you halve an arc, the chord length is closer to the arc length. In the algorithm the original arc  $x$  is halved 16 times; that is, divided by  $2^{16} = 65536$ . For that short an arc, clearly the chord is almost exactly equal in length and we take it to be equal. We square it<sup>10</sup> and use our second theorem sixteen times to double its length, noting that in the algorithm  $s$  represents both  $h^2$  and  $d^2$ . When we are back to the chord length

<sup>10</sup> That is where the 4294967296 comes from in the algorithm.  $(h/65535)^2 = h^2/4294967296$ .

(squared) corresponding to the arc  $x$ , we use the first theorem to get cosine  $x$ . This time our  $s$  is  $c^2$ .

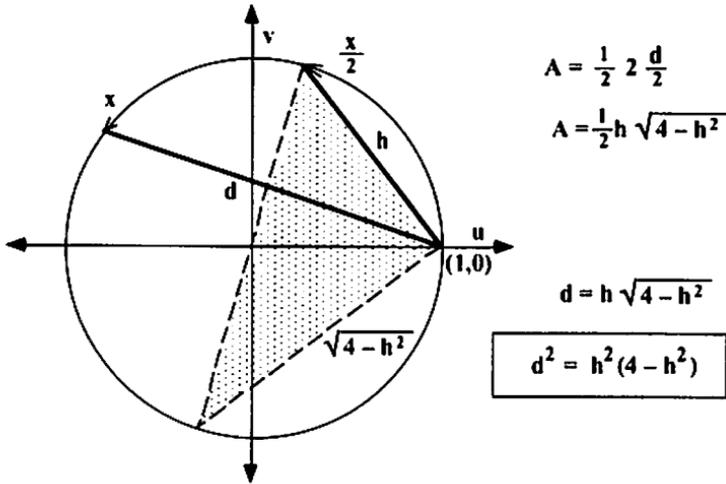


Figure 2.

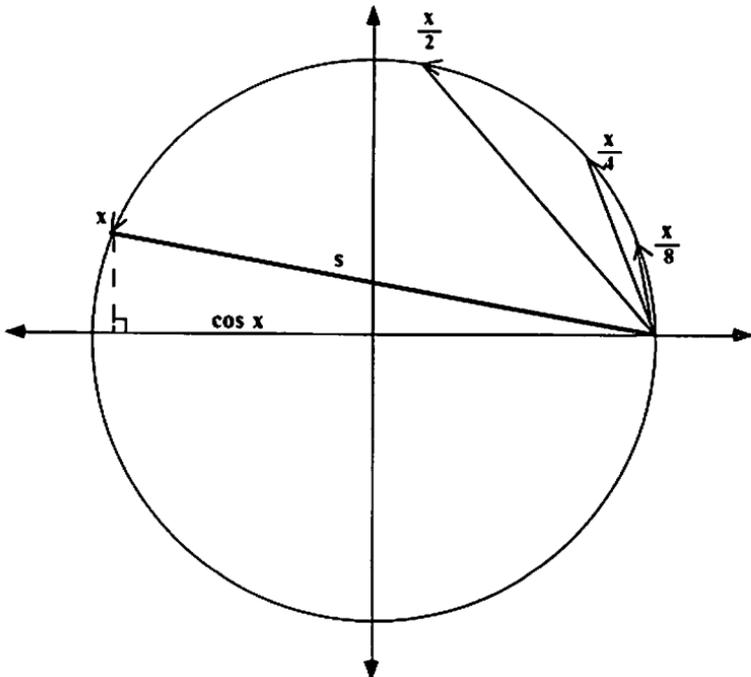


Figure 3.

## Logarithms.

Finally we turn to the calculation of logarithms.

To calculate  $\log_B X$ , where  $X > 1$ .

```

INPUT B, X
V2 ← 1 : L2 ← 0
1 V2 ← B × V2 : L2 ← L2 + 1
  IF X > V2 THEN GOTO 1
  V1 ← V2 / B : L1 ← L2 - 1
  FOR K = 1 TO 25
    V ← SQR (V1 × V2)
    L ← (L1 + L2) / 2
    IF X > V THEN V1 ← V : L1 ← L
    IF X ≤ V THEN V2 ← V : L2 ← L
  NEXT K
OUTPUT L : END

```

What is going on here? You should trace the process to gain insight into it. In doing so it will be well to remember two things: the  $V$ 's are values that correspond to the  $L$ 's, their logarithms; and the first five lines determine the whole number or characteristic part of the logarithm.

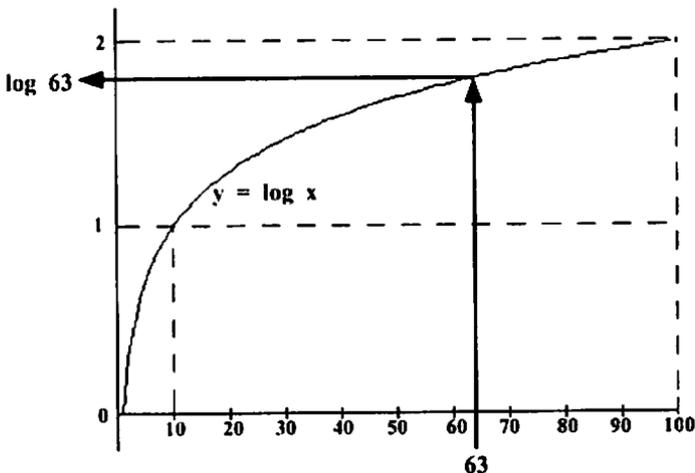


Figure 4.

This quite remarkable algorithm uses an identity that again relates, in a way different from our square root algorithm, the geometric and arithmetic means. That identity is

$$\log \sqrt{xy} = \frac{\log x + \log y}{2} .$$

Consider how this algorithm would go about calculating  $\log 63$  (with base  $B = 10$ ). First the program determines the characteristic. Clearly  $\log 63$  lies between  $\log 10 = 1$  and  $\log 100 = 2$  (see Figure 4). Next it squeezes in on  $\log 63$  through repeated use of our geometric mean-arithmetic mean relationship. Here is how the calculations work.

$$\log \sqrt{10 \times 100} = \frac{\log 10 + \log 100}{2}$$

$$\log \sqrt{1000} = \frac{1 + 2}{2}$$

$$\log 31.6227766 = 1.5 .$$

Now, since  $63 > 31.6227766$  we have reduced our problem to finding  $\log 63$  between  $\log 31.6227766 = 1.5$  and  $\log 100 = 2$ .

$$\log \sqrt{31.6227766 \times 100} = \frac{\log 31.6227766 + \log 100}{2}$$

$$\log \sqrt{3162.27766} = \frac{1.5 + 2}{2}$$

$$\log 56.23413252 = 1.75$$

and so on, each time using our square route algorithm on the left side of the equation and on the right merely averaging. A few more steps give

$$\log 74.98942093 = 1.875,$$

$$\log 64.93816316 = 1.8125 \text{ and}$$

$$\log 60.42963903 = 1.78125.$$

**Conclusion.**

If you have followed the development of the foregoing algorithms, you should come away with a better appreciation of the mathematical thinking that went into the development of the scientific calculator just twenty years ago. That thinking was at an elementary level but was still highly creative.

I leave you with two sides of the same question. The mathematicians of the 17th, 18th and 19th centuries, and even those of the early part of the 20th century, bogged down in calculation. The top side of my question is then: What wonderful things might those mathematicians have accomplished if they had been freed of the drudgery of computation by the simple machinery of today?

But when you look at the results those mathematicians came up with, you notice that they were often derived from their calculating experience. The obverse of my question is then: What might they have missed, and indeed what do you miss, being freed from that seemingly mindless labor?

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## The Problem Corner

Edited by Kenneth M. Wilke

*The Problem Corner* invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before 1 August 1994. Solutions received after the publication deadline will be considered also until the time when copy is prepared for publication. The solutions will be published in the Fall 1994 issue of *The Pentagon*, with credit being given to student solutions. Affirmation of student status and school should be included with solutions. Address all communications to Kenneth M. Wilke, Department of Mathematics, 275 Morgan Hall, Washburn University, Topeka, Kansas 66621.

### PROBLEMS 470-474.

*Problem 470.* Proposed by J. Sriskandarajah, University of Wisconsin Center-Richland, Richland Center, Wisconsin. Find all possible integers  $a$  and  $b$  such that  $a^b + b^a = 100$ .

*Problem 471.* Proposed by J. Sriskandarajah, University of Wisconsin Center-Richland, Richland Center, Wisconsin. Find all positive integers  $a$ ,  $b$  and  $c$  such that  $a^3 + b^3 + c^3 = abc$  where  $abc$  denotes a three digit number formed by concatenating  $a$ ,  $b$  and  $c$  in the order shown and not the product  $abc$ .

*Problem 472.* Proposed by Del Ebadi, Topeka West High School, Topeka, Kansas. Given that the absolute value of the average of two real numbers is  $4\sqrt{6}$ , find the absolute value of the difference of the two numbers without finding the numbers themselves.

*Problem 473.* Proposed by Del Ebadi, Topeka West High School, Topeka, Kansas. Two numbers  $a$  and  $b$  are selected randomly;  $a$  is chosen from the interval  $[0, 2]$  and  $b$  is chosen from the interval  $[0, 3]$ . Find the

probability that  $a > b$ . What is the probability if 2 and 3 are replaced by  $m$  and  $n$ , respectively?

*Problem 474.* Proposed by Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin. Evaluate

$$I = \int \frac{1}{1 - \sin(x)} dx$$

by converting the integrand to the form  $D_x h(x)$ .

*Please help your editor by submitting problem proposals.*

#### SOLUTIONS 461-463.

*Problems 460 and 464 remain open.*

*Problem 460.* Proposed by the Editor. The natural numbers 281 and 1926 have the property that

$$1926^2 + 5 \equiv 0 \pmod{281}$$

and

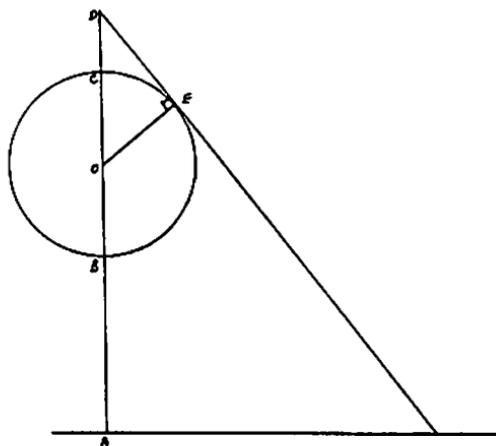
$$281^2 + 5 \equiv 0 \pmod{1926}.$$

Prove that there are an infinite number of pairs of natural numbers with this property and find an infinite family of solutions.

Since no solution has been received, this problem will remain open for another issue.

*Problem 461.* Proposed by Russell Euler, Northwest Missouri State University, Maryville, Missouri. A pole of length  $p$  is perpendicular to the level ground on which it stands. The pole is surmounted by a sphere with a radius of length  $r$ . A person is standing at the "north pole" of the sphere and the person's eyes are at a height  $m$  above the north pole. If all distances are measured in feet, find the area of the ground that cannot be seen by the person.

*Solution* by Fred A. Miller, Elkins, West Virginia.



In the figure above, let  $\overline{AB} = p$ ,  $\overline{BOC} = 2r$  and  $\overline{CD} = m$ . Then  $\triangle DOE$  is similar to  $\triangle DFA$ . Thus  $\overline{DE}/\overline{OE} = \overline{DA}/\overline{FA}$ . Hence  $\overline{DE}^2 = (r+m)^2 - r^2$ . Also,  $\overline{DA} = p + 2r + m$ . Let  $\overline{FA} = R$ . Thus  $\overline{DE}^2/r^2 = \overline{DA}^2/R^2$  where  $R^2 = (r^2(p+2r+m)^2)/((r+m)^2 - r^2)$ . The desired area is  $\pi R^2$ .

Also solved by Charles D. Ashbacher, Cedar Rapids, Iowa; J. Sriskandarajah, University of Wisconsin Center-Richland, Richland, Wisconsin; Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin; and the proposer.

**Problem 462.** Proposed by J. Sriskandarajah, University of Wisconsin Center-Richland, Richland Center, Wisconsin. Find the sum of the arithmetico-geometric series

$$ag + (a+d)gr + (a+2d)gr^2 + \cdots + (a+(n-1)d)gr^{n-1}$$

and also derive the sum of the corresponding infinite series when  $|r| < 1$  where  $|\cdot|$  denotes absolute value. Hence find the sum of the infinite series

$$-4 - 2 + 3 - \frac{5}{2} + \frac{7}{4} - \frac{9}{8} + \frac{11}{16} - \cdots$$

**Solution** by Bob Prielipp, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin.

Let

$$S = ag + (a+d)gr + (a+2d)gr^2 + \cdots + (a+(n-1)d)gr^{n-1}$$

If  $r = 1$ ,  $S = nag + dgn(n-1)/2$ . If  $r \neq 1$ , then

$$S = ag + agr + dgr + agr^2 + 2dgr^2 + \dots \\ + agr^{n-1} + (n-1)dgr^{n-1},$$

$$rS = agr + agr^2 + dgr^2 + \dots + agr^{n-1} + (n-2)dgr^{n-1} \\ + agr^n + (n-1)dgr^n$$

and so

$$(1-r)S = ag + dgr + dgr^2 + \dots + dgr^{n-1} - agr^n - (n-1)dgr^n \\ = ag + (dgr + dgr^2 + \dots + dgr^{n-1}) - (a + (n-1)d)gr^n.$$

Hence, if  $r \neq 1$ , then

$$S = \frac{ag}{1-r} + \frac{dgr(1-r^{n-1})}{(1-r)^2} - \frac{(a+(n-1)d)gr^n}{1-r} \\ = \frac{ag}{1-r} + \frac{dgr}{(1-r)^2} + \frac{dgr^n}{(1-r)^2} - \frac{(a+(n-1)d)gr^n}{1-r}.$$

If  $|r| < 1$ , it follows that for the infinite series,

$$ag + (a+d)gr + (a+2d)gr^2 + \dots = \frac{ag}{1-r} + \frac{dgr}{(1-r)^2}. \quad (*)$$

To generate the infinite series

$$-4 - 2 + 3 - \frac{5}{2} + \frac{7}{4} - \frac{9}{8} + \frac{11}{16} - \dots,$$

let  $a = -4$ ,  $g = 1$ ,  $d = 8$  and  $r = -1/2$ . From (\*), the sum of this infinite series is  $-40/9$ .

*Also solved by* Russell Euler, Northwest Missouri State University, Maryville, Missouri; Charles D. Ashbacher, Cedar Rapids, Iowa and the proposer. One incorrect solution was received.

**Problem 463.** Proposed by Lamarr Widmer, Messiah College, Grantham, Pennsylvania. This problem is proposed in honor of Volume 52 of *The Pentagon* which is being published in 1993. Find positive integers  $a, b, c$  and  $d$  such that

$$\frac{52}{1993} = \frac{1}{a + \frac{9}{b + \frac{9}{c + \frac{3}{d}}}}$$

*Solution* by Lawrence Skaggs, student, Eastern Kentucky University, Richmond, Kentucky (revised by the editor).

Throughout the solution  $a$ ,  $b$ ,  $c$  and  $d$  are positive integers. Using cross multiplication and subtracting  $a$  from both sides we obtain

$$\frac{1993 - 52a}{52 \times 9} = \frac{1}{b + \frac{9}{c + \frac{3}{d}}} < 1.$$

Hence  $1993 - 52a > 0$  implies that  $a < 38.327$  so  $a \leq 38$ . Using the same idea,

$$(1) \quad \frac{9}{c + \frac{3}{d}} = \frac{468}{1993 - 52a} - b > 0,$$

which implies that  $1 < b < 468/(1993 - 52a)$  which, in turn, implies that  $1993 - 52a < 468$  or  $a > 29.327$ . Hence

$$(2) \quad 30 < a < 38.$$

This relation also establishes the upper bound

$$(3) \quad b \leq \left[ \frac{468}{1993 - 52a} \right]$$

on  $b$  in terms of  $a$ , where  $[ ]$  is the greatest integer function. Noting that both  $c$  and  $d$  are positive integers, (1) also implies that

$$(4) \quad \frac{9}{\frac{468}{1993 - 52a} - b} > b$$

or

$$(5) \quad \frac{468}{1993 - 52a} - 9 > b,$$

which provides a nice lower bound for  $b$ ; i.e. using (3) and (4) for  $a = 38$ , we have  $18.529 < b < 27.52$ . (for  $a = 38$ , we have  $1993 - 52 \times 38 = 17$  so that  $b < 468/17 = 27.52$ ).

At this point, for each value of  $a$  and  $b$ , we use the following relation to check for possible values of  $c$  and  $d$  starting at the maximum possible value for  $c$  and determining the corresponding values of  $d$ . By decreasing the values of  $c$  until  $c = 1$  is reached, all possible cases are tested.

$$(7) \quad \frac{3}{d} = \frac{9}{\frac{468}{1993 - 52a} - b} - c.$$

Whenever possible values for either  $c$  or  $d$  become less than 1, we change to the next possible values of  $a$  and  $b$ . For example, consider  $a = 30$ . By (3),  $b$  must be 1. Then by (7),  $3/d = 3897/35 - c$ . Hence  $d = 105/(3897 - 35c)$ . Finally  $d \geq 1$  requires that  $105 \geq 3897 - 35c$  or  $c \geq 108.3$  while  $3897 - 35c > 0$  implies that  $c < 111.3$ . Testing the values of 109, 110 and 111 for  $c$  yields no integral values for  $d$ . Similar but tedious analysis for the other cases yields the seven solutions  $(a, b, c, d) = (34, 1, 8, 9)$ ,  $(34, 2, 111, 2)$ ,  $(34, 2, 112, 6)$ ,  $(37, 6, 10, 2)$ ,  $(37, 6, 11, 6)$ ,  $(38, 27, 14, 1)$  and  $(34, 27, 16, 3)$ .

*Solution* by Keith Pigeon, student, Eastern Kentucky University, Richmond, Kentucky (revised by the editor).

After finding five solutions using analysis similar to that used in prior solution, the following program in PASCAL found all seven solutions as listed above.

```
PROGRAM Credit (INPUT, OUTPUT);
VAR a,b,c,d,f:INTEGER;
    p,e,k,q:REAL;
BEGIN
  f:=0;
  e:= 1/100000000;
  FOR a:=1 TO 38 DO
    FOR b:=1 TO 27 DO
      FOR c:=1 TO 150 DO
        FOR d:=1 TO 20 DO
          BEGIN
            k:=52/1993;
            q:=1/(a+9/(b+9/(c+3/d)));
            p:=ABS(k-q);
            IF p < e THEN
              BEGIN
                f:=f+1;
                WRITELN(a,b,c,d);
```

```

                END;
            END;
        WRITELN(f);
    END.

```

Also solved by Dayong Li (2 solutions) [(38,27,14,1) and (38,27,16,3)], student, Eastern Kentucky University, Richmond, Kentucky; J. Sriskandarajah (2 solutions) [(38,27,14,1) and (38,27,16,3)], University of Wisconsin Center-Richland, Richland Center, Wisconsin; Jill Carnahan (1 solution) [(37,6,10,2)], student, Eastern Kentucky University, Richmond, Kentucky; Charles D. Ashbacher (1 solution) [(38,27,14,1)], Cedar Rapids, Iowa; Sheree Mathews (1 solution) [(38,27,16,3)], student, Central Michigan University, Mount Pleasant, Michigan; Sammy Yu and Jimmy Yu jointly (1 solution) [(34,1,8,9)], Vermillion Middle School, Vermillion, South Dakota; and the proposer (4 solutions) [(38,27,14,1), (38,27,16,3), (37,6,10,2) and (34,1,8,9)]. One incorrect solution was received.

*Editor's Comment.* This problem proved to be challenging because it can be attacked either by hand or by computer. One of our featured solvers performed lengthy analysis by hand and showed that there are exactly 7 solutions as given above while the other achieved the same result by computer. Sammy Yu is a seventh grader and Jimmy Yu is a sixth grader. Their analysis is very mature for their ages! We probably will hear more about them in the future. Following the lead given by our featured solver, one obtains the following conditions on the positive integers  $a, b, c$  and  $d$ .

$$(1) \quad 30 < a < 38,$$

$$(2) \quad 0 < b < \left[ \frac{468}{y} \right],$$

where  $y = 1993 - 52a$ ,

$$(3) \quad \frac{y(9 + 3b) - 1404}{z} < c < \frac{9y}{z},$$

where  $z = 468 - by$ , and

$$(4) \quad d = \frac{3z}{y(9 + bc) - 468c}$$

Since  $a, b, c$  and  $d$  are positive integers and we can have no zero denominators, the following simple computer program in BASIC does the necessary substitutions to find the solutions in a few seconds. Here  $ub, lb$

and  $uc$ ,  $lc$  are respective upper and lower bounds for  $b$  and  $c$ .

```

10 for a=30 to 38
20 y=1993-52*a:bu=int(486/y):lb=ub-9
30 if lb<1 then lb=1
40 for b=lb to ub
50 z=468-b*y:if z=0 then 140
60 uc=int((9*y)/z):lc=int((y*(9+3*b)1404)/z)
70 if lc<1 then lc=1
80 for c=lc to uc
90 if (9+b*c)*y-468*c=0 then 130
100 d=(3*z)/((9+b*c)*y-468*c)
110 if d<>int(d) then 130
120 print a,b,c,d
130 next c
140 next b
150 next a
160 end

```

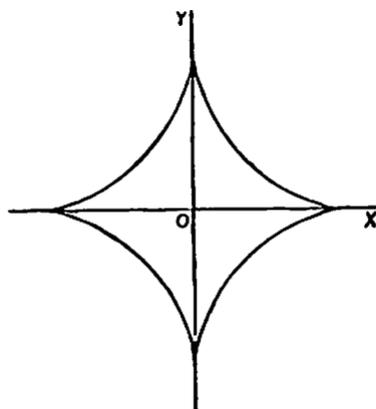


Figure 1.

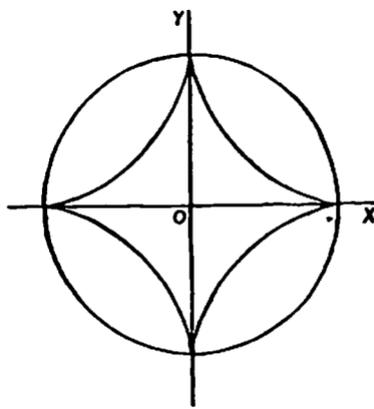


Figure 2.

**Problem 464.** Proposed by Mary Elick, Missouri Southern State College, Joplin, Missouri. Let  $C$  denote the curve given by the equation

$$x^{2/3} + y^{2/3} = 1$$

as shown in Figure 1. Suppose that the curve  $C$  is made from a flexible material which is attached to the coordinate axes at the points  $(0,1)$ ,  $(1,0)$ ,  $(0,-1)$  and  $(-1,0)$  and which may be moved without changing the

length of  $C$ . Let  $C_p$  denote the "companion curve" for  $C$  which is formed by inflating  $C$  with air until it "pops outward" as shown in Figure 2. (a) Find the equation of the companion curve  $C_p$ . (b) Find the derivative, if it exists, at the point  $(0,1)$  on the curve  $C_p$ . (c) Given that the curve  $C$  is the circle described by the equation  $x^2 + y^2 = 1$ , what is the equation of the "popped in" companion curve  $C_p$  resulting from "deflating" curve  $C$  appropriately?

Since no solution has been received, this problem will remain open for another issue.

## Convention Winners



National President Arnold D. Hammel (*far right*) poses with the authors of the best papers presented at the 29th Biennial Convention held 22-24 April 1993 at Niagara University (*left to right*): Rachel Zeller (Colorado Gamma), Mitch Richling (Kansas Alpha), Karen Brown (Iowa Alpha) and Jennifer Couter (California Gamma).

## Kappa Mu Epsilon News

Edited by Mary S. Elick, Historian

News of chapter activities and other noteworthy *KME* events should be sent to Mary S. Elick, Historian, Kappa Mu Epsilon, Mathematics Department, Missouri Southern State College, Joplin, Missouri 64801.

### INSTALLATION OF NEW CHAPTERS

#### *Louisiana Gamma*

Northwestern State University, Natchitoches

The installation of the Louisiana Gamma Chapter of Kappa Mu Epsilon was held on March 24, 1993, at the Landing Restaurant in Natchitoches, LA. Dr. Harold L. Thomas, National President of Kappa Mu Epsilon, conducted the installation ceremony. Professor Thomas Covington, faculty member of the Northwestern State University Department of Mathematics and a member of *KME* from Mississippi Gamma served as Conductor during the ceremony. Ten students and six faculty constituted the founding group of the new chapter at Northwestern State University. Those initiated were:

*Students:* Heather Chandler, Angela Dehart, Audra Delrie, Marianne Elliott, Nicholas Gajcowski, Donna Prothro, Shawn Rains, Stephanie Richardson, Samantha Smith and James Stamey.

*Faculty:* Dr. Stan Chadick, Dr. David Goloff, Dr. Tom Hanson, Prof. Ben Rushing, Dr. Frank Serio and Dr. Austin Temple.

Following the initiation ceremony, Dr. Thomas gave a brief history of honor societies in colleges and universities and, in particular, the founding of Kappa Mu Epsilon. Several Northwestern State University

administrators attended the installation ceremony which was preceded with an excellent steak dinner. Dr. Austin Temple, Mathematics Department Chair, and Dr. Robert Alost, Northwestern State University president, gave brief remarks to the group.

Officers installed during the ceremony were James Stamey, president; Heather Chandler, vice president; Samantha Smith, secretary; and Stephanie Richardson, treasurer. Faculty member Frank Serio accepted the responsibilities of the corresponding secretary and faculty sponsor.

## CHAPTER NEWS

### *California Gamma*

Chapter President – Eric Bauer

California Polytechnic State University, San Luis Obispo

30 actives

In an effort to increase fellowship among members and pledges, California Gamma instituted a Progressive Dinner Social which starts out at one house for appetizers, continues to another house for salad, then to another for the main course and finally to another house for dessert. The chapter held a marathon fund-raising booksale from April 12 through April 22. Also in April, Eric Bauer and Jennifer Courter attended the *KME* Biennial Convention. Jennifer presented a paper on search procedures, an elaboration of work she had initiated in the Undergraduate Seminar. Three members represented *KME* at the Annual Banquet of the College of Science and Mathematics in May. On June 4, California Gamma held its Annual Banquet at the Apple Farm Restaurant in San Luis Obispo. Twenty pledges were initiated and three awards were given. Doug Rosenfeld presented the Arthur Andersen Professional Performance Award from his company to Eric Emerton, an outstanding junior mathematics major. Dr. Stephen Weinstein, Chair of the Mathematics Department, attending with his wife, Eleanor, presented the W. Boyd Judd Scholarship to Cindy Hampton, who will be graduating at the end of the Fall 1993 quarter. Dr. Raymond D. Terry presented the *KME* Founders' Award to Jennifer Courter for contributions to the organization and functioning of the club. The banquet was also attended by Dr. and Mrs. Sabah Al-Hadad, long-time supporters of *KME*. Other chapter officers: Jennifer Courter, vice president; Henry Mesa, treasurer; Raymond D. Terry, corresponding secretary/faculty sponsor.

*Colorado Gamma*

Chapter President – Rachel Zeller

Fort Lewis College, Durango

12 actives

The chapter held two meetings during the semester and an initiation ceremony for eight new members. Professor Gibbs and three students, Susan Clinkenbeard, Rachel Zeller and Jim Zieff, attended the Biennial Convention at Niagara University and had a wonderful time. Rachel presented the first paper ever given by a Colorado Gamma member, placing third in the paper competition! Other chapter officers: Jim Zieff, vice president; Mary Wright, secretary; Stevan Scott, treasurer; Richard Gibbs, corresponding secretary; Deborah Berrier, faculty sponsor.

*Colorado Delta*

Chapter President – Rebecca Brown

Mesa State College, Grand Junction

20 actives

The chapter hosted a visiting lecturer from the Bureau of Land Management who spoke on Global Locater Systems. A field trip to the Grand Junction Office of the U. S. Weather Station to see mathematics and statistics in use was also sponsored by the group. Initiation ceremonies and dinner were held on April 22 with 10 new members being initiated. Other chapter officers: David Lister, vice president; Linda Brakelsberg, secretary; William Haworth, treasurer; Donna Hafner, corresponding secretary; Clifford Britton, faculty sponsor.

*Georgia Alpha*

Chapter President – Debbie Ingle

West Georgia College, Carrollton

25 actives

On June 2, ten new members were initiated and chapter officers were elected for the coming year. Following the election, a reception was held in honor of the 1993 initiates. At the reception, it was announced that four chapter members had been awarded scholarships for 1993-94: Debbie Ingle received the Crider Award in Computer Science; Bill Walker, the Crider Award in Mathematics; Polly Quertermus, the Whatley Scholarship; and Tracey Gore, the Cooley Scholarship. Other chapter officers: Michael Boleman, vice president; Gregg Dennis, secretary; Polly Quertermus, treasurer; Joe Sharp, corresponding secretary/faculty sponsor.

*Illinois Delta*

Chapter President – Mark Mitchell

College of St. Francis, Joliet

18 actives

A successful fund-raiser, held in January, helped pay the expenses of those who attended the national convention in Niagara in April. Twelve

new members were inducted in March. Other chapter officers: Molly Sullivan, vice president; Jennifer Hoffmann, secretary; Carrie Briscoe, treasurer; Sister Virginia McGee, corresponding secretary/faculty sponsor.

### *Illinois Zeta*

Rosary College, River Forest

Chapter President – Catherine Nemesnyik

20 actives

Other chapter officers: Sister Mary T. O'Malley, corresponding secretary/faculty sponsor.

### *Indiana Beta*

Butler University, Indianapolis

Chapter President – Robert Haack

23 actives

Other chapter officers: Sara Sterk, vice president; Julie Jones, secretary; Maggie Hyre, treasurer; Jeremiah Farrell, corresponding secretary/faculty sponsor.

### *Indiana Gamma*

Anderson University, Anderson

Chapter President – Gina Shellenbarger

24 actives

Other chapter officers: Ann Pratt, vice president; Shane Meyer, secretary/treasurer; Stanley L. Stephens, corresponding secretary/faculty sponsor.

### *Iowa Alpha*

University of Northern Iowa, Cedar Falls

Chapter President – Julie Beck

38 actives

The highlight of the spring semester was certainly the *KME* National Convention at Niagara University, April 22-24, 1993. Students Mary Bond, Karen Brown and Jason Sash, along with faculty advisor John Cross attended. Karen Brown presented her paper, "Error Correction for Basic Codes," to the convention on April 23. Karen made an excellent presentation, was awarded fourth place by the judging committee and received a check for \$40.00 and an HP48S calculator at the awards ceremony. All who attended the convention enjoyed the programs and sightseeing that followed. Students presenting papers at local *KME* meetings include: Karen Cannell on "Dynamic Programming," Megan Adams and Emily Eckman on "A Brief Exploration of Symmetry," and Dan Gruman on "The Gauss-Salamin Algorithm." Stephen Walk addressed the initiation banquet on April 27 on "Combinatorial Logic: or, Haskell Curry's Favorite Way to Break the Ice at Parties." Six new members were initiated during the spring semester. Other chapter officers: Ted Juhl, vice president; Jennifer Puffett, secretary; Karen Brown, treasurer; John S. Cross, corresponding secretary/faculty sponsor.

*Iowa Beta*

Chapter President – Chris Merton

Drake University, Des Moines

5 actives

Kendall Bailey gave a talk on "Potentially Stable Tree Sign Patterns with Five Vertices." Other chapter officers: H. D. Krishnapriyan, corresponding secretary; Lawrence Naylor, faculty sponsor.

*Iowa Gamma*

Chapter President – Brad Bock

Morningside College, Sioux City

8 actives

Three new members were initiated into Iowa Gamma this year and the chapter has hopes for a larger initiation in both the fall and the spring of the upcoming school year. A successful fund-raising campaign was recently held. Two new officers along with two returning officers will try to give new life and direction to the organization next year. Plans have been made for monthly meetings with an emphasis on problem solving. There are also plans for an aggressive recruitment of inter-departmental majors. In the past the Iowa Gamma Chapter of Kappa Mu Epsilon has excelled in the enrichment of mathematics; the chapter hopes to bring this tradition back to the Morningside campus. Other chapter officers: Dean Stevens, vice president; Taylor Guo, secretary; Mike Murray, treasurer; Steve Nimmo, corresponding secretary/faculty sponsor.

*Iowa Delta*

Chapter President – Van Beach

Wartburg College, Waverly

48 actives

The January meeting began with pizza and continued with fellowship and planning for the annual Math Field Day. In February, Van Beach presented "Domes, Homes and the Future," the paper he later presented at the national convention, and Melissa Doll talked on "Spatial Visualization," one of the papers listed as an alternate for the convention. The chapter co-sponsored the Sixteenth Annual Wartburg Math Field Day on February 20. On March 27, new officers were elected and seventeen new members initiated. Six members of the chapter and the corresponding secretary attended the national convention April 22-24. In addition to the papers mentioned above, Jo Steffenson also had a paper on the program entitled "Mathematics of the Spirograph." In May, the year-end picnic was held. Other chapter officers: Rebecca Hertenstein, vice president; Wendy Ahrendsen, secretary; Jodie Harper, treasurer; August Waltmann, corresponding secretary; Glenn Fenneman, faculty sponsor.

**Kansas Alpha**

Pittsburg State University, Pittsburg

Chapter President – Mitch Richling

65 actives

The spring semester activities started with a pizza party and initiation in February for sixteen new members. Following the initiation ceremony, the members made plans for attending the national convention in April. The program for the March meeting was given by Mitch Richling. He gave a trial run on his paper submitted for presentation at the national convention, "General Convex Solid Modeling." The chapter hosted a guest speaker at the regular April meeting. Dr. David Surowski from Kansas State University presented a most interesting program on "Orientable Space: Where We Live (And Where We Don't)." Nine students and two faculty made the long drive to Niagara University in April for the national convention. An enjoyable time was had by all – especially by Mitch Richling whose paper was selected for first place by the awards committee. Dr. Harold Thomas, Kansas Alpha corresponding secretary, also completed his term as national president at this convention. The chapter assisted the Mathematics Department faculty in administering and grading tests given at the annual Math Relays in late April. Several members also worked on the Alumni Association's Annual Phon-a-thon. The final meeting of the semester was an ice cream and cake social event held at the University Lake. Reports were given by those who attended the Niagara convention in April. Officers for the 1993-94 school year were elected. The annual Robert M. Mendenhall awards for scholastic achievement were presented to Jill Feiss and Jefry Simamora. Other chapter officers: Eddy Kuo, vice president; Kristi Simone, secretary; Sherry Brennon, treasurer; Harold Thomas, corresponding secretary; Bobby Winters, faculty sponsor.

**Kansas Beta**

Emporia State University, Emporia

Chapter President – Jason Henry

20 actives

Other chapter officers: Andi Blair, vice president; Ted Dawdy, secretary; Katherine Griffith, treasurer; Connie Schrock, corresponding secretary; Larry Scott, faculty sponsor.

**Kansas Gamma**

Benedictine College, Atchison

Chapter President – David Klenke

14 actives, 16 associates

Initiated into Kansas Gamma in February were Marty Kay Heideman, Meghan Herbert, Michael McGuire and Jodie Muhlbauer. Following initiation and while final preparations were being made for a spaghetti dinner, moderator Jo Ann Fellin, OSB, gave a slide presentation on the history of Kansas Gamma. In March the group

gathered for noon lunch with Benedictine alum Barbara Holder who shared information on her work as a computer specialist at Atchison Hospital. In April David Klenke and Chris Enyeart gave their presentations for the local chapter before departing for Niagara University where they were on the program for the national convention. Also attending the national meeting from Kansas Gamma was Sister Jo Ann who continues to serve the organization as the National Treasurer. A large group gathered for an "End-of-the-Year" breakfast at Paolucci's honoring the graduating seniors. The large group was due to the "wake-up" calls made by Tiffany Opsahl and David Klenke. Other chapter officers: Pamela Clearwater, vice president; Tiffany Opsahl, secretary/treasurer; Jill Weigand, Stugo Representative; Jo Ann Fellin, OSB, corresponding secretary/faculty sponsor.

### *Kansas Delta*

Chapter President - Shelley Bauman

Washburn University, Topeka

35 actives

Twelve new members were initiated at the Annual Spring Initiation Banquet held in February. Next year's officers were elected at the May meeting and plans for next year's activities were discussed. Other chapter officers: Kirk Drager, vice president; Kyndra Graybeal, secretary; David Brady, treasurer; Allan Riveland, corresponding secretary; Ron Wasserstein and Gary Schmidt, faculty sponsors.

### *Kansas Epsilon*

Chapter President - Donna Weninger

Fort Hays State University, Hays

29 actives

Semester activities included monthly meetings and a spring banquet. Other chapter officers: Marc Enyart, vice president; Anita Lessor, secretary/treasurer; Charles Votaw, corresponding secretary; Mary Kay Schippers, faculty sponsor.

### *Kentucky Alpha*

Chapter President - Eddie Robinson

Eastern Kentucky University, Richmond

36 actives

The annual initiation ceremony included a talk by Dr. Patricia Costello entitled "Statistical Surveys and Presidential Polls." The presentation contained both a history and the mathematics of polling techniques and their sometimes erroneous results. A party was held afterwards in the student center. The other major event of the semester was the national convention. Seven students accompanied Dr. C. to the convention and had a great time. The students chose to end the trip with a closeup (and wet) view of the Falls on the Maid of the Mist boat ride. At the last meeting of the semester a report of the trip was given and Dr.

C. was presented with a new umbrella to replace the "holey" one he took on the trip. Also, new officers were elected and installed. Other chapter officers: Mike Mattingly, vice president; Becky Taylor, secretary; Crystal Pendencygraft, treasurer; Pat Costello, corresponding secretary; Kirk Jones, faculty sponsor.

### *Maryland Alpha*

Chapter President – Susan Miller Burgee

College of Notre Dame of Maryland, Baltimore

12 actives

On May 13, the Maryland Alpha Chapter held its annual dinner meeting which was well attended by chapter members and members of the Mathematics Society. Following dinner, initiation of new members was held and an address was given by Danielle Kulick, '93, on the topic of "Ethnomathematics." Other chapter officers: Laura Saffran, vice president; Sharon Pesto, secretary; Pamela Rick, treasurer; Sister Marie Dowling, corresponding secretary; Joseph DiRienzi, faculty sponsor.

### *Maryland Beta*

Chapter President – Brenton Squires

Western Maryland College, Westminster

22 actives

In April Maryland Beta sponsored a successful Career Night, inviting back four former mathematics majors to speak about their work experiences. In May the chapter hosted, in conjunction with the Math Department, the annual math majors picnic. Other chapter officers: Sun Yee Wu, vice president; Todd Wizotsky, secretary; William Yankosky, treasurer; James Lightner, corresponding secretary; Linda Eshleman, faculty sponsor.

### *Maryland Delta*

Chapter President – John Hughes

Frostburg State University, Frostburg

46 actives

Nineteen new members were inducted in February. At the induction, Dr. Steve Luzader of the Physics Department presented a talk entitled "Chaos Rules!" At other times during the semester, the chapter enjoyed presentations by Dr. John Biggs and student Tom Currier. Biggs spoke on "Columbus and the Short Route to Cipangu;" Currier's topic was "Mathematics of Navigation." Other chapter officers: Kileen Baker, vice president; Karl Streaker, secretary; Melissa Thomas, treasurer; Edward White, corresponding secretary; John Jones, faculty sponsor.

### *Massachusetts Alpha*

Assumption College, Worcester

13 actives

Eleven new members were initiated on March 3. Following a dinner in honor of the new members, Prof. Charles Brusard of the Assumption

faculty spoke on "The Gibbs Phenomenon in Fourier Series." Charles Brusard is corresponding secretary/faculty sponsor.

### *Michigan Beta*

Central Michigan University, Mt. Pleasant

Chapter President - Erica Hall

25 actives

Twelve new members joined the Michigan Beta chapter at our spring initiation. Professor Tom Miles was our speaker and gave a delightful talk entitled "Kappa, Mu and Epsilon." He looked at some topics in mathematics and physics which use the symbols  $\kappa$ ,  $\mu$  and  $\epsilon$  for notation. Tom did a fine job of relating these three Greek letters to what membership in an organization involves. Some of his comments are as follows.  $\kappa$  is often used to denote curvature. In life draw your circle of friends wide so make  $\kappa$  small. In *KME* you have a smaller circle of people with common interests, so your radius  $R$  is smaller with  $\kappa = 1/R$  bigger.  $\mu$  is often used to denote the coefficient of friction. May things go smoothly for you so  $\mu$  will be low. When you attach a problem, stick with it — keep  $\mu$  high. And remember that  $\mu$  depends on the material.  $\epsilon$  is used to mean "is an element of." You  $\epsilon$  *KME*. Participate and be active so there is no danger of confusion about whether you  $\epsilon$  *KME* or you  $\notin$  *KME*. Four members and their advisor attended the National Convention at Niagara University. All had a great time meeting students and faculty from other chapters, listening to student talks and visiting the Falls area. Thank you, New York Eta! Our president, Matt Ayotte, gave a talk at one of our local meetings. The semester concluded with a picnic with members of the Actuarial Club, Gamma Iota Sigma. Other chapter officers: Sara Meese, vice president; Jen Blake, secretary; Mark Anderson, treasurer; Arnold Hammel, corresponding secretary/faculty sponsor.

### *Mississippi Alpha*

Mississippi University for Women, Columbus

Chapter President - Rebecca Cagle

20 actives

Other chapter officers: Leah O'Gwynn, vice president; Emily Corely, secretary; Leigh Blair, treasurer; Jean Ann Parra, corresponding secretary; Shaochen Yang, faculty sponsor.

### *Missouri Alpha*

Southwest Missouri State University, Springfield

Chapter President - Susan Gibiser

55 actives

In addition to regular monthly meetings, Missouri Alpha held a very successful joint spring banquet with the student chapter of MAA. The event was attended by approximately 70 students, faculty and friends. Honored at the banquet were graduating seniors, freshman math

awardees, *KME* merit winners and Math Department scholarship awardees. Twelve student members and two faculty attended the national convention at Niagara Falls. Other chapter officers: Mike Jones, vice president; Christine Hixon, secretary; Mae Rivera, treasurer; Ed Huffman, corresponding secretary; Mike Awad, faculty sponsor.

### *Missouri Beta*

Central Missouri State University, Warrensburg

Chapter President – Jay Rowland

25 actives

Missouri Beta held spring initiation ceremonies for eighteen members and nine associate members. Lynn Gerner, a former graduate of CMSU who is now working for NASA at Goddard Air Force Base, was this year's Klingenberg Lecturer. The annual Spring Banquet was held in March. In conjunction with a pizza party, elections were held in April. Eight students and one sponsor attended the national convention in Niagara. Moss Presill presented his paper, "Planes, Spheres, Lines, Circles and Distance," at the convention. Other chapter officers: Jennifer Ritzo, secretary; Tracy Rouchka, treasurer; Rhonda McKee, corresponding secretary; Larry Dilley and Phoebe Ho, faculty sponsors.

### *Missouri Gamma*

William Jewell College, Liberty

Chapter President – Tracey Sterbenz

23 actives

Nine new members were initiated at the annual banquet on April 6. Guest speaker for the event was Steve Chiaparrri of Avila College in Kansas City, who spoke on "Conway's Game of Life." Other chapter officers: Kristin Hedberg, vice president; Joe Pierce, secretary; Joseph T. Mathis, corresponding secretary/faculty sponsor.

### *Missouri Epsilon*

Central Methodist College, Fayette

Chapter President – Mary Ann Neal

8 actives

Other chapter officers: Roselyn Magosha, vice president; Ed LaValle, secretary; Holly Toler, treasurer; William D. McIntosh, corresponding secretary/faculty sponsor; Linda O. Lembke, faculty sponsor.

### *Missouri Zeta*

University of Missouri at Rolla, Rolla

Chapter President – Brad Gardner

The chapter initiated fifteen students during the spring semester. Other chapter officers: Shawn Van Asdale, vice president; Melissa Ince, secretary; Doug Sept, treasurer; Roger Hering, corresponding secretary; James Joiner, faculty sponsor.

**Missouri Eta**

Chapter President - Jason Lott

Northeast Missouri State University, Kirksville

30 actives

Senior presentations were the featured programs for regular monthly meetings. Special tee-shirts bearing the message "GO FIGURE" were sold by the pledge class. Ten students and two faculty attended the national convention in Niagara. At the convention Judy Allen presented a paper entitled "Exploring Braess' Paradox." Other chapter officers: Scott Niemeyer, vice president; Deanne Reber, secretary; Angela Hahn, treasurer; Mary Sue Beersman, corresponding secretary; Shelle Palaski, faculty sponsor.

**Missouri Theta**

Chapter President - Stan Yoder

Evangel College, Springfield

12 actives

Three students and one faculty member attended the National Convention at Niagara Falls. Past chapter president Kevin Wilson presented a paper which was subsequently accepted for publication in *The Pentagon*. Other chapter officers: Don Tosh, corresponding secretary/ faculty sponsor.

**Missouri Iota**

Chapter Presidents - Laura Jay and Diane Hoch

Missouri Southern State College, Joplin

25 actives

Missouri Iota sponsored a talk by Dr. Juan Vasquez on Diophantine Equations. The chapter petitioned Student Senate for funds to help with expenses for student member Cindy Cummins to attend the national convention in Niagara, New York. Fifteen students were initiated into the chapter in late March. Other chapter officers: Kim Tarnowiecky, secretary/treasurer; Mary Elick, corresponding secretary; Linda Noel, faculty sponsor.

**Missouri Lambda**

Chapter President - Shawn Crawford

Missouri Western State College, St. Joseph

46 actives

In addition to regular meetings, Missouri Lambda held two bake sales, initiated 19 new members in March and participated in Mathematics Awareness Week. The chapter's delegation at the national convention at Niagara University was one of the largest with thirteen students and three faculty attending. The group also enjoyed a spring picnic held jointly with Math Club. Other chapter officers: Tammy Resler, vice president; Tracy Schemmer, secretary; Denise Fuller, treasurer; John Atkinson, corresponding secretary; Jerry Wilkerson, faculty sponsor.

**Nebraska Alpha**

Chapter President – Leslie Iwai

Wayne State College, Wayne

Throughout the semester, club members monitored the Math-Science Building in the evenings as a money-making activity. The club administered the competitive examination to identify the outstanding freshman in mathematics for the 1992-93 year. The winner of the competition, Nathan Bartsch of LaMars, Iowa, will have his name engraved on a permanent plaque, will have his national *KME* dues paid by the local chapter and will receive one year's honorary membership at the local level. Former *KME* member Steve Gedwillo, Applications Programmer for Woodmen of the World Insurance located in Omaha, Nebraska, was the featured speaker at the annual Math-Science Building Banquet in April. Members Susan Sorenson and Jeff Schneider were awarded the \$25.00 book scholarships which are given to *KME* members each year by the club. Five student members and faculty member John Fuelberth attended the national convention at Niagara University in Niagara Falls in April. Other chapter officers: Susan Sorenson, vice president; Pat Spieler, secretary/treasurer; Jennie Hartman, historian; Fred Webber, corresponding secretary; Jim Paige and Hilbert Johs, faculty sponsors.

**Nebraska Beta**

Chapter President – Valarie Klein

University of Nebraska at Kearney, Kearney

24 actives

Nebraska Beta held installation of thirteen new members in March. A reception honoring May math and statistics graduates was held at the Alumni House. The chapter registered participants and corrected papers for Math Counts, a math skills competition sponsored by an organization of Nebraska Engineers. During National Math Awareness Week, posters were designed and displayed. A math tutoring service for elementary through adult was also implemented. Other chapter officers: Jenae Waters, vice president; Christina Lynes, secretary; Douglas Blum, treasurer; Charles Pickens, corresponding secretary; Lutfi A. Lutfiyya, faculty sponsor.

**Nebraska Delta**

Chapter President – George Wahle

Nebraska Wesleyan University, Lincoln

23 actives

The semester got underway with a get-acquainted mixer featuring a scavenger hunt for items around campus. A panel of alumni provided a program on careers, including suggestions for career preparation and job-search techniques. Another program was given by student members who discussed issues and possibilities in computer networking. Other chapter

officers: Shawn Clymer, vice president; Allison Hurt, secretary; Michael Dempsey, treasurer; Muriel Skoug, corresponding secretary/faculty sponsor.

### *New Hampshire Alpha*

Keene State College, Keene

Chapter President – Eileen DePecol

29 actives

Installation of New Hampshire Alpha was held February 16 at Keene State College. On April 12 an initiation of six additional members was held with Ockle Johnson giving a talk on an application of calculus to the longest and shortest days of the year. Bethany Andrews, Eileen DePecol, Shayne Noyes and Tracey Thibeault attended the national convention at Niagara University. Other chapter officers: Tracey Thibeault, vice president; Bethany Andrews, secretary; Elise Lachance, treasurer; Charles Riley, corresponding secretary; Ockle Johnson, faculty sponsor.

### *New Mexico Alpha*

University of New Mexico, Albuquerque

Chapter President – David Black

30 actives

In addition to four meetings, two banquets were held and a delegation of two attended the national convention in Niagara. Other chapter officers: Terry Lynn Vigil, vice president; Patricia Lucker, secretary; Wil Grover, treasurer; Richard Metzler, corresponding secretary/faculty sponsor.

### *New York Alpha*

Hofstra University, Hempstead

Chapter President – Lee Ann Moltzen

Other chapter officers: Jason Lieberman and John Cullen, vice presidents; Jodi Witek, secretary; Kaliope Gravanis, treasurer; Aileen Michaels, corresponding secretary/ faculty sponsor.

### *New York Eta*

Niagara University, Niagara

Chapter President – Paul Schreiner

15 actives

The entire spring semester was devoted to planning and implementing the 29th Biennial Convention for which New York Eta served as host. The chapter members were pleased to meet so many other students and faculty from various locations around the country. Other chapter officers: Rich Inserra, vice president; Lisa Maselli, secretary/treasurer; Robert Bailey, corresponding secretary; Kenneth Bernard, faculty sponsor.

**New York Kappa**

Pace University, New York

Chapter President – Paula Murray

25 actives

An induction of new members was held on April 26. Other chapter officers: Ricky Gocool, vice president; Eileen Lawrance, secretary; Geraldine Taiani, corresponding secretary; John Kennedy and Blanche Abramov, faculty sponsors.

**New York Lambda**

C. W. Post Campus of Long Island University, Brookville

Chapter President – Myleen S. Rojano

31 actives

The annual banquet and initiation of new members was held at the Roslyn Café on the evening of March 18 and marked the tenth anniversary of the installation of the New York Lambda Chapter. Ten new members were initiated by the chapter officers and Dr. Eric Posmentier from Southampton College of Long Island University spoke on “Dynamical Chaos and Its Implications for Climate Change.” Student members Eileen K. Sipperley and Khaled Amleh attended the national convention at Niagara University along with alumnus Kevin O’Reilly and faculty members Dr. Kunoff and Dr. Rockett. Traveling by car, the trip also included an incredible April snow storm near Binghamton and a side trip to Letchworth State Park. Other chapter officers: Suzanne Hecker, vice president; Nicholas Ramer, secretary; Lisa Evans, treasurer; Andrew M. Rockett, corresponding secretary; Sharon Kunoff, faculty sponsor.

**North Carolina Gamma**

Elon College, Elon College

Chapter President – Varnn Rao

26 actives

Guest speaker for the initiation of 17 new members on April 21 was Dr. Virginia Knight of Meredith College. Other chapter officers: Miguel Johnston, vice president; Kristie Collins, secretary; Charles Touron, treasurer; Jeff Clark, corresponding secretary; Graham Gersdorff, faculty sponsor.

**Ohio Zeta**

Muskingum College, New Concord

Chapter President – Janet Gongola

21 actives

In January Heyes Dean presented a paper entitled “Composition of Reflections in Two Intersecting Lines: Rotation.” The paper was later presented at the national convention. Induction of three new members, Barb Miller, Ed Redder and Brad Stagmyer, occurred in February. In March the Chapter, in conjunction with the education honor society, hosted Dr. Joel Schneider from New York City’s SQUARE ONE TV of Children’s Television Network. Four students and two faculty attended the national convention in Niagara Falls. Other chapter officers: Brad

Stagmyer, vice president; Barbara Miller, secretary; Jim Buddenberg, treasurer; Russ Smucker, corresponding secretary; Javad Habibi, faculty sponsor.

### *Oklahoma Alpha*

Northeastern State University, Tahlequah

Chapter President - Denise Sturgeon

36 actives

Oklahoma Alpha continues to have joint activities with NSU's student chapter of the MAA. The organization participated in "The Problem Solving Competition," a series of math problems prepared each month by Richard S. Neal of Oklahoma University. The initiation of five students was held in the banquet room of a local restaurant. All were saddened in January by the death of Mr. Mike Reagan, who was sponsor of the Oklahoma Alpha Chapter for many years. In April members provided refreshments after a talk by Dr. James R. Choike, Oklahoma State University, on "Applications of Mathematics in Aeronautics." The Chapter sponsored several activities during National Mathematics Awareness Week. Dr. Jeanne Agnew, Emeritus Professor of Mathematics of Oklahoma State University, gave an informal presentation, entitled "A Visit With Dr. Agnew." Dr. Agnew is one of the author's of the linear algebra book currently being used by NSU. Following her lively and informative talk, the chapter sponsored the annual ice cream social. Dr. Deborah Carment was chosen "Mathematics Teacher of the Year" by the combined *KME* chapter and the local student MAA chapter. Other chapter officers: Allison Mohr, vice president; Joni Nichols, secretary; Jennifer Beals, treasurer; Joan E. Bell, corresponding secretary/faculty sponsor.

### *Oklahoma Gamma*

Southwestern Oklahoma State University, Weatherford

Chapter President - Joe Shepherd

20 actives

Other chapter officers: Cath Lair, vice president; Stephanie Hill, secretary/treasurer; Wayne Hayes, corresponding secretary; Radwan Al-Jarrah, faculty sponsor.

### *Oklahoma Delta*

Oral Roberts University, Tulsa

Chapter President - Brian Augenstein

22 actives

Oklahoma Delta helped host the regional MAA meeting that was held on the ORU campus in March. Member Jason Billions presented his senior paper at that meeting. In conjunction with the honorary science groups, the chapter also sponsored the pre-graduation hooding ceremony for the mathematical and natural sciences. Other chapter officers: Stephanie Wall, vice president; Amy Amsler, secretary; Lisa Brecheisen,

treasurer; Debbie Oltman, corresponding secretary; Roy Rakestraw, faculty sponsor.

*Pennsylvania Beta* Chapter President – Dannielle Ambrosini  
La Salle University, Philadelphia 15 actives

The chapter sponsored a stimulating talk by Dr. Errol Pomerance entitled "Black Holes, White Holes, Wormholes – How to Get There from Here." Other chapter officers: Richard Wojnar, vice president; Angela Rowbottom, secretary; Anne Hofmann, treasurer; Hugh N. Albright, corresponding secretary; Stephen Andrilli, faculty sponsor.

*Pennsylvania Gamma* Chapter President – Christy Barclay  
Waynesburg College, Waynesburg 12 actives

Other chapter officers: Michelle Markle, vice president; Bob McNulty, secretary; Pete Massung, treasurer; A. B. Billings, corresponding secretary/faculty sponsor.

*Pennsylvania Delta* Chapter President – Kelly Curtin  
Marywood College, Scranton. 9 actives

The chapter had no new inductees during this school year but will during the Fall '93 semester. Two members attended a PCTM Conference in March. Other chapter officers: Alice Ward, vice president; Marsha Galgon, secretary; Kathleen Hanlon, treasurer; Sister Robert Ann von Ahnen, corresponding secretary/faculty sponsor.

*Pennsylvania Epsilon* Chapter President – Lyn Mugridge  
Kutztown University, Kutztown 20 actives

Thirteen new members were initiated on April 16. Other chapter officers: Stephanie Schweyer, vice president; Brandy Thiele, secretary; Heidi Leiby, treasurer; Cherry Mauk, corresponding secretary; Randy Schaeffer, faculty sponsor.

*Pennsylvania Eta* Chapter President – Denise Good  
Grove City College, Grove City 32 actives

Initiation of new members and election of officers were held on March 17. On March 24 Dr. Dale McIntyre's wife gave a talk on Graph Theory. The *KME* Spring Picnic was held in May at the Grove City Community Park. *KME's* selection for the Outstanding Freshman Mathematics Student was announced at the Parent's Day Award Ceremony on May 2. Other chapter officers: Carolyn Luccheta, vice

president; Lara Skirpan, secretary; Heather Menzies, treasurer; Marvin C. Henry, corresponding secretary; Dan Dean, faculty sponsor.

### *Pennsylvania Iota*

Shippensburg University, Shippensburg

Chapter President - Jason Baker

20 actives

Pennsylvania Iota co-sponsored with Math Club the "Adopt a Highway" project. They also co-sponsored the Mathematics Colloquim Series. The annual Initiation Banquet was held in April. Other chapter officers: Angela Foltz, vice president; Christine Tipa, secretary; Fred Nordai, treasurer; Jenna Hooperarther, historian; Mike Seyfried, corresponding secretary/faculty sponsor.

### *Pennsylvania Kappa*

Holy Family College, Philadelphia

Chapter President - Dawn McDermond

10 actives

The tutoring provided by the chapter was in increased demand. Eight new members were initiated March 15. Sister Marcella Louise Wallowicz gave a slide presentation at the initiation and former *KME* members, Margaret Jankowski '75, Frances Laukagalis '64, Christine Michaels '83 and Susan Murray '78, shared some thoughts with those assembled. Other chapter officers: Mary Beth Emery, vice president/secretary; Daniel Lubicky, treasurer; Sister Marcell Louise Wallowicz, corresponding secretary; Sister M. Grace Kuzawa, faculty sponsor.

### *Pennsylvania Mu*

Saint Francis College, Loretta

Chapter President - Patricia D. George

37 actives

Twelve students were inducted on February 16, bringing total membership in Pennsylvania Mu to 127. The induction ceremony followed mass and dinner. Seven students, accompanied by Dr. Skoner, traveled by van to the national convention. Although the trip was long and tiresome, a great time was had by all. Chapter member Kyle Salisbury presented a paper at the convention entitled "Forming An Eye For Fractals." The annual picnic was held on April 28 with 45 members and guests participating. Other chapter officers: Scott Beers; vice president; Gerry Albright, secretary; Paul Schiele, treasurer; Peter Skoner, corresponding secretary; Adrian Baylock, faculty sponsor.

### *South Dakota Alpha*

Northern State University, Aberdeen

Chapter President - Ann Vidoloff

10 actives

South Dakota Alpha initiated three new members during the spring semester. They also celebrated Math Awareness Week and Faculty

Appreciation Week and sponsored pizza and ice cream parties. Other chapter officers: Joe Brooks, vice president; Marci Liebermann, secretary; Brenda Rook, treasurer; Raj Markanda, corresponding secretary/faculty sponsor.

### *Texas Alpha*

Texas Tech University, Lubbock

Chapter President – Troy Smith

40 actives

Twenty-seven new members were initiated on April 13. Other chapter officers: Chris Norden, vice president; Nina Nelson, secretary; Brian Ashcraft, treasurer; Robert Moreland, corresponding secretary; Gary Harris, faculty sponsor.

### *Texas Iota*

McMurry University, Abilene

Chapter President – Lon Outland

Thirteen new members were added to chapter roles at a banquet held April 18. Elaine White presented a student paper at the meeting of the Texas Section of the MAA in April. A reunion is planned for Possum Kingdom Lake this summer. Other chapter officers: John Paul Huber, vice president; Dana Maynard, secretary/treasurer; Bill Dulin, corresponding secretary; Dianne Dulin, faculty sponsor.

### *Texas Kappa*

University of Mary Hardin-Baylor, Belton

Chapter President – Thomas Ross

12 actives

Other chapter officers: Elizabeth Bartlett, vice president; Kori Whatley, secretary; Nathan Hagemann, treasurer; Peter H. Chen, corresponding secretary; Maxwell Hart, faculty sponsor.

### *Virginia Gamma*

Liberty University, Lynchburg

Chapter President – Nicole Boodram

12 actives

The first meeting of the spring semester was held in February. The program was presented by three high school and middle school teachers who shared with the chapter their experiences as teachers. The March meeting featured pizza and a problem solving fun time. An initiation meeting was held in April. Other semester activities included a visit from an alum who discussed his graduate school experience and his work experience after completing his Ph. D. Other chapter officers: Tracy Grissinger, vice president; Jeff Curtin, secretary; Michael McCleery, treasurer; Gly K. Wooldridge, corresponding secretary; Sandra Rumore, faculty sponsor.

*Wisconsin Alpha*

Mount Mary College, Milwaukee

Chapter President – Jill Rogahn

6 actives

Students Erin Hein, Silvia Navarro and Shima Tsujimoto, accompanied by Sister Adrienne Eickman, attended the national convention at Niagara University April 22-24. Other chapter officers: Jill Rogahn, secretary/treasurer; Sister Adrienne Eickman, corresponding secretary/faculty sponsor.

*Wisconsin Beta*

University of Wisconsin, River Falls

Chapter President – Dixie Carroll

35 actives

Spring semester began with an election to fill the vacancy left by Vice President Greg Redding. Members suited up for spring with new tee shirts and sweatshirts bearing the UWRF/KME logo. The chapter held a bake/hot dog sale in March, with proceeds going toward expenses of the trip to the annual math conference at St. John's University in Collegeville, MN. Throughout the spring semester several members tutored high school and junior high math students in area schools on a volunteer basis. During Math Awareness Week an information session was held for students who had recently been invited to membership in KME. Also during this week, the organization provided table tents of interesting math facts for the Student Center and assisted faculty in manning a booth featuring several challenging math puzzlers. Other April activities included attendance at a professional hockey game in Minneapolis, participation in a campus wide Hunger Clean-Up and, at the end of the month, sponsoring the annual Spring Picnic with the Computer Science Club. Semester activities ended with a Senior Send-Off Party at Pizza Hut with many faculty and KME members in attendance. Other chapter officers: Michael Weber, vice president; Janet Pugh, secretary; Timothy Stroth, treasurer; Robert Coffman, corresponding secretary/faculty sponsor.

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The following Fall report was inadvertently omitted from Spring 1993 issue.

*Colorado Gamma*

Fort Lewis College, Durango

Chapter President – Rachel Zeller

25 actives

Two business meetings were held and eight new members were inducted during the fall semester. Club members sponsored a combined pizza party, hexaflexagon construction event and participated in the annual Alumni Phon-A-Thon fund raiser. Other 1992-93 officers: Mary Wright, secretary; Stevan Scott, treasurer; Richard Gibbs, corresponding secretary; Deborah Berrier, faculty sponsor.

## Report on the 29th Biennial Convention

The Twenty-Ninth Biennial Convention of *Kappa Mu Epsilon* was held April 22-24, 1993 on the campus of Niagara University, New York. Portions of the Convention were also held at the Inn at the Falls in nearby Niagara Falls, New York with New York Eta serving as the host chapter at both sites.

On Thursday evening, April 22, a registration/mixer was held at the Inn at the Falls and the nearby Winter Garden. The National Council and the Regional Directors met in the Whitney Room.

On Friday morning, April 23, registration continued at Dunleavy Hall on the campus of Niagara University. The first general session was held in 127 Dunleavy and also in 227 Dunleavy, which served as an "overflow" room for all general sessions during the convention. The session commenced at 8:30 a.m. with Harold L. Thomas of Kansas Alpha, National President, presiding. Rev. Brian J. O'Connell, C.M., President of Niagara University, gave an address of welcome and Arnold D. Hammel of Michigan Beta, National President-Elect, responded for the Society. Paul Schreiner, President of New York Eta, presented a greeting to the delegates.

A roll call of the chapters was made by Robert L. Bailey of New York Eta, National Secretary. Thirty-four chapters and about 220 members were in attendance. Travel vouchers were filed and delegate voting cards were issued. The following new chapters installed during the 1991-93 biennium were recognized: South Carolina Delta at Erskine College, installed April 28, 1991; South Dakota Alpha at Northern State University, installed May 3, 1992; New York Nu at Hartwick College, installed May 14, 1992; New Hampshire Alpha at Keene State College, installed February 16, 1993. In addition, Louisiana Gamma at Northwestern State University was installed March 24, 1993 and so is included in the next biennium. South Dakota Alpha and New Hampshire Alpha were represented by delegations at this convention.

A petition for a new chapter at Cumberland College in Williamsburg, Kentucky, was presented by Harold L. Thomas, National President. It was moved and seconded that this petition be accepted and the delegates voted unanimously in favor of establishing the new chapter. President Thomas indicated that this chapter will be installed on May 3 by Pat Costello of KY Alpha, Director of Region 3.

The Nominating Committee report was presented by James Smith of Ohio Zeta, chair. The committee nominated Pat Costello of Kentucky Alpha and Sr. Adrienne Eickman of Wisconsin Alpha to the office of President-Elect, and Mary Elick of Missouri Iota to the office of Historian. The nominees were introduced to the delegates and additional nominations were requested from the floor. There being none, nominations were closed.

Kenneth J. Wilke, Problem Editor for *The Pentagon* and a special guest at the convention was introduced, along with his wife, to those assembled. Mr. Wilkie was recognized for his excellent work with the Problem Section. He encouraged all delegates to read this section of *The Pentagon* and submit solutions as well as new problems to the magazine.

During the coffee break, the Awards Committee met in 205 Dunleavy Hall.

Arnold D. Hammel of Michigan Beta, National President-Elect, presided during the presentation of the following student papers:

- 1) *Planes, Spheres, Lines, Circles and Distance*  
MOSS PREWITT, Missouri Beta  
Central Missouri State University
- 2) *Exploring Braess' Paradox*  
JUDY ALLEN, Missouri Eta  
Northeast Missouri State University
- 3) *Pythagorean Triples*  
DAVID J. KLENKE, Kansas Gamma  
Benedictine College
- 4) *Error-Correction for Basic Codes*  
KAREN S. BROWN, Iowa Alpha  
University of Northern Iowa
- 5) *Mathematics of the Spirograph*  
BILLIE JO STEFFENSON, Iowa Delta  
Wartburg College

Convention Committees met during lunch at Clet Dining Hall. Following lunch, a group picture was taken in front of the Castellani Art Museum.

The convention reconvened at 1:30 p.m. in 127 Dunleavy Hall. Arnold D. Hammel of Michigan Beta, National President-Elect, again presided during the presentation of the following student papers:

- 6) *Search Procedure*  
JENNIFER COURTER, California Gamma  
California Polytechnic State University
- 7) *Raisin Bread versus Balloons*  
WILLIAM B. DAVIS, Missouri Kappa  
Drury College
- 8) *Take a Guess - A Look at Statistical Inference*  
CHRISTINE A. HIXON, Missouri Alpha  
Southwest Missouri State University

At 2:45 p.m., a student section met in 127 Dunleavy Hall with Paul Schreiner, President of New York Eta, presiding. A faculty section met in 227 Dunleavy Hall with Harold L. Thomas of Kansas Alpha, National President, presiding.

At 3:45 p.m., the convention reconvened for the presentation of papers with Arnold D. Hammel, National President-Elect, presiding. The following papers were presented:

- 9) *Even the Least of These*  
KEVIN WILSON, Missouri Theta  
Evangel College
- 10) *Composition of Reflections in Two Intersecting Lines: Rotation*  
HEYES DEAN, Ohio Zeta  
Muskingum College
- 11) *Forming an Eye for Fractals*  
KYLE SALISBURY, Pennsylvania Mu  
Saint Francis College

At 7:00 p.m., the convention banquet was held in the Ballroom of the Inn at the Falls in Niagara Falls, New York with Paul Schreiner of New York Eta as master of ceremonies. The invocation was given by Fr. Louis Trota of Niagara University, and following dinner the keynote address was given by Dr. Gerald R. Rising, Distinguished Teaching Professor Emeritus of the State University of New York at Buffalo. Musical selections were then provided by the Niagara University Players. Arnold D. Hammel of Michigan Beta, National President-Elect, proceeded to explain the criteria for the selection of the recipient of the George R. Mach Distinguished Service Award which is given each biennium to the person who has made major contributions to *Kappa Mu*

*Epsilon*. The award originated in 1987 and prior winners have been Laura Green, Wilbur Waggoner, Fred Lott and Sr. Helen Sullivan. This biennium's recipient is James E. Lightner, the citation for whom reads as follows:

CITATION FOR DR. JAMES E. LIGHTNER  
THE GEORGE R. MACH DISTINGUISHED SERVICE AWARD  
APRIL 23, 1993

In 1987, James Lightner informed the *KME* 26th Biennial Convention in San Luis Obispo, California, of the establishment of the George R. Mach Distinguished Service Award. Little did he realize that he would be the recipient of this important honor in 1993.

Dr. Lightner's service to *Kappa Mu Epsilon* began in 1965 when he founded the Maryland Beta Chapter at Western Maryland College in Westminster, Maryland. Since the chapter was installed on May 30, 1965, he has served continuously as the corresponding secretary and often fills the dual role of faculty sponsor.

Dr. Lightner's talents were first noted at the national *KME* level in his appointment as the first director of Region 1 in 1971. He was then elected National Vice-President in 1973, prior to his election to the National Presidency for two terms from 1977 to 1981. His capable leadership brought our Society through some difficult times to make it the sound organization we experience today. Through the years, Jim has always had a great deal of enthusiasm and affection for *Kappa Mu Epsilon*. He is primarily responsible for the development of the brochure we use to tell the *KME* story to prospective members and chapters. He has also served as the installing officer for several of our active chapters.

Although Dr. Lightner is no longer involved at the national level with *KME*, he continues to teach and inspire students in mathematics as Professor of Mathematics and Education at Western Maryland College.

We take great pleasure in recognizing Dr. Lightner's many years of service to *Kappa Mu Epsilon* by presenting to him the George R. Mach Distinguished Service Award.

The delegates of the convention were especially fortunate that Dr. Lightner was in attendance at the banquet and so was able to receive this

award in person. An award of \$100 will be given to Maryland Beta, the chapter with which James Lightner has been associated since 1965.

The convention reconvened on Saturday, April 24 at 8:30 a.m. in 127 Dunleavy Hall. Arnold D. Hammel of Michigan Beta, National President-Elect, presided during the presentation of the following papers:

- 12) *General Convex Solid Modeling*  
MITCH RICHLING, Kansas Alpha  
Pittsburg State University
- 13) *Tesselations: A Perfect Fit*  
CHRISTOPHER M. ENYEART, Kansas Gamma  
Benedictine College
- 14) *Two Interesting Approaches to Counting the Number of  
Spanning Trees of the Complete n-Partite Graph*  
RACHEL ZELLER, Colorado Gamma  
Fort Lewis College
- 15) *Domes, Homes, and the Future*  
VAN BEACH, Iowa Delta  
Wartburg College

During the coffee break, the Awards Committee met in 204 Dunleavy Hall at 10:00 a.m.

The second business meeting was held at 10:15 a.m. in 127 Dunleavy with Harold L. Thomas, National President, presiding. The following national officers presented reports:

Business Manager, *The Pentagon* — Sharon Kunoff, New York Lambda  
Editor, *The Pentagon* — Andrew Rockett, New York Lambda  
National Historian — Mary Elick, Missouri Iota  
National Treasurer — Sr. Jo Ann Fellin, Kansas Gamma  
National Secretary — Robert L. Bailey, New York Eta  
National President-Elect — Arnold Hammel, Michigan Beta  
National President — Harold L. Thomas, Kansas Alpha

Paul Schreiner of New York Eta reported for the student section meeting, while Mary Sue Beersman reported for the faculty section meeting.

Sr. Adrienne Eickman, Chair of the Auditing Committee, reported that the National Treasurer's records were found to be accurate and in good order.

James Pomfret, Chair of the Resolutions Committee, reported for the Committee. The following resolutions were adopted:

Whereas Niagara Falls and Niagara University have proven to be the perfect surroundings for this meeting, be it resolved:

1. That the 29th Biennial Convention express its gratitude to New York Eta Chapter for the wonderful arrangements they have made for this convention.
2. That the 29th Biennial Convention recognize and thank Fr. Brian J. O'Connell, President of Niagara University, Paul Schreiner, President of New York Eta, Robert Bailey, Corresponding Secretary of NY Eta, Kenneth Bernard, Faculty Advisor of New York Eta and all the members of New York Eta who have devoted countless hours to making this meeting a success.

Whereas the success of any organization is directly proportional to the dedication of its leaders, be it resolved that:

1. The 29th Biennial Convention express its gratitude to Harold L. Thomas who has ably served as National President for the past four years and under whose stewardship *Kappa Mu Epsilon* has installed 12 new chapters.
2. The 29th Biennial Convention express thanks to the other national officers, Arnold Hammel, President-Elect, Robert Bailey, Secretary, Sister Jo Ann Fellin, Treasurer, and Mary Elick, Historian for their devotion and hard work.
3. The 29th Biennial Convention thank the staff of *The Pentagon*, Andrew Rockett, editor, Sharon Kunoff, business manager, and Kenneth M. Wilke, problem editor, for their dedication to quality in publication.
4. The 29th Biennial convention express thanks and support for the 6 Regional Directors.
5. The Convention express thanks to the faculty and students who served on the Auditing, Awards, Nomination, Resolutions, and Selection committees.

Whereas the primary purpose of *Kappa Mu Epsilon* is to

encourage participation in mathematics be it resolved that:

1. The students who prepared and submitted papers be given special commendation by this convention for their enthusiasm for and dedication to mathematics.
2. The Convention express gratitude to Gerald R. Rising for his speech and shared wisdom at the Friday evening banquet.
3. The Convention recognize the support of Hewlett Packard, Texas Instruments, Niagara Falls Convention and Visitors Bureau, CWM Chemical Services, and Officemax, Inc., all of whom contributed to the success of this meeting.

President Harold Thomas presided during the election of officers for 1993-1995. Ballots were cast and counted for the office of National President-Elect and President Thomas announced that the winner was Pat Costello of Kentucky Alpha. Since no other nominations were presented from the floor for the office of National Historian, a unanimous ballot was cast for a second term for the incumbent, Mary Elick of Missouri Iota.

No invitations were received to host the 1995 Biennial Convention, but delegates were reminded that such invitations can be extended by any chapter to the National Council any time during the next few months.

Pat Roden of Alabama Beta, Chair of the Awards Committee, reported for the committee. Certificates of participation were presented to all 15 student speakers. In addition, each speaker received a calculator courtesy of either Hewlett Packard or Texas Instruments. The following awards for student papers were announced and presented:

- First Place (\$100) — Mitch Richling, Kansas Alpha
- Second Place (\$80) — Jennifer Courter, California Gamma
- Third Place (\$70) — Rachel Zeller, Colorado Gamma
- Fourth Place (\$40) — Karen S. Brown, Iowa Alpha

The following officers were installed by Harold L. Thomas, National President for the term 1993-97:

- National Historian — Mary Elick, Missouri Iota
- National President-Elect — Pat Costello, Kentucky Alpha
- National President — Arnold Hammel, Michigan Beta

Arnold D. Hammel, newly installed National President, presented an engraved plaque to retiring National President, Harold L. Thomas, in recognition of his 22 years of service to the Society.

Arnold Hammel also announced that Carol Harrison, PA Theta, has accepted appointment as Director of Region 1 for 1993-97 and that Richard Gibbs, CO Gamma, has accepted reappointment as Director of Region 5 for the same time period.

Convention evaluation forms which had been distributed to the delegates earlier were collected by the host chapter. Copies of reports of the national officers and *The Pentagon* staff were made available to each chapter. Delegates were also reminded about the sightseeing bus tour of the Niagara Falls area which would take place immediately following lunch.

Travel allowances were paid to the delegates by Sr. Jo Ann Fellin of Kansas Gamma, National Treasurer. The convention was adjourned at 12:05 p.m.

Robert L. Bailey

## Report of the President

During my second biennium as your National President, we have installed five new chapters, which brings the total number of active chapters to 114. I am pleased to report that none of our chapters were placed on inactive status during my tenure as president. New chapters added this biennium are South Carolina Delta at Erskine College (installed by Don Aplin, April 28, 1991); South Dakota Alpha at Northern State University (installed by Harold Thomas, May 3, 1992); New York Nu at Hartwick College (installed by Robert Bailey, May 14, 1992); New Hampshire Alpha at Keene State College (installed by Charles Brusard, February 16, 1993); and Louisiana Gamma at Northwestern State University (installed by Harold Thomas, March 24, 1993). At this convention, we are acting on the petition from Cumberland College at Williamsburg, Kentucky. In addition, six other institutions have received petition forms this biennium but have not as yet returned them. I have also corresponded with twenty-three colleges and universities that have indicated an interest in *Kappa Mu Epsilon*. If you have friends and colleagues at schools that do not have a *KME* chapter, but are interested, have them contact Dr. Hammel.

The National Council continues to support the regional structure of *KME*. Please refer to the report by President-Elect Arnold Hammel for the summary of regional conventions held in 1992. With much gratitude,

we recognize the work and efforts of our Regional Directors. These people have served our society well and deserve the thanks of each of us. I would especially cite James Pomfret, Region 1 Director, whose term expires with this convention. Since 1982, Jim has been diligently looking after *KME* interests in Region 1. Thanks, Jim, for a job well done.

A special thanks is extended to each of the faculty who serve as corresponding secretaries and faculty sponsors with our active chapters. I know many of you played an important role in assisting your students in the preparation of the excellent papers we have on the convention program this year. Furthermore, we all express our gratitude to each of the students who did the work, endured the stress, and prevailed in submitting and presenting a paper at this convention. Without the student papers, the major focal point of the convention does not exist. We are further indebted to all of the individuals who did all of the work necessary to bring this convention to fruition and to those who have served on the convention committees.

Almost without exception, everyone who was asked to serve on a convention committee willingly agreed to do so. This kind of response, both at convention time, and throughout my tenure, has made the privilege of being your President much easier. To all of you at New York Eta (under the capable direction of Bob Bailey and Kenneth Bernard), and to each committee member, please accept our most sincere thanks for jobs well done.

During the past biennium, I have represented *Kappa Mu Epsilon* at the annual meeting of the Association of College Honor Societies (ACHS). It has been very helpful to meet with officers of the other 58 honor societies that are members of ACHS for the purpose of exchanging ideas and acquiring suggestions as to how we can possibly improve on the programs we currently have in place. The National Council will, in later meetings, focus on implementing some of these ideas to help *Kappa Mu Epsilon* move forward towards an even better and stronger national honor society. President-Elect Arnold Hammel attended the 1993 ACHS meeting with me to become more knowledgeable about this organization as he makes the transition to being our next *KME* President.

I next want to recognize the fantastic job which is being done by those who work with, manage, write for, and produce our journal, *The Pentagon*. We are most appreciative of the editorial leadership of Andrew Rockett (NY Lambda) and the sound business management given us by Sharon Kunoff (NY Lambda). We thank these two, and all the people staffing the low profile jobs who truly give us a journal which is respected nationwide by our professional mathematics community. We would especially recognize the outstanding service Kenneth Wilke has given *The Pentagon* for many years as Associate Editor for "The

Corner." We are most pleased that Ken and his wife, Janet, were able to attend this convention.

I also want to recognize and applaud the outstanding efforts put forth by other members of the National Council in their respective areas of responsibility. We have special indebtedness to these very capable and conscientious individuals who so unselfishly give of their time and efforts in making our honor society the very best it can possibly be.

Finally, I would like to express my appreciation to the Mathematics Department at Pittsburg State University for the strong support they have given me during my tenure. Both moral and monetary help have been extended these past four years by my department to assist me in doing a better job as National President. I would especially laud Dr. Elwyn Davis, departmental chair, and Joyce Kovacic, secretary, for the respective roles they played in helping me fulfill the responsibilities related to this position.

In summary, I can honestly say that I have thoroughly enjoyed the privilege of serving *Kappa Mu Epsilon* at the national level the past several years. Under the capable leadership of Dr. Arnold Hammel and the rest of the National Council, I know that *Kappa Mu Epsilon* will continue to grow and occupy a position of pride in our hearts and minds because of our affiliation with this very special honor society. Best wishes to each of you as you return to your respective homes. My sincere thanks to all for your participation in this 29th Biennial Convention.

Harold L. Thomas

### Report of the President-Elect

One of the responsibilities of the President-Elect is to serve as coordinator of regional activities of the Society through the regional directors. During the Spring of 1992, there were five regional conventions held in:

Region I at Pennsylvania Mu  
Saint Francis College, March 13-14  
Two chapters, 27 participants  
James Pomfret, Regional Director

Region II at Ohio Alpha  
Bowling Green State University, April 10-11  
Seven chapters, 46 participants  
Sister Adrienne Eickman, Regional Director

Region III at Kentucky Alpha  
Eastern Kentucky University, April 11  
One chapter  
Patrick Costello, Regional Director

Region IV at Kansas Beta  
Emporia State University, April 10-11  
Fifteen chapters, 94 participants  
Mary Sue Beersman, Regional Director

Region V at Oklahoma Gamma  
Southwestern Oklahoma State University, March 20-21  
Three chapters, 22 participants  
Dick Gibbs, Regional Director

Programs at the regional conventions included student papers, guest talks, banquets and lunches, faculty discussions, student discussions and good social times. We extend our sincere thanks to the host chapters, regional directors, and all who participated in this regional activity. We also appreciate the efforts of the Region VI Director, Raymond Terry, in attempting to have a regional convention in that region.

It is another of the President-Elect's responsibilities to make arrangements for the presentation of student papers at the National Convention. I am pleased to report that eighteen students, representing fourteen chapters and eight states, submitted papers for this convention. Seventeen of the papers were written by undergraduate students and one by a graduate student. Fifteen undergraduate papers are being presented at this convention. I want to express our sincere thanks to the eighteen students who prepared and submitted papers. These papers form an important component of a successful convention.

On behalf of our entire Society, I want to extend special thanks to the members of the Paper Selection Committee who read and ranked the papers:

Professor John Kubicek (Missouri Alpha)  
Professor Richard Metzler (New Mexico Alpha)  
Professor Peter Skoner (Pennsylvania Mu)

All three were very receptive to assisting the organization in this endeavor. The criteria used by the Paper Selection Committee in judging the papers are as follows and also appear in the Spring 1992 *Pentagon* Announcement of the National Convention as well as in a Fall 1992

letter to local chapters: (A) The paper will be judged on (1) topic originality; (2) appropriateness to the meeting and audience; (3) organization; (4) depth and significance of the content; and (5) understanding of the material. Special thanks also goes to the Awards Committee, chaired by Patricia Roden (Alabama Beta). Their much appreciated effort this weekend leads to the choosing of the four papers that receive awards. This committee is composed of four students and four faculty members representing as many chapters as possible. The Awards Committee used the following criteria in evaluating the presentations: (B) The presentation will be evaluated on (1) style of presentation; (2) maintenance of interest; (3) use of audio-visual materials (if applicable); (4) enthusiasm for the topic; (5) overall effect; and (6) adherence to the time limit.

Another responsibility of the President-Elect is the coordination of the selection of the George R. Mach Distinguished Service Award. I want to thank those corresponding secretaries and faculty sponsors who made nominations for the Mach Award. The recipient of the 1993 award will be announced at the Friday evening banquet.

As I close my four year term of President-Elect, I would like to give my appreciation to 1) the members of the National Council, 2) the Regional Directors, 3) the Corresponding Secretaries and Faculty Sponsors who assist their students, region and national with dedication, time and energy, and 4) the members of our local chapters. Thank you for jobs well done.

Arnold D. Hammel

### Report of the National Secretary

During the last biennium, four new chapters of *Kappa Mu Epsilon* were installed as listed below.

Chapter	Institution	Installation Date
South Carolina Delta	Erskine College	4/28/91
South Dakota Alpha	Northern State University	5/3/92
New York Nu	Hartwick College	5/14/92
New Hampshire Alpha	Keene State College	2/16/93

During the last biennium 2,574 members were initiated. The 113 active chapters have a combined membership of 56,838 and the 29 inactive chapters have a combined membership of 6,390, making the total membership of *Kappa Mu Epsilon* 63,228 at the end of the biennium.

The newest chapter is Louisiana Gamma at Northwestern State University which was installed on March 24, 1993. Since this date falls in the next biennium, the data above do not reflect this latest addition to *KME*.

As National Secretary, I maintain permanent files on all active and inactive chapters, including reports of all initiations. I order membership certificates and jewelry for all new members and I stock all supplies, including forms, invitations, and jewelry. I assist corresponding secretaries in any ways that I can and I take minutes of National Council meetings and Biennial Conventions.

Robert L. Bailey

**Financial Report of the National Treasurer**  
1991-93 Biennium (March 12, 1991 through March 22, 1993)

Assets at beginning of biennium \$61,661.79

**RECEIPTS**

Receipts from Chapters		
Initiates (2574)	38610.00	
Jewelry	1971.50	
Supplies	293.20	
Interest	6034.23	
Miscellaneous	2046.75	
Certificate/Crest replacements	56.75	
Overpays/handling	249.25	
Insuff funds reimbursement	270.00	
New Chapter Fees	1149.45	
<i>Pentagon</i> (advance returned)	300.00	
Capital Gain	21.30	
Total Receipts during biennium	48955.68	
Receipts plus Assets		110,617.47

**EXPENDITURES**

Jewelry	2787.94
Printing	11088.31
<i>Pentagon</i>	14350.00
Travel (Regional Directors 548; ACHS 1318)	1866.00
Conventions/meetings	16677.56
1991 Biennial Convention	12280.82
1992 Regional meetings	1188.97
National Council Meeting 1991	1888.83

National Council Meeting 1992	590.50	
1993 Convention advance	340.44	
ACHS meeting registration fees	388.00	
ACHS dues		167.00
60th anniversary Special Project		4781.00
Miscellaneous		3119.09
Supplies	321.82	
Telephone	135.82	
Postage	1360.07	
Chapter installations	604.13	
Insufficient funds	270.00	
Overpay/handling refunds	232.25	
Bonding of Treasurer	195.00	
Total Expenditures		54836.90
Assets at end of biennium		55,780.57
Exchange National Bank #346896	19487.58	
Treasury Note #24131	9995.40	
World Savings CD #62539739	26297.59	

Jo Ann Fellin, OSB

### Report of the National Historian

The files of the National Historian are being maintained and continually updated with records received from the chapters about their events and activities; with information received from Regional Directors about regional conventions and items of interest related to the regions; and with material received from the National Officers which has historical significance.

News items are solicited from the corresponding secretaries semi-annually. The responses to these requests are then edited for publication in the Chapter News Section of *The Pentagon*.

During the past biennium 87 of the 114 active chapters responded at least once to the chapter news request. Special mention goes to the following 37 chapters for their cooperation in responding to all four inquiries: CA Gamma, CO Gamma, GA Alpha, IL Beta, IA Alpha, IA Delta, KS Alpha, KS Beta, KS Gamma, KS Epsilon, KY Alpha, MD Beta, MD Delta, MI Beta, MO Beta, MO Gamma, MO Epsilon, MO Eta, MO Iota, MO Lambda, NE Alpha, NE Delta, NY Alpha, NC Gamma, OH Alpha, OK Alpha, PA Alpha, PA Beta, PA Delta, PA Eta, PA Iota, PA Kappa, PA Nu, TN Alpha, TX Eta, TX Kappa and WI

Gamma. I would urge chapters to reply to the requests for chapter news even if it is just to identify chapter officers. This would provide chapters with a permanent record of their local officers in the event they do not retain that information within their own chapter.

I wish to thank all those with whom I have corresponded relative to this office — The National Officers, The Regional Directors, the Editor of *The Pentagon*, Corresponding Secretaries, and individual *KME* members. I have enjoyed serving all of you in this capacity.

Mary S. Elick

### Report of the Editor of *The Pentagon*

Volumes 51 and 52 of *The Pentagon* featured twelve student papers, three faculty papers, a review of the Canadian problem journal *Mathematical Mayhem* and abstracts of five student papers presented at the 1992 Region IV convention. "The Problem Corner" and "Kappa Mu Epsilon News" sections appeared in each issue and thus continued to form the core of our journal. "The Hexagon" section has been discontinued.

Unsolicited manuscripts are refereed by faculty volunteers. The efforts of seven such individuals were acknowledged in the Spring 1992 issue and a similar listing will appear in the Spring 1994 issue. They have been a great help to me and I am sure that the authors also found their constructive comments most useful.

The continued efforts and prompt attention to details by associate editors Kenneth M. Wilke and Mary S. Elick have greatly simplified my tasks, as have the support and assistance of Sharon Kunoff and the *KME* national officers.

Andrew M. Rockett

### Report of the Business Manager of *The Pentagon*

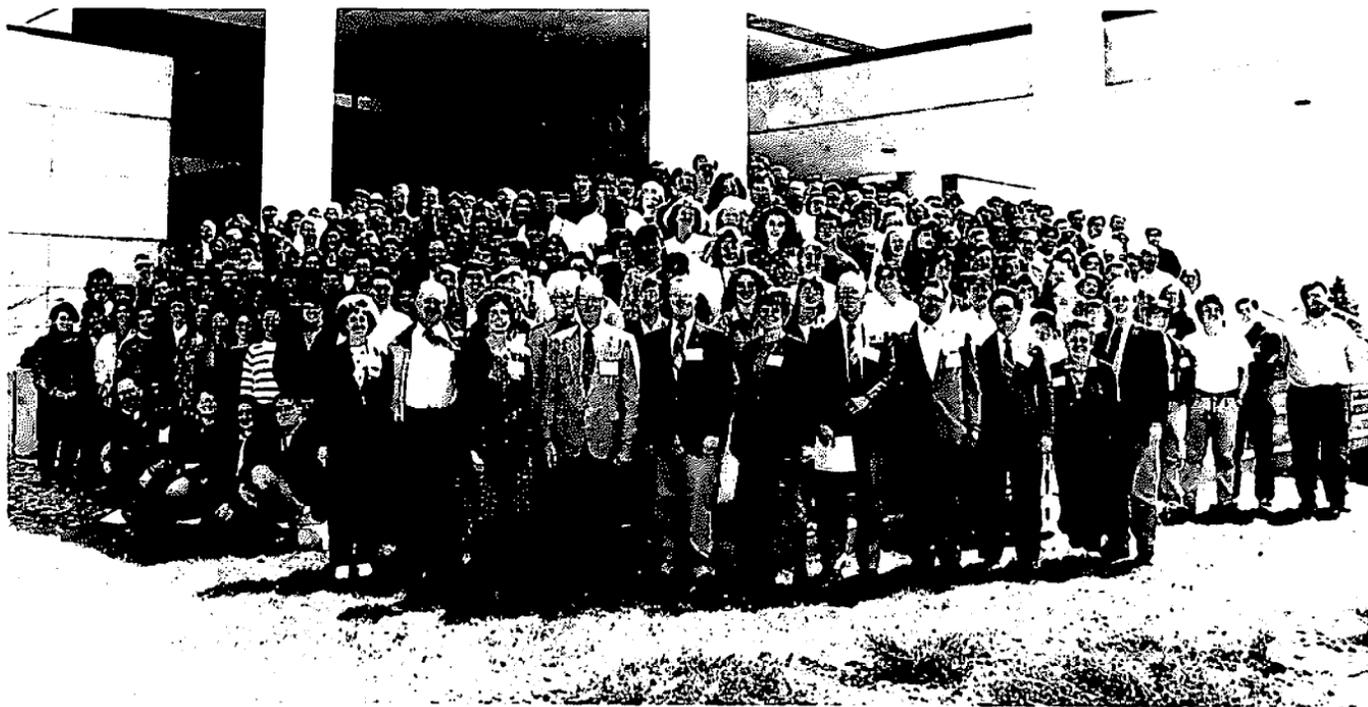
It is hard to believe that I have been your business manager for four years. Within the next six weeks we will be mailing the eighth issue of *The Pentagon* since I assumed my position. I want to thank those of you who have made my job easier by filling out *The Pentagon* subscription card properly, making sure that the address given is a permanent address. *The Pentagon* is mailed at the bulk mail rate and so the U.S. Postal Service will not forward copies. Parental addresses, as a rule, are more dependable!

At the last biennial convention, I announced that our subscription rate had held steady at about 2800 for a few years but that your editor and I were working on various means to increase our readership. I have been mailing renewal notices to people as their subscriptions expire and have found that this picks up about 70-80 renewals that might otherwise not occur. I appreciate those of you who respond promptly and who note change of name and address with along with the old information, which makes my job much simpler. Any of you here who have let your subscription lapse, or have friends or colleagues in that position, are encouraged to "come back." At present our subscription rate is still \$5.00 for 2 years. However, with the postage increases we have been experiencing yearly I am going to recommend to your executive board that they consider an increase.

Since Dr. Rockett and I assumed the reins of *The Pentagon*, we have made all issues uniform in size and color (although I was forced to change printers a year and a half ago). This has enabled us to send, along with our bulk mailing of the new magazines, copies of a previous issue (marked sample copy) to chairpersons of mathematics departments at schools which we feel might be candidates for *Kappa Mu Epsilon* membership. This has brought us a few new subscriptions, but more importantly, it has brought inquiries for membership to your president which has increased our chapter rolls and has thereby brought additional subscribers. I am pleased to announce that our last issue had almost 3200 subscribers and this number appears to be our roster for Vol. 52-2 as well. This includes approximately 40 foreign subscribers. Foreign inductees who will be returning to their home countries after graduation will continue to receive *The Pentagon* if I am notified of their new addresses and if they renew on expiration in American dollars!

Be on the lookout for the Spring issue of *The Pentagon* sometime in June. If you do not receive yours or if there is a problem with your address, please notify me and I will make the necessary changes. I aim to please.

Sharon Kunoff



Twenty-Ninth Biennial Convention of *Kappa Mu Epsilon*, 22-24 April 1993,  
at Niagara University, Niagara University, New York.

Front row (left to right): M.S. Beersman, R.A. Gibbs, M.S. Elick, [A. Eickman], A.D. Hammel,  
[P.J. Costello], H.L. Thomas, J.A. Fellin, R.L. Bailey, K.M. Wilke, A.M. Rockett, S. Kunoff, P. Schreiner.

## Kappa Mu Epsilon National Officers

Arnold D. Hammel

*President*

Department of Mathematics  
Central Michigan University, Mt. Pleasant, Michigan 48859

Patrick J. Costello

*President-Elect*

Department of Mathematics, Statistics and Computer Science  
Eastern Kentucky University, Richmond, Kentucky 40475

Robert L. Bailey

*Secretary*

Department of Mathematics  
Niagara University, Niagara University, New York 14109

Jo Ann Fellin

*Treasurer*

Mathematics and Computer Science Department  
Benedictine College, Atchison, Kansas 66002

Mary S. Elick

*Historian*

Department of Mathematics  
Missouri Southern State College, Joplin, Missouri 64801

*Kappa Mu Epsilon*, Mathematics Honor Society, was founded in 1931. The object of the Society is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics due to its demands for logical and rigorous modes of thought; to provide a Society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; and to disseminate the knowledge of mathematics and familiarize the members with the advances made in mathematics. The official journal of the Society, *The Pentagon*, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the Chapters.

## Active Chapters of Kappa Mu Epsilon

*Listed by date of installation.*

Chapter	Location	Installation Date
OK Alpha	Northeastern Oklahoma State University, Tahlequah	18 April 1931
IA Alpha	University of Northern Iowa, Cedar Falls	27 May 1931
KS Alpha	Pittsburg State University, Pittsburg	30 Jan 1932
MO Alpha	Southwest Missouri State University, Springfield	20 May 1932
MS Alpha	Mississippi University for Women, Columbus	30 May 1932
MS Beta	Mississippi State University, Mississippi State College	14 Dec 1932
NE Alpha	Wayne State College, Wayne	17 Jan 1933
KS Beta	Emporia State University, Emporia	12 May 1934
NM Alpha	University of New Mexico, Albuquerque	28 March 1935
IL Beta	Eastern Illinois University, Charleston	11 April 1935
AL Beta	University of North Alabama, Florence	20 May 1935
AL Gamma	University of Montevallo, Montevallo	24 April 1937
OH Alpha	Bowling Green State University, Bowling Green	24 April 1937
MI Alpha	Albion College, Albion	29 May 1937
MO Beta	Central Missouri State University, Warrensburg	10 June 1938
TX Alpha	Texas Tech University, Lubbock	10 May 1940
TX Beta	Southern Methodist University, Dallas	15 May 1940
KS Gamma	Benedictine College, Atchison	26 May 1940
IA Beta	Drake University, Des Moines	27 May 1940
TN Alpha	Tennessee Technological University, Cookeville	5 June 1941
NY Alpha	Hofstra University, Hempstead	4 April 1942
MI Beta	Central Michigan University, Mount Pleasant	25 April 1942
NJ Beta	Montclair State College, Upper Montclair	21 April 1944
IL Delta	College of St. Francis, Joliet	21 May 1945
KS Delta	Washburn University, Topeka	29 March 1947
MO Gamma	William Jewell College, Liberty	7 May 1947
TX Gamma	Texas Woman's University, Denton	7 May 1947
WI Alpha	Mount Mary College, Milwaukee	11 May 1947
OH Gamma	Baldwin-Wallace College, Berea	6 June 1947
CO Alpha	Colorado State University, Fort Collins	16 May 1948
MO Epsilon	Central Methodist College, Fayette	18 May 1949
MS Gamma	University of Southern Mississippi, Hattiesburg	21 May 1949
IN Alpha	Manchester College, North Manchester	16 May 1950

PA Alpha	Westminster College, New Wilmington	17 May 1950
IN Beta	Butler University, Indianapolis	16 May 1952
KS Epsilon	Fort Hays State University, Hays	6 Dec 1952
PA Beta	LaSalle University, Philadelphia	19 May 1953
VA Alpha	Virginia State University, Petersburg	29 Jan 1955
IN Gamma	Anderson University, Anderson	5 April 1957
CA Gamma	California Polytechnic State University, San Luis Obispo	23 May 1958
TN Beta	East Tennessee State University, Johnson City	22 May 1959
PA Gamma	Waynesburg College, Waynesburg	23 May 1959
VA Beta	Radford University, Radford	12 Nov 1959
NE Beta	Kearney State College, Kearney	11 Dec 1959
IN Delta	University of Evansville, Evansville	27 May 1960
OH Epsilon	Marietta College, Marietta	29 Oct 1960
MO Zeta	University of Missouri - Rolla, Rolla	19 May 1961
NE Gamma	Chadron State College, Chadron	19 May 1962
MD Alpha	College of Notre Dame of Maryland, Baltimore	22 May 1963
IL Epsilon	North Park College, Chicago	22 May 1963
OK Beta	University of Tulsa, Tulsa	3 May 1964
CA Delta	California State Polytechnic University, Pomona	5 Nov 1964
PA Delta	Marywood College, Scranton	8 Nov 1964
PA Epsilon	Kutztown University of Pennsylvania, Kutztown	3 April 1965
AL Epsilon	Huntingdon College, Montgomery	15 April 1965
PA Zeta	Indiana University of Pennsylvania, Indiana	6 May 1965
AR Alpha	Arkansas State University, State University	21 May 1965
TN Gamma	Union University, Jackson	24 May 1965
WI Beta	University of Wisconsin - River Falls, River Falls	25 May 1965
IA Gamma	Morningside College, Sioux City	25 May 1965
MD Beta	Western Maryland College, Westminster	30 May 1965
IL Zeta	Rosary College, River Forest	26 Feb 1967
SC Beta	South Carolina State College, Orangeburg	6 May 1967
PA Eta	Grove City College, Grove City	13 May 1967
NY Eta	Niagara University, Niagara University	18 May 1968
MA Alpha	Assumption College, Worcester	19 Nov 1968
MO Eta	Northeast Missouri State University, Kirksville	7 Dec 1968
IL Eta	Western Illinois University, Macomb	9 May 1969
OH Zeta	Muskingum College, New Concord	17 May 1969
PA Theta	Susquehanna University, Selinsgrove	26 May 1969
PA Iota	Shippensburg University of Pennsylvania, Shippensburg	1 Nov 1969
MS Delta	William Carey College, Hattiesburg	17 Dec 1970
MO Theta	Evangel College, Springfield	12 Jan 1971
PA Kappa	Holy Family College, Philadelphia	23 Jan 1971

CO Beta	Colorado School of Mines, Golden	4 March 1971
KY Alpha	Eastern Kentucky University, Richmond	27 March 1971
TN Delta	Carson-Newman College, Jefferson City	15 May 1971
NY Iota	Wagner College, Staten Island	19 May 1971
SC Gamma	Winthrop University, Rock Hill	3 Nov 1972
IA Delta	Wartburg College, Waverly	6 April 1973
PA Lambda	Bloomsburg University of Pennsylvania, Bloomsburg	17 Oct 1973
OK Gamma	Southwestern Oklahoma State University, Weatherford	1 May 1973
NY Kappa	Pace University, New York	24 April 1974
TX Eta	Hardin-Simmons University, Abilene	3 May 1975
MO Iota	Missouri Southern State College, Joplin	8 May 1975
GA Alpha	West Georgia College, Carrollton	21 May 1975
WV Alpha	Bethany College, Bethany	21 May 1975
FL Beta	Florida Southern College, Lakeland	31 Oct 1976
WI Gamma	University of Wisconsin - Eau Claire, Eau Claire	4 Feb 1978
MD Delta	Frostburg State University, Frostburg	17 Sept 1978
IL Theta	Illinois Benedictine College, Lisle	18 May 1979
PA Mu	St. Francis College, Loretto	14 Sept 1979
AL Zeta	Birmingham-Southern College, Birmingham	18 Feb 1981
CT Beta	Eastern Connecticut State University, Willimantic	2 May 1981
NY Lambda	C. W. Post Center of Long Island University, Brookville	2 May 1983
MO Kappa	Drury College, Springfield	30 Nov 1984
CO Gamma	Fort Lewis College, Durango	29 March 1985
NE Delta	Nebraska Wesleyan University, Lincoln	18 April 1986
TX Iota	McMurry College, Abilene	25 April 1987
PA Nu	Ursinus College, Collegeville	28 April 1987
VA Gamma	Liberty University, Lynchburg	30 April 1987
NY Mu	St. Thomas Aquinas College, Sparkill	14 May 1987
OH Eta	Ohio Northern University, Ada	15 Dec 1987
OK Delta	Oral Roberts University, Tulsa	10 April 1990
CO Delta	Mesa State College, Grand Junction	27 April 1990
NC Gamma	Elon College, Elon College	3 May 1990
PA Xi	Cedar Crest College, Allentown	30 Oct 1990
MO Lambda	Missouri Western State College, St. Joseph	10 Feb 1991
TX Kappa	University of Mary Hardin-Baylor, Belton	21 Feb 1991
SC Delta	Erskine College, Due West	28 April 1991
SD Alpha	Northern State University, Aberdeen	3 May 1992
NY Nu	Hartwick College, Oneonta	14 May 1992
NH Alpha	Keene State College, Keene	16 Feb 1993
LA Gamma	Northwestern State University, Natchitoches	24 March 1993