

## THE PENTAGON

*A Mathematics Magazine for Students*

Volume 61 Number 1

Fall 2001

## Contents

In Defense of Euclid <i>Andy Nichols</i>	3
Quadratic Varieties <i>Joseph Bohanon</i>	16
Generalized Pascal Triangles of Finite Groups with Two Generators <i>Tim Pierce</i>	21
<i>The Problem Corner</i>	48
<i>Kappa Mu Epsilon News</i>	54
Report on the 33 <sup>rd</sup> Biennial Convention	xx
<i>Kappa Mu Epsilon Officers</i>	69
Active Chapters of <i>Kappa Mu Epsilon</i>	70

The Pentagon (ISSN 0031-4870) is published semiannually in December and May by Kappa Mu Epsilon. No responsibility is assumed for opinions expressed by individual authors. Papers written by undergraduate mathematics students for undergraduate mathematics students are solicited. Papers written by graduate students or faculty will be considered on a space-available basis. Submissions should be typewritten (double spaced with wide margins) on white paper, standard notation conventions should be respected any special symbols not typed should be carefully inserted by hand in black ink. All illustrations must be submitted on separate sheets and drawn in black ink. Computer programs, although best represented by pseudocode in the main text, may be included as an appendix. Graphs, tables or other materials taken from copyrighted works **MUST** be accompanied by an appropriate release form from the copyright holder permitting their further reproduction. Student authors should include the names and addresses of their faculty advisors. Final versions on 3.5 inch disk in "text only" (ASCII) format are appreciated. Contributors to The Problem Corner or Kappa Mu Epsilon News are invited to correspond directly with the appropriate Associate Editor.

Individual domestic subscriptions: \$5.00 for two issues (one year) or \$10.00 for four issues (two years); individual foreign subscriptions: \$7.00 (US) for two issues (one year). Library rate: \$10.00 (US) for two issues (one year) or \$20.00 (US) for four issues (two years). Correspondence regarding subscriptions, changes of address or back copies should be addressed to the Business Manager. Copies lost because of failure to notify the Business Manager of changes of address cannot be replaced. Microform copies are available from University Microfilms, Inc., 300 North Zeeb Road, Ann Arbor, Michigan 48106-1346 USA.

---

Editor: Steven D. Nimmo  
Mathematical Sciences Department  
Morningside College, Sioux City, IA 51106  
nimmo@morningside.edu

Associate Editors:  
The Problem Corner: Kenneth M. Wilke  
Department of Mathematics  
Washburn University of Topeka, Topeka, Kansas 66621  
xxwilke@acc.wuacc.edu

Kappa Mu Epsilon News: Connie Schrock  
Department of Mathematics  
Emporia State University, Emporia, Kansas 66801  
schrockc@emporia.edu

Business Manager: Larry Scott  
Division of Mathematics and Computer Science  
Emporia State University, Emporia, Kansas 66801  
scottlar@esumail.emporia.edu

## In Defense of Euclid

Andy Nichols, *students*

Tennessee Gamma

Union University  
Jackson, TN 38305

Presented at the 2001 National Convention and awarded "top four" status by the Awards Committee

"A unit is that by virtue of which each of the things that exist is called one."

*The Elements*, book VII, definition 1 (Heath, 1956, p.277)

Thus, Euclid begins his introduction to the ancient Greek theory of numbers. The poetry of these words reminds the reader of the beauty inherent in mathematics. It is that beauty that draws us to investigate the intricacies of Euclid's number theory and determine for ourselves the strength or weakness of Euclid's work. Historically, commentary on Euclid's *Elements* has cycled through periods of high praise and sharp criticism. Many historians cite an alleged lack of generality and the supposed limitations of his geometric language as reasons that *The Elements* are inferior to more modern works. However, it can be demonstrated through an examination of Euclid's number theory in books VII-IX of *The Elements* that Euclid's methods are strong enough to prove the *Fundamental Theorem of Arithmetic*, the gem of elementary number theory.

First, it is necessary to introduce some of the basic language and vocabulary used by Euclid and to relate it to our modern terminology. In Greek number theory, the numbers are 2, 3, 4 and unity as defined above (Hartshorn, 2000). The operations on these integers were limited to addition, subtraction (the smaller from the larger), and multiplication. Numbers may be related as "equal," "greater than," or "less than" (Grattan-Guinness, 1996). Also very important to this investigation are the phrases that Heath translates as "A is measured by B" or equivalently "B measures A." With little controversy, we can interpret these as B divides A or symbolically  $B|A$  (Collison, 1980). Note that, since Euclid does not conceive of rational numbers, this operation is not equivalent to division: A can be written as a multiple of B, so  $A = k \cdot B$  for some positive integer k. No algebraic notation was available, therefore Euclid wrote all definitions and propositions in long form (Knorr, 1976). For example, VII;16<sup>1</sup> states "If

<sup>1</sup> VII; 16 refers to book VII, proposition 16 in the *Elements*. This notation will be used throughout.

two numbers by multiplying one another make certain numbers, the numbers so produced will be equal to one another" (Heath, 1956, p.316). According to Heath, this seemingly complex statement can be written simply in algebraic notation:  $ab = ba$ , the commutative property of multiplication. Although Euclid's terminology and phrasing are very different from our own, it is not difficult to grasp the concepts that he has proven.

Among the critics of the *Elements*, many cite a lack of generality as a major flaw. For instance, Euclid's proof of the infinitude of primes (IX;20) begins: "Let  $A, B, C$  be assigned prime numbers" (Heath, 1956, p.412) In fact, the *Elements* are not rigorously general, but in most cases, this does not significantly damage Euclid's proof. If the propositions and proofs in the *Elements* are not rigorously general, one must inquire as to why they are so. Did Euclid not recognize the need for a generalized method to prove a proposition for all integers? Knorr believes that Euclid did recognize the need, but the lack of algebraic notation limited him. In modern notation, a proof of the infinitude of primes would begin: let  $p_1, p_2, p_3, \dots, p_n$  be distinct primes... In this manner, the terms are generalized and the number of terms is indefinite. Does this lack of generality damage Euclid's proof? Not according to the ancient commentator Proclus who writes in *A Commentary on the First Book of Euclid's Elements*:

...We may not suppose that the result is confined to the particular instance. This procedure is justified, since for the demonstration they use the objects set out in the diagram not as these particular figures, but as figures resembling others of the same sort (Knorr, 1976, p.357).

By choosing representative cases, which are characteristic of the whole, Euclid is able to construct valid proofs. It is interesting to note that with the simple addition of "etc.." to the list  $A, B, C$ , Gauss has avoided all such criticism through the centuries. Similarly, one should not discount the proofs of Euclid on such an insignificant point (Knorr, 1976).

Euclid may have chosen to limit his generality for another reason: pedagogy. The *Elements* is a pedagogical text, and books VII-IX are a summary of elementary number theory. When teaching, a loss of generality is acceptable for purposes of simplification, especially when that loss does not significantly damage the concepts or procedures used (Knorr, 1976). It is common in classrooms today for teachers to use an explicit number of terms to demonstrate the behavior of a larger set.

Euclid's commentators have often cited his 'geometric arithmetic' as a major barrier to a more generalized method. Those critics usually name two major examples as a basis of the limitation. First, Euclid represents integers as line segments and, second, classifies numbers in geometric terms:

square, plane, solid, cube, etc. The representation of integers as line segments is, in fact, well established in books VII-IX; Knorr notes that linear representations are utilized in less than one fourth of the propositions in those books. However, Euclid never utilizes line segments or the properties of line segments in the proof of any proposition. The second argument, the geometric classification of numbers, has Pythagorean roots. Yet, by the time of Plato, philosophers only used these classifications as analogies. Euclid defines the classifications, but he does so in purely numerical terms. If geometric reasoning is assumed, then no powers higher than the third would be conceivable in three dimensions, but Euclid defines higher powers using recursive multiplication so that  $A^n$  is generated by the  $n + 1^{\text{st}}$  term of the sequence  $1, A, A \cdot A, A \cdot (A \cdot A)$ , etc. Furthermore, Euclid always represents squares and cubes in linear terms. These two features conflict with a strict interpretation of the number theory as geometric (Knorr, 1976).

It will be helpful to examine a specific proposition and proof to illustrate that Euclid's number theory is not geometric. Recall that VII;16 states the commutative property of multiplication: "If two numbers by multiplying one another make certain numbers, the numbers so produced will be equal to one another." Euclid's proof is as follows.

Let  $A, B$  be two numbers, and let  $A$  by multiplying  $B$  make  $C$ , and  $B$  multiplying  $A$  make  $D$ ; I say that  $C$  is equal to  $D$ . For since  $A$  by multiplying  $B$  had made  $C$ , therefore  $B$  measures  $C$  according to the units in  $A$ . But the unit  $E$  also measures the number  $A$  according to the units in it; therefore the unit  $E$  measures  $A$  the same number of times that  $B$  measures  $C$ . Therefore, alternately, the unit  $E$  measures the number  $B$  the same number of times that  $A$  measures  $C$  [VII;15]. Again, since  $B$  by multiplying  $A$  has made  $D$ , therefore  $A$  measures  $D$  according to the units in  $B$ . But the unit  $E$  also measures  $B$  according to the units in it; therefore the unit  $E$  measures the number  $B$  the same number of times that  $A$  measures  $D$ . But the unit  $E$  measured the number  $B$  that same number of times that  $A$  measures  $C$ ; therefore  $A$  measures each of the numbers  $C, D$  the same number of times. Therefore  $C$  is equal to  $D$ . Q.E.D. (Heath, 1956, p.316)

Paraphrasing, if  $AB = C$  and  $BA = D$ , then  $A|C, B$  times and  $A|D, B$  times. Thus,  $C = D$  since  $A$  divides them both  $B$  times. If Euclid had intended his number theory to be geometric, he would likely have proven this proposition on the commutative property of multiplication using area and the axioms of superposition and invariance of rectangles under translations (Knorr, 1976). Instead, Euclid uses a purely arithmetic

approach based on the definition of divisibility.

It has been shown that *The Elements* are not as flawed as many commentators have concluded them to be. The lack of generality, due to the absence of an algebraic notation, and a supposed geometric reasoning do not seriously damage Euclid's proofs in books VII-IX. It is now possible to move on to a discussion of Euclid's number theory in relation to the *Unique Factorization Theorem*, or as many label it, the *Fundamental Theorem of Arithmetic (FTA)*.

"If a number be the least that is measured by prime numbers, it will not be measured by any other prime number except those originally measuring it," Book IX, proposition 14 (Heath, 1956, p.402). Heath paraphrases IX;14: "a number can be resolved into prime factors in only one way" (1956, p.403) thus claiming that IX;14 is Euclid's equivalent of the *Unique Factorization Theorem (FTA)*.<sup>2</sup> Since Heath first made this claim in his 1906 translation, many critics have come forth to disagree. One of these critics, M.D. Hendy, completely discredits Euclid in this regard claiming that IX;14 is not equivalent to the *FTA* and furthermore, that the *FTA* is beyond the reach of Euclid entirely (1975). Hendy and others point out that IX;14 provides only for the unique prime factorization of integers with a square free factorization, and indeed, this is true. However, one can show that Euclid did indeed grasp the essence of the *FTA* and that its proof was within his reach.

If it is true that Euclid understood the essence of the *FTA*, why can its complete proof not be found in *The Elements*? Since the time of Gauss, the *Unique Factorization Theorem* has carried the label of a fundamental theorem in number theory. This classification was due to the structure that Gauss built for his number theory. Beginning with unique factorization, Gauss develops the properties of relatively prime numbers; thus the *FTA* is central. However, Euclid structures his theory differently. His foundation is the division algorithm from which he constructs a working definition of relatively prime numbers (Knorr, 1976). Euclid proceeds to obtain results on prime numbers such as VII;30 (paraphrased): "If  $c$ , a prime number, measures  $ab$ , then  $c$  will measure either  $a$  or  $b$ ," (Heath, 1956, p.332) which is used in the proof of IX;14. Thus, Euclid's theory places unique prime factorization on the level of a corollary. It is also important to note that Euclid does not use IX;14 to verify any previous result; it is stated late in the last book of Euclid's number theory; and its proof utilizes only VII;30

<sup>2</sup> "...the least that is measured by prime numbers..." refers to the least common multiple of a list of primes. If  $\text{lcm}(a,b,c) = A$  and we find  $a', b', c'$  such that  $A = \text{lcm}(a',b',c')$ , then by IX;14 every element in  $a,b,c$  must equal exactly one element in  $a',b',c'$  and vice versa. Thus IX;14 provides for square-free prime factorizations (Knorr, 1976)

and 36. Furthermore, IX;14 is never used in the proof of a later theorem. IX;14 may have only been included because of Euclid's familiarity with the concept in arithmetic and because of the suitability of his number theory to prove it (Knorr, 1976).

By combining the content of VII;30 and IX;14, Collison claims that the essence of the *FTA* is present in *The Elements*. It is only because of notational limitations that Euclid does not combine the two into a stronger statement (1980). By overcoming the notational limitations of his time, could Euclid have proven, using only the content of his *Elements*, the full *Fundamental Theorem of Arithmetic*? Knorr believes that Euclid could and demonstrates the following method by which it could have been done. *The Fundamental Theorem of Arithmetic* will develop as a corollary to the following statement:

Let  $N$  equal the least common multiple of  $A, B, C, \dots$  where  $A, B, C, \dots$ , are powers of distinct primes; let  $p$  be a prime such that  $p$  divides  $N$ ,  $p^n$  divides  $N$ , but  $p^{n+1}$  does not divide  $N$ ; then  $p^n$  is exactly one element of  $A, B, C, \dots$  (Knorr, 1976, p.364)

The proof of this statement relies on the following three Euclidean propositions:

(i.) IX;11:  $A^n = A^r \cdot A^{n-r}$ , where  $r \in \{0, 1, 2, \dots, n\}$ .

(ii.) IX;12: if a prime  $p$  divides  $A^n$ , then  $p$  divides  $A$ .

(iii.) IX;13: if  $p$  is a prime, then the set  $1, p, p^2, p^{n-1}$  contains all proper divisors of  $p^n$ .

These direct corollaries will also be needed:

(iv.) Corollary to IX;13: if  $p$  and  $q$  are prime and  $p \neq q$ , then  $p^n$  and  $q^m$  are relatively prime for all positive integers  $m$  and  $n$ .

(v.) Corollary to IX;14: if  $N = \text{lcm}(A, B, C, \dots)$  where  $A, B, C, \dots$  are relatively prime, then there exist no divisor of  $N$  that is relatively prime to every element of  $\{A, B, C, \dots\}$ .

(vi.) Corollary to IX;14: if a prime  $p$  divides  $N$  where  $N = \text{lcm}(A, B, C, \dots)$  and  $A, B, C, \dots$  are relatively prime, then  $p$  divides exactly one term of  $\{A, B, C, \dots\}$

Proof: Let  $N$  equal the least common multiple of  $A, B, C, \dots$  where  $A, B, C, \dots$  are powers of distinct primes; let  $p$  be a prime such that  $p$  divides  $N$ ,  $p^n$  divides  $N$ , but  $p^{n+1}$  does not divide  $N$ . Since  $A, B, C, \dots$  are powers of distinct primes, they all relatively prime to one another by (iv). This meets the conditions for (v) and (vi). By (vi), since  $p$  divides  $N$ ,  $p$  divides exactly one term of the set  $\{A, B, C, \dots\}$ , say  $A$ . By (ii),  $A$  is a power of  $p$ . We must show that  $A = p^n$ . If  $A \neq p^n$ , then by (i) one must divide the other:

1. Suppose  $p^n$  divides  $A$ . By definition of divides,  $A = p^{n+q}$  for some  $q$ ,

thus  $q|A$ , and by (iii)  $q \in \{1, p, p^2, \dots, p^{n-1}\}$ . Recall that  $A$  divides  $N$  since  $N = \text{lcm}(A, B, C, \dots)$  which implies that  $p^n q$  divides  $N$ . Thus a power of  $p$  greater than or equal to  $(n + 1)$  divides  $N$ . This is a contradiction.

- Suppose  $A$  divides  $p^n$ . Thus  $p^n = A \cdot q$  and by (iii)  $p$  divides  $q$  as before. Now set  $N = p^n \cdot M$ .  $B$  divides  $N$  and by (iv) is relatively prime to  $p^n$ ; therefore,  $B$  divides  $M$ . The same follows for  $C$  and all other elements that originally divided  $N$  (other than  $A$ ). Hence  $A \cdot M$  is a common multiple of  $A, B, C, \dots$ . But if  $A$  divides  $p^n$ , then  $(A \cdot M)$  divides  $(p^n \cdot M)$  and  $(p^n \cdot M) = N$ . This contradicts the construction of  $N$  as the least common multiple of  $A, B, C, \dots$

Thus  $p^n = A$ . Q.E.D. (Knorr, 1976, p. 363-364)

The *Fundamental Theorem of Arithmetic* follows as a corollary: let  $N = \text{lcm}(A, B, C, \dots)$  where  $A, B, C, \dots$  are powers of distinct primes. Suppose  $N = \text{lcm}(A', B', C', \dots)$  where  $A', B', C', \dots$  are powers of distinct primes. By the previous result, every element of  $A', B', C', \dots$  equals exactly one element of  $A, B, C, \dots$  and vice versa. So the two sets are identical (Knorr, 1976). We have factored  $N$  into a unique list of distinct powers of primes.

It has been shown that Euclid's theorems and techniques are quite adequate for proving a full version of the *Fundamental Theorem of Arithmetic*. What reasons do we have then for criticizing Euclid's number theory? We have shown that the lack of generality is undamaging, we have shown that Euclid's number theory is not geometric, and we have demonstrated its strength by proving the unique prime factorization of the integers. Why then has it been the habit of so many mathematical historians to tear down the accomplishments of this great work? In fact, great praise is in order. Let us quickly compare Euclid's treatment of unique factorization to those of Euler, Legendre, and Gauss, three of the greatest mathematicians of the modern age. In *Elements of Algebra*, Euler simply assumes the unique prime factorization of the integers and uses the fact in many proofs. Legendre also lets the statement go without proof in his *Théorie des Nombres*, treating it as self-evident. In notation, Euler and Legendre were still somewhat handicapped, but this does not explain a complete absence of an argument. On the other hand, Gauss developed an adequate notation to represent the statement in his *Disquisitiones Arithmeticae*. Most give credit for the *Fundamental Theorem of Arithmetic*'s first proof to him, but in fact, in this work Gauss proves only the uniqueness of prime factorizations and assumes their existence without proof. Euclid deserves the largest amount of credit for the unique factorization theorem. Euclid's statement may not

have included all possible cases, but he overcame a limited notation and constructed a proof that was both rigorous and complete (Collison, 1980).

### References

1. Collison, M.J., "The Unique Factorization Theorem: From Euclid to Gauss," *Mathematics Magazine*, 1980, 53 (2), 96-100.
2. Grattan-Guinness, I., "Number, Magnitudes, Ratios, and Proportions in Euclid's Elements: How Did He Handle Them?" In K.H. Parshall (Ed.), *Historia Mathematica*, 1996, 23, (pp.355-375). Great Britain: Academic Press.
3. Hartshorn, R., "Teaching Geometry According to Euclid," *Notices of the American Mathematics Society*, 2000, 47 (4), 460-465.
4. Heath, T.L., "The Thirteen Books of Euclid's Elements" (2nd edition unabridged), 2000, New York: Dover.
5. Hendy, M.D., "Euclid and the Fundamental Theorem of Arithmetic," In K.O. May (Ed.), *Historia Mathematica*, 1975, 2, (pp.189-191). Great Britain: Academic Press.
6. Knorr, W., "Problems in the Interpretation of Greek Number Theory: Euclid and the 'Fundamental Theorem of Arithmetic,'" *In Studies in History and Philosophy of Science*, 1976, 7 No. 3 (pp.353-368). Tarrytown, NY: Pergamon.

---

*Kappa Mu Epsilon*, Mathematics Honor Society, was founded in 1931. The object of the Society is fivefold: to further the interests of mathematics in those schools which place their primary emphasis on the undergraduate program; to help the undergraduate realize the important role that mathematics has played in the development of western civilization; to develop an appreciation of the power and beauty possessed by mathematics due to its demands for logical and rigorous modes of thought; to provide a Society for the recognition of outstanding achievement in the study of mathematics at the undergraduate level; and to disseminate the knowledge of mathematics and familiarize the members with the advances being made in mathematics. The official journal of the Society, *The Pentagon*, is designed to assist in achieving these objectives as well as to aid in establishing fraternal ties between the Chapters.

## Quadratic Varieties

Joseph Bohanon, *student*

Missouri Theta

Evangel University  
Springfield, MO 65802

Presented at the 2001 National Convention.

In high school algebra courses we are introduced to conic sections, and in multi-variate calculus courses we are introduced to the natural extension of these: quadric surfaces. However, for mathematicians, curiosity overcomes us and we want to know about such figures in 4-, 5-, ...  $n$ -dimensional spaces as well as spaces with an infinite number of dimensions (Hilbert spaces). This generalization is called a quadratic variety, which is the scope of this paper. In this paper, we will look at the quadratic varieties in four dimensions from a three-dimensional perspective and look at what naturally comes next in five, six, ...  $n$  dimensions. We will also look at some properties of the generalization of the sphere in higher dimensions.

First we need to establish a few definitions. A quadratic variety is a polynomial equation mapping a set  $\{x_1, \dots, x_n\}$  into  $R^n$  such that the maximum of the degree over all the variables is two. So an example of a quadratic variety is

$$f(x_1, x_2, \dots, x_n) = x_1^2 + x_2^2 + \dots + x_n^2 - 1$$

We are concerned with the set of points such that

$$f(x_1, x_2, \dots, x_n) = 0$$

In other words a set of numbers such that at least one of them is squared, but none of them has degree greater than two. The next concept is not quite as important, but it greatly simplifies work in the classification of these varieties, namely homeomorphism. The actual definition says that homeomorphism is a continuous mapping of one variety onto another without tearing or sewing, which has a continuous inverse. In other words, we can stretch one variety into the other. In this respect, a circle and an ellipse will be considered the same variety because you can stretch one into the other without tearing or sewing. Similarly some of the equations we get for quadratic varieties are just affine transformations of others. An affine transformation is basically shifting the entire space in one direction. It can be easily seen that this need not map the origin to the origin.

Before going on to the quadratic varieties we would normally recognize, we should look at what these varieties are in  $R^0$  and  $R^1$ . We can consider the  $R^0$  set, with only one element, namely 0. Since we have no variables to choose from, this quadratic variety is just a constant. If the constant is non-zero, the variety is empty; if it is zero, it is all of  $R^0$ . In  $R^1$  the general quadratic variety would be of the form  $x^2 = k$ . For positive  $k$  the only points are  $\pm\sqrt{k}$  on the  $x$ -axis. If  $k = 0$  we yield what is called a degenerate case, with only one solution, namely  $x = 0$ . If  $k$  is negative the variety is non-existent in  $R^1$  (it can be done with complex numbers, but we restrict our domain to the real numbers). Here's the first place where homeomorphism comes into play. We might as well restrict the values of  $k$  to be  $-1, 0$ , and  $1$ , because if we have  $\pm\sqrt{k}$  on the  $x$ -axis we can just shrink the values until we get to one, and keep all of the same general properties. We need to consider  $k = 0$  because this will yield a degenerate case, and in some cases with multiple variables we will need to consider  $k = -1$ , because this is not always non-existent.

So now we need to consider all of the possible quadratic functions in  $R^2$  (quadratic curves). First we consider the cases where all of the variables are squared. This gives us  $x^2 \pm y^2$ . Next we have to consider the cases where only one variable is squared, which gives  $x \pm y^2$  and  $y \pm x^2$ , but the first is just a rotation of the second so they are homeomorphic and we might as well consider the second. Also, as we learned in Algebra, the plus or minus sign only determines the direction the curve opens which means that the plus case is homeomorphic to the minus case, so we might as well just consider the minus. This leaves us with three general cases of quadratic curves. In algebra we were taught that the general form also included multipliers of  $x$  and  $y$ , but once again, through homeomorphism we can just get rid of them.

First, let us consider the case  $y - x^2 = k$ . We learned in algebra that  $k$  just determines how high above the axis the graph is, so we can just let this be zero. Here we get what is called a parabola. One of the ways to view this from a one-dimensional perspective is to see what happens if we pass it through (take slices parallel to) the  $x$ -axis or the  $y$ -axis. With the  $x$ -axis, first a point would appear, and then it would shift into two points, which slowly accelerated away from each other to infinity. With the  $y$ -axis a point would appear at infinity, accelerate toward the middle of the axis, and return back to infinity.

Next consider the case where  $x^2 + y^2 = 1$ . This gives us a circle. If we pass this through (i.e. take slices parallel to) the  $x$ -axis we have a point that suddenly appears, then turns into two points which spread apart, then come together, then turn into a single point, which finally vanishes. The same

happens with the  $y$ -axis. If we replace the 1 by  $-1$  we get an equation that has no solutions in the real numbers, so it is non-existent. If we replace the 1 by 0, we get only one point satisfying the equation, namely  $(0, 0)$ . This is the degenerate form of a circle.

Finally consider the case where  $x^2 - y^2 = 1$ . This gives us a hyperbola. Passing this through the  $x$ -axis, we first start the axis as far down as we wish. This gives us two points near the edge of the graph which close in toward each other as the slices get closer to the  $x$ -axis. Somewhere near the axis the two points become one, then it disappears for a small portion of the graph. The point then re-appears, changes to two points, which both go to infinity in opposite directions. If we let change the 1 to a  $-1$  this just flips the hyperbola along one of the axes, but this is just homeomorphism (passing this through the  $x$ -axis is the same as what would happen passing the hyperbola through the  $y$ -axis). If we change 1 to 0 and solve for  $y$  we get  $y = x$  or  $y = -x$ . The graph of this just looks like a giant X across the entire  $xy$ -plane. This completely classifies the quadratic varieties in  $R^2$  up to homeomorphism.

Moving into  $R^3$ , things get a little trickier. To view these "quadric surfaces" from a two-dimensional perspective we need to look at what happens when it passes through the  $xy$ -,  $yz$ -, and  $xz$ -plane. In addition there are many more possibilities for our polynomial equations. First we should consider the equations where all three variables are squared. This gives us

$$x^2 + y^2 + z^2, x^2 + y^2 - z^2$$

and

$$x^2 - y^2 - z^2$$

Now we consider the cases where two of the variables are squared. As in the case of two variables, the sign of the term that isn't squared isn't important since it just results in a flip. This gives  $x^2 + y^2 - z$  and  $x^2 - y^2 + z$ . As in the case with the parabola, it doesn't really matter which two of our variables is squared, because in  $R^3$  we can just rotate along a plane and yield the same variety.

Finally we should consider the equations where only one term is squared. Once again, the signs in front of the two terms that aren't squared and the particular term that is squared aren't important so we only have  $x^2 - y - z$ . Consider this one first, letting the entire equation be equal to zero. Let  $z$  be a constant that can vary. This is the equivalent of passing our surface through the  $xy$ -plane. On this plane we get a parabola opening up, and  $z$  changes our height. Similarly if  $y$  is a constant we get a parabola on the  $xz$ -plane. If  $x$  is a constant we have plane which is the equivalent of taking

the line  $y = z$  and stacking it on top of itself in the  $x$  direction. More generally, this surface is called a parabolic cylinder, and looks like a parabola that has been extended across all of space.

Next consider the first of the cases where two terms are squared and let this be equal to zero. Letting  $z$  be a constant gives us a circle that increases in radius as the constant gets larger. Letting  $x$  or  $y$  be a constant gives us a parabola opening up. This shape is called a paraboloid and is the equivalent of taking a parabola and rotating it around its axis of symmetry. However, if we fix  $z$  we get a circular cylinder, which looks like a bunch of circles stacked on top of each other.

Now consider the second surface with two terms squared and set it equal to zero. Letting  $z$  be a constant gives us a hyperbola. If  $y$  is constant we get a parabola opening up. If  $x$  is constant we get a parabola opening down. This shape is called a hyperbolic paraboloid and can be visualized as the graphical equivalent of a saddle.

Finally we can consider the three surfaces with all three variables squared. Looking at the first of these we can set it equal to one and no matter which variable we let be a constant we get a circle of increasing radius. This surface is called a sphere and is a fundamental surface in the study of quadratic varieties. It looks like a circle revolved around one of its diameters. As in the case with two dimensions, if the equation is equal to zero we get a single point at the origin, and if the equation is equal to a negative number it is non-existent.

Next look at the second of the surfaces where all three variables are squared but there is one minus sign and set this equal to 1. If  $z$  is a constant we get a circle with a radius that gets larger if we go up or down from the origin. If  $y$  or  $x$  is a constant we get hyperbolas opening in different directions. This surface is called a hyperboloid of one sheet. If we fix  $z$  here we obtain a hyperbolic cylinder that looks like a bunch of hyperbolas stacked on top of each other. To picture this we should first look at the degenerate case where the whole equation is equal to zero. This looks like two cones meeting at the origin, one opening up, and one opening down. The general form involves taking the center point and making it a circle while keeping the cones intact.

Finally consider the last form where all three variables are squared but two are negative. Letting  $z$  or  $y$  be a constant gives us a hyperbola. Letting  $x$  be a constant gives us a degenerate case if  $x = 0$  and non-existent otherwise. This shape is called a hyperboloid of two sheets and is like taking the two cones from above, but pulling them away from each other. If we let the equation be equal to a negative number we just get a hyperboloid of one sheet. This completely classifies the quadratic varieties in  $R^3$  up to

homeomorphism.

Before we proceed on to the fourth dimension, we should look at what happened when we jumped from two dimensions to three. Of the six general types, five were just easy extensions of the three curves in two-space. The sphere is obtained by revolving a circle about its diameter, the paraboloid by rotating a parabola about its axis of symmetry, and both hyperboloids by rotating a hyperbola about its two axes. The cylinders are just obtained by stretching the circle, paraboloid, and the hyperboloid. The one surface that is not exactly a rotation or stretch is the hyperbolic paraboloid and naturally we will encounter things like it in four dimensions. The basic idea is that a multiple number of the slices parallel to planes are the same, but in different directions, and this introduces hyperbolas.

Now, to make the jump into four dimensions we have to think abstractly. We will still stretch the quadratic surfaces, but in a dimension which we cannot visualize. However, we can still look at cross-sections of the variety that are parallel to three-dimensional spaces.

As before, it will be best to first consider all the possibilities for the left side of our equation. First we have the equations where all four terms are squared, namely

$$x^2 + y^2 + z^2 + w^2, x^2 + y^2 + z^2 - w^2$$

$$x^2 + y^2 - z^2 - w^2$$

and

$$x^2 - y^2 - z^2 - w^2$$

Secondly, we have the equations where three terms are squared. These consist of

$$x^2 + y^2 + z^2 + w$$

$$x^2 + y^2 - z^2 + w$$

and

$$x^2 - y^2 - z^2 + w$$

Now consider the equations where two variables are squared:

$$x^2 + y^2 + z + w$$

and

$$x^2 - y^2 + z + w$$

Finally we have the equation where only one term is squared:

$$x^2 + y + z + w$$

Since we don't have a clear-cut way of visualizing the fourth dimension, it will be easiest to look at slices along "hyper-planes" first.

Below is a table of what the slices of the ten four-dimensional quadratic varieties look like along the  $xyz$ -,  $xyw$ -,  $xzw$ -, and  $yzw$ -hyper-planes.

Equation	$xyz$ -hyper-plane	$xyw$ -hyper-plane	$xzw$ -hyper-plane	$yzw$ -hyper-plane
$x^2 + y^2 + z^2 + w^2$	Sphere	Sphere	Sphere	Sphere
$x^2 + y^2 + z^2 - w^2$	Sphere	Hyperboloid of one sheet	Hyperboloid of one sheet	Hyperboloid of one sheet
$x^2 + y^2 - z^2 - w^2$	Hyperboloid of one sheet	Hyperboloid of one sheet	Hyperboloid of two sheets	Hyperboloid of two sheets
$x^2 - y^2 - z^2 - w^2$	Hyperboloid of two sheets	Hyperboloid of two sheets	Hyperboloid of two sheets	Sphere
$x^2 + y^2 + z^2 + w$	Sphere	Paraboloid	Paraboloid	Paraboloid
$x^2 + y^2 - z^2 + w$	Hyperboloid of one sheet	Paraboloid	Hyperbolic Paraboloid	Hyperbolic Paraboloid
$x^2 - y^2 - z^2 + w$	Hyperboloid of two sheets	Hyperbolic Paraboloid	Hyperbolic Paraboloid	Paraboloid
$x^2 + y^2 + z + w$	Paraboloid	Paraboloid	Parabolic Cylinder	Parabolic Cylinder
$x^2 - y^2 + z + w$	Hyperbolic Paraboloid	Hyperbolic Paraboloid	Parabolic Cylinder	Parabolic Cylinder
$x^2 + y + z + w$	Parabolic Cylinder	Parabolic Cylinder	Parabolic Cylinder	Plane

Now we must examine each of these more carefully. Edwin A. Abbott's book *Flatland* showed us that we can visualize higher-dimensional objects in our world by seeing what they look like when we pass them through our world. In other words, if we live in a universe with  $x$ -,  $y$ -, and  $z$ -axes, since  $w$  is a variable, consider what our surface looks like as  $w$  goes from minus infinity to infinity. Likewise, if we live in a universe with any three of the four axes, we can consider what happens when the missing axis is changed to a variable constant.

Consider first, the most fundamental of these hyper-surfaces in which all the terms are squared and the sign in front of all four are positive. As in lower dimensions, the degenerate form is a singular point at the origin and if our equation is equal to a negative number, the hyper-surface does not exist. Analogous to what we saw when passing a sphere through two-dimensional planes, when passing this hyper-sphere through any of the

four hyper-planes we see a single point, which grows into a larger and larger sphere and, after reaching the critical radius, shrinks to a point and disappears.

Now consider the hyper-surface in which all terms are squared, but only three terms are positive. In degenerate form, this is a four-dimensional cone consisting of a single point at the origin and sphere stacked on each other in four-space extending infinitely in both directions. If the equation is equal to a negative number then we can rearrange it to yield another hyper-surface with which we'll deal later. If the number is positive the visualization is analogous to our view of the hyperboloid of one sheet. On the  $xyz$ -plane we will see a sphere of radius 1 which opens both up and down as spheres of increasing radius.

Next let's look at the hyper-surface with all terms squared, but three of the terms are negative. The degenerate form of this figure looks the same as the degenerate for the previous example. If the equation is equal to a negative number it is the same as the previous hyper-surface. When the equation is equal to 1, consider what happens when we let  $x$  vary. If  $x = 0$  we get a non-existent sphere. If  $x = \pm 1$  we get the degenerate sphere. Any value of  $x$  greater than one yields a sphere, as does any value of  $x$  less than negative one. This is analogous to the hyperboloid of two sheets. At one unit above, and below the origin, there is a single point, which grows into increasingly larger spheres in each direction.

Lastly, consider the figure where all terms are squared, but two terms are positive. The degenerate form of this is like a cone made out of tori (donuts). At plus and minus infinity there are infinitely large tori, and as each gets closer to zero the inner radius of the tori get infinitely close to zero, and the limiting process eventually condenses them down to a point. In the non-degenerate case we can rewrite this equation as  $y^2 - z^2 - w^2 = 1 - x^2$ . If  $|x| > 1$ , this is a hyperboloid of one sheet; if  $|x| = 1$ , it's a cone; if  $|x| < 1$ , it's a hyperboloid of two sheets.

Now consider the forms where three terms are squared. If all of them are positive, this is just analogous to the paraboloid, except we use spheres instead of circles. The other two forms with three terms squared are really the same thing. Looking at the first we see that if we move  $w$  to the right side of the equation we get a hyperboloid of two sheets above the axis, a cone at the origin, and a hyperboloid of one sheet below the axis. If we look at the second we see that it is the same as the first, only reflected across the axis.

The other forms are mostly analogous to lower dimensional forms in that we have more than one linear term. For the form where two terms are squared, both positive, we have a paraboloid that has been slid in two

dimensions. For the form where two terms are squared and one of them is positive, we have a hyperbolic paraboloid that has been slid in two dimensions. Finally the form where only one term is squared we have a parabola that has been slid in three dimensions.

As one can imagine, as the dimensions increase, the forms get more and more difficult to categorize. In general there are at most  $\frac{n^2+n}{2}$  unique forms in each dimension. However, as seen above, two of the ten forms in four dimensions are homeomorphic, so there is probably an error term that needs to be taken off of the total, which is zero is one, two, or three dimensions.

Although most of the higher dimensional varieties are too complex to describe, we can see what some of them look like. First, the general  $n$ -dimensional form with all of its terms squared and positive is called the  $(n-1)$ -hypersphere (the  $n-1$  is a consequence of topology and not really important here). Analogous to lower dimensions, the  $n$ -hypersphere is just a collection of  $(n-1)$ -hyperspheres such that if you pass it through  $(n-1)$  dimensional space, you see a point that expands to an  $(n-1)$ -hypersphere of radius 1 and shrinks to a point. These forms have more interesting properties as we will see a little later.

The forms having some linear terms are analogous to lower dimensions where we take lower dimensional forms and stretch them in multiple dimensions, depending on how many linear terms we have. The forms where all terms are squared, but not all of them are positive are a little more confusing.

In general, the forms where only one term is negative or positive, are analogous to the hyperboloids of one or two sheets respectively, using different dimensional hyperspheres instead of circles. If we allow more than one negative or positive term, this becomes a family of lower dimensional forms where not all terms are positive.

One interesting consequence of higher dimensions is what the degenerates look like. First, we need to know what a Cartesian product looks like. In general, you will have one set of numbers  $A$ , and another set of numbers  $B$ , and you consider  $A \times B$  to be the set of all ordered pairs where the first number is from  $A$  and the second number is from  $B$ . Now, define the  $n$ -hypersphere of radius 1  $S_n$ . Let's look at what  $S_1 \times S_1$  is. We want a form where every point can be described uniquely as part of two different circles. Such a form is the torus, mentioned above. If we pick a certain point on a torus, we have one circle containing the point that goes around the hole in the middle, and one circle that contains the point, but doesn't contain the hole in the middle. Now, define the form with all terms squared, with  $m$  positive terms, and  $n$  negative terms  $(m, n)$ . To get

the degenerate form for  $(m, n)$ , take  $S_{m-1} \times S_{n-1}$  and make a cone out of these figures. In other words, from both sides of infinity to the origin, you have these figures, decreasing in size, and condensing to a single point. One consequence of this is that degenerate hyperspheres are a single point.

We can also look at some examples of degenerates in lower dimensions. Consider the hyperbola, labeled  $(1, 1)$ . So we cross the 0-sphere with itself. The 0-sphere is the set of point on the  $x$ -axis that are length 1 from the origin, which consist of 1 and  $-1$ . When we cross two of these spheres together, we get just the set of points such that each is a set of different 0-spheres. It turns out that this is just a 0-sphere. Now, consider a cone made out of 0-spheres, and you get the degenerate hyperbola. Similarly, consider the hyperboloid of one sheet. This is labeled  $(2, 1)$ . Therefore, we cross a circle with a 0-sphere, which in turn gives us a circle. Making a cone out of these gives us the degenerate hyperboloid of one sheet. Another interesting point is that up to homeomorphism, the Cartesian product is commutative, so the degenerate of  $(m, n)$  is the same as the degenerate of  $(n, m)$ . The next step in examining quadratic varieties is to look at Hilbert spaces (those with infinite dimensions).

As a side note, we should look at one of the practical applications of hyperspheres. When you were first introduced to complex numbers, you were shown that you can look at the  $x$ -axis as the real part of the number, and the  $y$ -axis as the imaginary part of the number. One of the natural questions to follow from this is, is there a set of numbers such that we can visualize them as 3-dimensional space? The answer to this question is no. However, if we look to 4 dimensions, and 8 dimensions, the answer is yes.

One may also ask is this true for  $n$  dimensions, where  $n$  is a power of two. As the dimensions go past 8, it turns out that there are certain structural properties that start to fall apart. First, let's look at some of the properties we lose when we go from smaller sets to larger sets. The real numbers are a subset of the complex numbers. One property the real numbers have is that if we pick two distinct numbers,  $a$  and  $b$ , then  $a < b$  or  $b < a$ . When we expand the real numbers to include the complex numbers we lose this property.

Now, consider the next set up from the complex numbers called the quaternions. This is the set of numbers  $a1 + bi + cj + dk$  where  $1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ,  $i = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$ ,  $j = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ , and  $k = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$ . One of the first things we learn when learning matrices is that matrix multiplication is not commutative. Since the quaternions are just matrices, they are not commutative. When we expand even further, we get a set called Cayley's numbers. These are written in the form  $a1 + bi_0 + ci_1 + di_2 + ei_3 +$

$fi_4 + gi_5 + hi_6$  where  $1, i_n, i_{n+2}, i_{n+3}$  behave like the quaternions. As you can probably imagine, the multiplication tables for these number are very complex and lengthy to do. One property that these numbers lack is that multiplication is not even associative, in other words  $a(bc)$  is not equal to  $(ab)c$ . If we try to expand Cayley's numbers, we get an even larger set that isn't even well-defined (this was proven by James Milnor).

Now, let's look at a problem that seems unrelated, but in fact is a direct consequence of the fact that there are only 2-, 4-, and 8-dimensional complex numbers. Consider a round disc with little hairs sticking perpendicularly along the outside. We can take a comb and in one continuous motion comb all the hair on the circle such that we can start at one place, and return to it, having combed all the hair. In fact, we can do this in two different ways. This turns out to be a consequence of imaginary numbers, and the fact that we can do it two different ways is analogous to taking the complex conjugate of a number.

Now, if we try to do the same thing on a sphere, we'll discover that it is not possible to do so. In fact, you cannot do it without leaving a bald spot. This is a consequence of the fact that there are no 3-dimensional imaginary numbers. Although the consequences are theoretical, because there are only 4- and 8-dimensional complex numbers, you can only comb the hair without bald spots on a 3-sphere, and a 7-sphere, in 4 and 8 different ways, respectively. Proofs of this result are far too complex for the scope of this paper.

Getting back on track, let's look at some non-topological properties of spheres, namely volume and surface area. The reasoning behind the following formula takes a little multi-dimensional calculus to understand, but we will state it without explanation. If  $S_n$  is the hyper-surface area of the  $n$ -

hypersphere of radius 1, then it turns out that  $S_n = \frac{\left( \int_{-\infty}^{\infty} e^{-x^2} dx \right)^n}{\int_0^{\infty} e^{-r^2} r^{n-1} dr}$ . Evaluating

these integrals takes some techniques taught in multi-dimensional calculus, but in the end, we get that  $S_n = \frac{2\pi^{\frac{n}{2}}}{\Gamma(\frac{n}{2})}$  where  $\Gamma(n) = \int_0^{\infty} x^{n-1} e^{-x} dx$ .

If you do integration by parts once you get that  $\Gamma(n) = (n-1)\Gamma(n-1)$ , and you can prove by induction that  $\Gamma(n) = (n-1)!$ . Using another more advanced technique, we can get that  $\Gamma(\frac{1}{2}) = \sqrt{\pi}$ . Since the dimension of a hypersphere is an integral value, we need not worry about any other values of the gamma function, and using the identities above, we can get the surface area for all unit hyperspheres. It turns out that when we let the hypersphere have radius  $R$ , the surface area becomes  $S_n R^{n-1}$ . Using more advanced techniques, we can show that if we take the general formula for

the surface area of a hypersphere with radius  $R$  and integrate with respect to  $R$  that we get the general formula for the volume of the hypersphere with radius  $R$ . After algebraic manipulation, we get that the volume of a hypersphere in  $n$ -dimensions with radius  $R$  is  $\frac{\pi^{\frac{n}{2}} R^n}{\Gamma(\frac{n+2}{2})}$ . If we let  $n = 2$ , we get  $\frac{\pi R^2}{\Gamma(2)} = \pi R^2$ , as expected.

Similarly if we let  $n = 3$  we get  $\frac{\pi^{\frac{3}{2}} R^3}{\Gamma(\frac{5}{2})} = \frac{\pi^{\frac{3}{2}} R^3}{\frac{3\sqrt{\pi}}{4}} = \frac{4}{3} \pi R^3$ , as expected.

At first it would seem that as the dimensions increase, so would the hypervolume (or content). However, if we fix the radius and plot the dimensions against the content, we see that the content peaks somewhere between five and six dimensions, and the surface area peaks somewhere between seven and eight dimensions. If you take the limit of the content or hyper-surface area of a hypersphere of finite radius, you get that the infinite dimensional hypersphere has no content or hyper-surface area. This only changes if the radius is infinite, in which content and hyper-surface area proves to be infinite as well.

So we see that when we look at things in higher dimensions they are just as simple to view as analogous examples in lower dimensions. Although this paper contains several points about higher-dimensional "conic sections", it is certainly not complete. I pose the following questions as a conclusion to this paper:

1. Is there a simple formula for determining the number of quadratic varieties (up to homeomorphism) in a given dimension?
2. What sort of properties to higher dimensional analogies of the complex numbers lose as we continue doubling dimensions?
3. If the infinite hypersphere has no volume or surface area, what does it look like?

*Acknowledgments.* I would like to thank my advisor, Dr. Don Tosh, and SMSU professor Dr. Les Reid for all their help with this paper.

### *What's in a Name*

"It seems that Laplace transforms, like many other things bearing the names of other persons, are inventions of Euler. In any case, Euler used the transforms to solve differential equations when Laplace was - 7 years old, and he did it very neatly."

-Ralph Agnew

# Generalized Pascal Triangles of Finite Groups with Two Generators

Tim Pierce, *student*

Kansas Alpha

Pittsburg State University

Pittsburg, KS 66762

Presented at the 2001 National Convention.

## Introduction

The purpose of this project is to generate generalized Pascal Triangles of finite groups with two generators. Other code has been done to create Pascal Triangles of cyclic groups; we wish to extend this capability to groups with two non-trivial generators. One generator will be represented on the left side of a Pascal triangle and the other generator will be shown on the right side. We chose Maple to accomplish this task. Although Maple is slower and more idiosyncratic than other languages such as Pascal or C, Maple has a wealth of data structures that have useful properties in accomplishing this task that made this decision relatively easy.

> restart;

Number of rows for the *Generalized Pascal Triangle*

> Rows := 25;

## Defining the Group

Any group can be completely defined in terms of its generators and defining equations. The defining equations tell how the generators behave among themselves. If a finite group has  $n$  generators, we need  $n$  trivial such equations to tell us the degree of the generator and some nontrivial equations to show how generators interact with one another. In our case with  $n = 2$ , we need two trivial and one nontrivial equations. For example,  $Z_3 \times Z_4$  with generators  $x$  and  $y$  can be defined by the equations:  $x^3 = e, y^4 = e, xy = yx$ . The first two equations tell us that the orders of the two generators are 3 and 4, respectively. The last equation determines that the group is abelian. Any group whose generators satisfy these equations will be isomorphic to  $Z_3 \times Z_4$ . Our code requires that the desired group be entered in terms of its defining equations.

Any group is isomorphic to a group of permutations, which itself is a subgroup of a symmetric group. We use this fact to turn our given group, called  $g$  in the code, into an isomorphic permutation group, appropriately labeled  $pg$ . This new group is not stored in terms of all its elements but

only the  $n$  of  $S_n$  and the two generators as permutations (represented as disjoint cycles) are stored.

Group definitions

```
> with(group):
```

```
with(plots):
```

```
> g := grelgroup({a,b}, {[a,a], [b,b,b,b,b,b], [a,b,a,1/b]});
```

```
grouporder(g);
```

```
> pg := permrep(subgrel({},g));
```

```
pg := permgroup(op(1,pg),{seq(op(2,op(2,pg))[i]),i=1..nops(op(2,pg))});
```

$$g := \text{grelgroup} \left( \{a, b\}, \{[a, a], \left[ a, b, a, \frac{1}{b} \right], [b, b, b, b, b, b]\} \right)$$

14

```
pg := permgroup(14, {[[1, 4, 5, 6, 7, 8, 9], [2, 3, 10, 11, 12, 13, 14]],
```

```
[[1, 2], [3, 4], [5, 10], [6, 11], [7, 12], [8, 13], [9, 14]]})
```

Generators of the group

```
> gorder := grouporder(pg):
```

```
generators := op(2,pg):
```

```
> for i from 1 to nops(generators) do
```

```
print(i,generators[i]);
```

```
od;
```

```
1, [[1, 4, 5, 6, 7, 8, 9], [2, 3, 10, 11, 12, 13, 14]]
```

```
2, [[1, 2], [3, 4], [5, 10], [6, 11], [7, 12], [8, 13], [9, 14]]
```

### Generating the Elements of the Group

We use an algorithm named Depth First Search to generate all of the elements of the group. Sahni defines a Depth First Search (dfs) in this manner: starting at a vertex  $v$ , a dfs proceeds as follows. The vertex  $v$  is marked as visited and an unvisited vertex  $u$  adjacent from  $v$  is selected. If such a vertex does not exist, the search terminates. Assume  $u$  does exist. A dfs from  $u$  is now initiated. When this is completed, we select another unvisited vertex adjacent to  $v$ . If such a vertex does not exist, the search terminates. If it does, a dfs begins from this new vertex, and so

on. In our code, we start at the identity element  $e$ , denoted [ ] in the code. At each new element there are two paths we can take, since our group is generated by two elements: multiplying the element by the first generator,  $x$ , or multiplying the element by the second generator,  $y$ . In our example of  $Z_3 \times Z_4$ , we start with  $e$ , and then multiply by  $x$ , yielding  $x$ . Again we multiply by  $x$ , giving  $x^2$ . Once again, we multiply by  $x$  and get  $x^3$ . But by the defining equations,  $x^3 = e$ . So we have reached an element that we have already visited. By our algorithm we backtrack one level and take the other branch. In this case we go to  $x^2$  and multiply by  $y$ , giving us  $x^2y$ , which is an element we have not yet visited. We continue in this manner until we have no new paths to take. Depth First Search is generally used to traverse a data structure, such as a graph, but we actually use it to generate the data. Here we have no need to store the arcs of the graph since we have the rules to go from one element to another: we simply multiply either by  $x$  or  $y$ . All that we store are the vertices, or elements. This is analogous to instead of solving a maze by holding a map of it in your hand, to just knowing a set of rules to implement at each intersection in the maze (for example, mark the intersection with chalk and go right, straight, and then left).

We place all of these elements in a set called *visited*. We need to assign each element an arbitrary number so we place the elements of *visited* in an array named  $G$ . We store them in such a way that  $e$  is the first element, the first generator is the second element, and the second generator is the third element. We then create a group multiplication table, *GroupTable*, and display it.

Procedure that generates the elements of the group using a *Depth First Search* of the *Cayley graph* of the group with the given generators

```
> dfs := proc(x)
local y, z;
global generators, i, visited;
for z in generators do
  y := mulperms(x,z);
  if not member(y,visited) then
    print(i,y);
    i := i + 1;
    visited := visited union {y};
    dfs(y);
  fi;
od;
end;
i := 2;
```

```

visited := {}:
print(l,[]);
dfs([]);
G:=[[[],op(generators),op((visited minus generators) minus {})]]:

```

```

1, [ ]

```

```

2, [[1, 4, 5, 6, 7, 8, 9], [2, 3, 10, 11, 12, 13, 14]]

```

```

3, [[1, 5, 7, 9, 4, 6, 8], [2, 10, 12, 14, 3, 11, 13]]

```

```

4, [[1, 6, 9, 5, 8, 4, 7], [2, 11, 14, 10, 13, 3, 12]]

```

```

5, [[1, 7, 4, 8, 5, 9, 6], [2, 12, 3, 13, 10, 14, 11]]

```

```

6, [[1, 8, 6, 4, 9, 7, 5], [2, 13, 11, 3, 14, 12, 10]]

```

```

7, [[1, 9, 8, 7, 6, 5, 4], [2, 14, 13, 12, 11, 10, 3]]

```

```

8, [[1, 14, 8, 12, 6, 10, 4, 2, 9, 13, 7, 11, 5, 3]]

```

```

9, [[1, 2], [3, 4], [5, 10], [6, 11], [7, 12], [8, 13], [9, 14]]

```

```

10, [[1, 3, 5, 11, 7, 13, 9, 2, 4, 10, 6, 12, 8, 14]]

```

```

11, [[1, 10, 7, 14, 4, 11, 8, 2, 5, 12, 9, 3, 6, 13]]

```

```

12, [[1, 11, 9, 10, 8, 3, 7, 2, 6, 14, 5, 13, 4, 12]]

```

```

13, [[1, 12, 4, 13, 5, 14, 6, 2, 7, 3, 8, 10, 9, 11]]

```

```

14, [[1, 13, 6, 3, 9, 12, 5, 2, 8, 11, 4, 14, 7, 10]]

```

Generate the multiplication table

```

>Lookup :=proc(a)
global G;
local i;

```

```

for i from 1 to nops(G) do
  if a = G[i] then
    RETURN(i);
  fi;
od;
end:
> for i from 1 to gorder do
  for j from 1 to gorder do
    GroupTable[i,j] := Lookup(mulperms(G[i],G[j]));
  od;
od;

```

### The Multiplication Table

```

> print(array([seq([seq(GroupTable[i, j], j = 1..gorder)], i = 1..gorder)]));

```

1	2	3	4	5	6	7	8	9	10	11	12	13	14
2	10	4	5	8	9	3	6	7	11	13	14	12	1
3	4	1	2	10	13	14	11	12	5	8	9	6	7
4	5	2	10	11	12	1	13	14	8	6	7	9	3
5	8	10	11	13	14	2	12	1	6	9	3	7	4
6	9	13	12	14	2	11	1	10	7	3	5	4	8
7	3	14	1	2	11	12	10	13	4	5	6	8	9
8	6	11	13	12	1	10	14	2	9	7	4	3	5
9	7	12	14	1	10	13	2	11	3	4	8	5	6
10	11	5	8	6	7	4	9	3	13	12	1	14	2
11	13	8	6	9	3	5	7	4	12	14	2	1	10
12	14	9	7	3	5	6	4	8	1	2	11	10	13
13	12	6	9	7	4	8	3	5	14	1	10	2	11
14	1	7	3	4	8	9	5	6	2	10	13	11	12

### Generating the Pascal Triangle

We use two different functions to give us information about a given node in the Pascal Triangle. The function *Pascal* returns the number of the element that was somewhat arbitrarily assigned in  $G$ . The function *Pascal2* returns the  $(x, y)$  coordinate of the center of the node. When we graph the Pascal Triangle later, we choose the top of the triangle to be located at  $(0, 0)$ , but since no axes are displayed, this could easily be changed. Thus, the  $(x, y)$  coordinate returned by *Pascal2* is based on the top of the triangle being located at  $(0, 0)$ .

The function *Colors* assigns a color to each element in the group. Maple has a color palette named *HUE* whose values range from 0 to 1. For example *HUE(1)* would be red and *HUE(0.1)* would be a shade of yellow. We assign black to the identity element. We then divide the color spectrum into equal partitions, where the number of partitions equals the order of the group, and assign each element the color that falls on the edge of one of these partitions. Since we placed the two generators adjacent to each other, the color of each generator would be very similar to the other. For example, the first generator might be colored a very light yellow, while the second generator would be colored just a slightly darker yellow. Since this might not be aesthetically pleasing and each color would be very prominent along the edges of the triangle, we swapped the color of the second generator and the last element of the group. Thus, the right side of all our triangles will always be colored red.

We choose to draw the nodes of the Pascal Triangle as hexagons. Maple has a function *polygonplot* that is able to graph any convex quadrilateral given its vertices. The function *P* actually creates a hexagon for a given position in the triangle.

Routines to generate a numerical representation of the *Generalized Pascal Triangle*

```
> Pascal := proc(m, n)
option remember;
  if n = 1 then
    RETURN(2);
  elif n = m then
    RETURN(3);
  else
    RETURN(GroupTable[Pascal(m-1,n-1),Pascal(m-1,n)]);
  fi;
end:
> sqrt3 := evalf(sqrt(3));
> Pascal2 := proc(m,n)
option remember;
global sqrt3;
if m = 1 then
  RETURN([0, 0])
else
if n = 1 then
  RETURN([Pascal2(m-1,n)[1]-1, Pascal2(m-1,n)[2]-sqrt3]);
else
RETURN([Pascal2(m,n-1)[1]+2, Pascal2(m,n-1)[2]]);
```

```
fi;
fi;
end:
```

Procedure that assigns a color to each group element

```
> Colors := proc(x)
global gorder;
if x = 1 then
    RETURN(black);
elif x = 3 then
    RETURN(COLOR(HUE,1));
elif x = gorder then
    RETURN(COLOR(HUE,evalf(3/gorder)));
else
    RETURN(COLOR(HUE,evalf(x/gorder)));
fi;
end:
```

Procedure that generates the hexagon in position (m, n)

```
> a := evalf(2/sqrt(3));
adiv2 := evalf(a/2);
P := proc(m,n)
    global a, adiv2;
    local h, k;
    h := Pascal2(m,n)[1];
    k := Pascal2(m,n)[2];
    RETURN(polygonplot([[h, k+a], [h+1, k+adiv2], [h+1, k-adiv2], [h,
k-a], [h-1, k-adiv2], [h-1, k+adiv2]], color=Colors(Pascal(m,n))));
end:
```

### Printing the Color Group Table

At this point, we take a small tangent and redisplay our group table. But this time we display it in color. We have a function *Square* that is similar to *P* but which creates a colored square at a given position. Maple has another convenient data structure called a sequence. This function creates finite sequences with given parameters. We use a sequence to create an array *Table* which stores all of the colored squares in the group table. To print the group table we simply display *Table*.

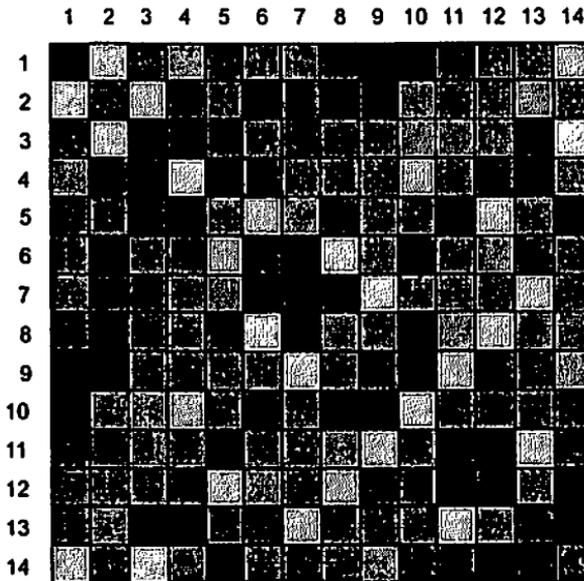
Routines to generate a colored version of the multiplication table  
*GroupTable*

```
> Square := proc(x,y)
    global gorder;
```

```

local h, k;
h := x;
k := gorder - y;
RETURN(polygonplot([[h+0.5, k+0.5],[h+0.5, k-0.5], [h-0.5, k-0.5],
[h-0.5, k+0.5]], color=Colors(GroupTable[x,y]]));
end:
> Table := [seq(seq(Square(m,n), n = 1 .. gorder), m = 1 .. gorder),
seq(textplot([i, gorder convert(i, string)], align = ABOVE), i = 1 ..
gorder), seq(textplot([0, gorder-i, convert(i,string)], align = LEFT),i = 1 ..
gorder)]:
> display(Table, axes=none, scaling=CONSTRAINED);
Table := 'Table':

```



### Printing the Generalized Pascal Triangle

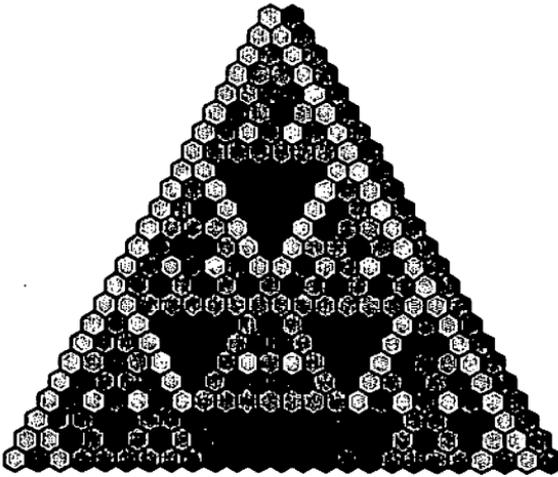
We also use a sequence to create the array  $S$ , which stores all of the hexagons for the entire triangle. After this is accomplished, we display  $S$  and the Pascal Triangle is printed.

Generalized Pascal Triangle

```

> S := [seq(seq(P(m,n), n=1..m), m=2..Rows)];
forget(Pascal, Pascal2);
display(S, axes=none, scaling=CONSTRAINED);

```



## Conclusions

We think the above project has allowed us to achieve many things and has uncovered some interesting questions. We learned how to use Maple V for nontrivial tasks and how to apply our knowledge of groups (generators, relations, and permutation groups). We have developed a powerful tool that will allow us to study in a semi-experimental manner the properties of these generalized Pascal Triangles. A study could be done concerning the effect on the patterns in the triangles when one of the order of one generator is significantly higher than the order of the other generator. Also, in our limited number of examples, it appears that every element of the group appears in the triangle, even in the first 75 rows. We might try to prove this observation. Further study is definitely warranted.

*Acknowledgments.* I would like to thank Dr. Chandra, Pittsburg State University CSIS dept., Dr. Woodburn, and Dr. Figueroa, both Pittsburg State University Math dept., for all their help in completion of this project.

## References

1. Maple V. Vers. 5. Computer Software. Waterloo Maple Inc, 1999.
2. Pinter, Charles C. A Book of Abstract Algebra. New York: McGraw-Hill, Inc., 1990.
3. Sahni, Sartaj. Data Structures, Algorithms, and Applications in C++. Boston: WCB/McGraw-Hill, 1998.
4. Shannon, Kathleen M. PascGalois Triangles & Hexagons and other Group-related Cellular Automata. <http://faculty.ssu.edu/~kmshanno/pascal/>. Verified on 03/28/2001.

### *Henkel's Names of the Periods in Numbers*

Millions (1), Billions (2), Trillions (3), Quadrillions (4), Quintillions (5), Sextillions (6), Septillions (7), Octillions (8), Nonillions (9), Decillions (10), Undecillions (11), Duodecillions (12), Tertiodecillions (13), Quarto-decillions (14), Vigillions (20), Primo-vigillions (21), Secundo-vigillions (22), Tertio-vigillions (23), Quarto-vigillions (24), Trigillions (30), Quadragillions (40), Quinquagillions (50), Sexagillions (60), Septuagillions (70), Octogillions (80), Nonagillions (90), Centillions (100), Decimo-centillions (110), Vigesimo-centillions (120), Trigesimo-centillions (130), Quadragesimo-centillions (140), Quinquagesimo-centillions (150), Sexagesimo-centillions (160), Septuagesimo-centillions (170), Octogesimo-centillions (180), Nonagesimo-centillions (190), Ducentillions (200), Trecentillions (300), Quadrigentillions (400), Quingentillions (500), Sexcentillions (600), Septingentillions (700), Octingentillions (800), Nongentillions (900), Millillioins (1000), Decimillillions (10,000), Centi-millillions (100,000), Milli-millillions (1,000,000). - Edward Brooks, *Philosophy of Arithmetic*, 1880.

-*The Pentagon*, Spring 1952 (pp.99-100)

### *Starting a KME Chapter*

For complete information on starting a KME chapter, contact the National President. Some information is given below.

An organized group of at least ten members may petition through a faculty member for a chapter. These members may be either faculty or students; students must meet certain coursework and g.p.a. requirements.

The financial obligation of new chapters to the national organization includes the cost of the chapter's charter and crest (approximately \$50) and the expenses of the installing officer. The individual membership fee to the national organization is \$20 per member and is paid just once, at that individual's initiation. Much of the \$20 is returned to the new members in the form of membership certificates and cards, keypin jewelry, a two-year subscription to the society's journal, etc. Local chapters are allowed to collect semester or yearly dues as well.

The petition itself, which is the formal application for the establishment of a chapter, requests information about the petitioning group, the academic qualifications of the eligible petitioning students, the mathematics faculty, mathematics course offering and other facts about the institution. It also requests evidence of faculty and administrative approval and support of the petition. Petitions are subject to approval by the National Council and ratification by the current chapters.

## The Problem Corner

Edited by Kenneth M. Wilke

*The Problem Corner* invites questions of interest to undergraduate students. As a rule the solution should not demand any tools beyond calculus. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following problems should be submitted on separate sheets before July 1, 2002. Solutions received after the publication deadline will be considered also until the time when copy is prepared for publication. The solutions will be published in the Fall 2002 issue of *The Pentagon*, with credit being given to the student solutions. Affirmation of student status and school should be included with solutions. Address all communications to Kenneth M. Wilke, Department of Mathematics, 275 Morgan Hall, Washburn University, Topeka, Kansas 66621 (e-mail: xxwilke@acc.wuacc.edu).

### PROBLEMS 550-554

*Problem 550.* Proposed by Jose Luis Diaz, Universitat Politecnica de Catalunya, Terrassa, Spain.

If  $a$ ,  $b$ , and  $c$  are distinct nonzero complex numbers, determine

$$\frac{bc(a+1)}{(a-b)(a-c)} + \frac{ac(b+1)}{(b-a)(b-c)} + \frac{ab(c+1)}{(c-a)(c-b)}$$

*Problem 551.* Proposed by Carol Browning, Greg Eastman, and John House (jointly).

The divisibility test for 3 is simple: a natural number is divisible by 3 if and only if the sum of its digits is divisible by 3. The same test works for 9. Notice that the test seems to work for 3 in base 4 but fails for 3 in base 5. Give conditions on divisor  $d$  and base  $b$  (with  $d < b$ ) for which  $d$  divides  $n$  if and only if  $d$  divides the sum of the digits of  $n$  expressed in base  $b$ .

**Problem 552.** Proposed by Albert White, St. Bonaventure University, St. Bonaventure, New York.

Determine if each of the following two series  $\sum_{j=1}^{\infty} \frac{1}{x_j}$  are convergent or divergent. If a series is convergent, find the sum of the series.

(a) Define  $x_1 = 1, x_n = \sum_{j=1}^{n-1} jx_j, j \geq 2$ . Find  $\sum_{j=1}^{\infty} \frac{1}{x_j}$ .

(b) Define  $x_1 = 1, x_n = \sum_{j=1}^{n-1} \frac{1}{j}x_j, j \geq 2$ . Find  $\sum_{j=1}^{\infty} \frac{1}{x_j}$ .

**Problem 553.** Proposed by Albert White, St. Bonaventure University, St. Bonaventure, New York.

Al spotted a squirrel in the family room. He had Samantha stand in the doorway so that the squirrel would not go into the kitchen. Then Al opened the back door so that the squirrel would run outside. Using the handle of a broom, he poked under the entertainment center until the squirrel ran out.

The squirrel ran a series of straight lines from (0,0) to (15,-9) to (19,-5) to (23,-5) where the back door is located. When the squirrel was located at (15,-9), Samantha abandoned her post and ran in the opposite direction along the path  $y = 10 - (x - 16)^2$ . Samantha started at (16, -10) and stopped at (21,-35). Assuming that the squirrel ran at a constant speed of 110 feet per minute while Samantha ran at a constant speed of 150 feet per minute, does the squirrel reach the back door before Samantha reaches her stopping point?

**Problem 554.** Proposed by the editor.

Let  $f$  be defined by the relation  $f = 2 + 2\sqrt{44g^2 + 1}$  where  $g$  is a positive integer. Show that if  $f$  is an integer  $f$  is a perfect square.

Please help your editor by submitting problem proposals.

### SOLUTIONS 537, 538, 541 – 544

**Problem 537.** Proposed by the editor.

Let  $r, s, t, u$  and  $v$  be integers such that both their sum and the sum of their squares are divisible by an odd prime  $p$ . Prove that  $p$  also divides the quantity  $r^5 + s^5 + t^5 + u^5 + v^5 - 5rstuv$ .

**Solution** by Ovidiu Furdui, Western Michigan University, Kalamazoo, Michigan.

Consider the following notations: let  $p_i = r^i + s^i + t^i + u^i + v^i$  for  $i \geq 1$  and  $S_i$  denote the symmetric function of degree  $i$  in terms of  $r, s, t, u$ , and  $v$ . In other words,  $S_1 = r + s + t + u + v$ ,  $S_2 = rs + rt + ru + rv +$

$st + su + sv + tu + tv + uv$ ,  $S_3 = rst + rsu + rsv + rtu + rtv + ruv + stu + stv + suv + tuv$ ,  $S_4 = rstu + rstv + rsuv + rtuv + stuv$  and  $S_5 = rstuv$ .

The following are known as Newton's Formulas:

$$p_1 - S_1 = 0 \quad (1)$$

$$p_2 - S_1p_1 + 2S_2 = 0 \quad (2)$$

$$p_3 - S_1p_2 + S_2p_1 - 3S_3 = 0 \quad (3)$$

$$p_4 - S_1p_3 + S_2p_2 - S_3p_1 + 4S_4 = 0 \quad (4)$$

$$p_5 - S_1p_4 + S_2p_3 - S_3p_2 + S_4p_1 - 5S_5 = 0 \quad (5)$$

From (1) we get  $S_1 = p_1$  so that  $p_1$  divides  $S_1$ . From (2) we get  $2S_2 = S_1p_1 - p_2$ . Then since  $p$  divides both  $p_1$  and  $p_2$  and since  $p$  is an odd prime,  $p$  divides  $S_2$  since  $(p, 2) = 1$ . Finally from (5) we have  $p_5 - 5S_5 = S_1p_4 - S_2p_3 + S_3p_2 - S_4p_1$ . Now since  $p$  divides each of  $S_1, S_2, p_1$  and  $p_2$ ,  $p$  divides the quantity  $(p_5 - 5S_5)$ .

*Problem 538.* Proposed by the editor.

An eccentric gardener with a mathematical penchant has a group of gardens which have the following common properties: each garden has a triangular shape such that the area of the garden is twice the perimeter; each side is an integral number of feet; and in each garden two sides are consecutive integers. How many gardens does the eccentric gardener have and what are the dimensions of each garden?

*Solution* by the editor.

Let  $x$ ,  $x + 1$  and  $2c - 1$  be the sides of a garden where  $x$  and  $c$  are both positive integers. (This is required since both the area and perimeter of each garden is an integer.) Then the perimeter of the garden is  $2(x + c)$ . By Heron's Formula the area of the garden is given by the equation

$$A^2 = [4(x + c)]^2 = (x + c)(c)(c - 1)(x - c + 1) \text{ or} \\ 16(x + c) = (c^2 - c)(x - c + 1) \quad (1)$$

Equation (1) can be rewritten as

$$\frac{16}{c^2 - c} = \frac{x - c + 1}{x + c} = 1 - \frac{2c - 1}{x + c}$$

so that

$$\frac{x+c}{2c-1} = \frac{c^2-c}{c^2-c-16} = 1 + \frac{64}{(2c-1)^2-65}$$

and finally

$$x = c - 1 + \frac{64(2c-1)}{(2c-1)^2-65} \quad (2)$$

Since  $x$  is an integer, the quantity  $\frac{64(2c-1)}{(2c-1)^2-65}$  is also an integer.

Since  $(2c-1) - \frac{65}{2c-1} > 64$  for  $c > 33$  and since  $(2c-1)^2 - 65 < 0$  for  $c < 5$ , we must have  $5 \leq c \leq 33$ . Integral solutions occur for  $c = 5, 7, 8$  and  $33$ . The corresponding triangular gardens, their perimeters and areas are given below:

$c$	$x$	$x+1$	$2c-1$	perimeter	area
5	40	41	9	90	84
7	14	15	13	42	84
8	13	14	15	42	84
33	33	34	65	132	264

Thus there are three distinct gardens having the desired properties.

Also solved by Ovidiu Furdui, Western Michigan University, Kalamazoo, Michigan.

**Problem 540.** Proposed by the Robert Rogers, SUNY College at Fredonia, Fredonia, New York.

Given a quintic polynomial  $f(x)$  with exactly one inflection point at  $x = 0$ , one maximum at  $x = -1$ , and one minimum at  $x = m$ , what is the maximum value the polynomial can attain? [Note: For a cubic polynomial,  $m = 1$ .]

Since no solutions have been received, this problem will remain open for another issue.

**Problem 541.** Proposed by the Alma College Problem Solving Group, Alma College, Alma, Michigan.

Find a closed form for  $\sum_{k=0}^n k^5 \binom{n}{k}$

**Solution** by Carl Libis, Richard Stockton College of New Jersey, Pomona, New Jersey.

For positive integers  $n$ , the binomial theorem yields  $(1+x)^n = \sum_{k=0}^n \binom{n}{k} x^k$ . Differentiating both sides with respect to  $x$  and then multiplying by  $x$ , we

get  $nx(1+x)^{n-1} = \sum_{k=0}^n k \binom{n}{k} x^k$ . Repeating this process four more times yields,

$$nx(1+nx)(1+x)^{n-2} = \sum_{k=0}^n k^2 \binom{n}{k} x^k$$

$$nx[1+(3n-1)x+n^2x^2](1+x)^{n-3} = \sum_{k=0}^n k^3 \binom{n}{k} x^k$$

$$nx[1+(7n-4)x+(6n^2-4n+1)x^2+n^3x^3](1+x)^{n-4} \\ = \sum_{k=0}^n k^4 \binom{n}{k} x^k$$

$$nx \left[ 1 + (15n-11)x + (25n^2-30n+11)x^2 \right. \\ \left. + (10n^3-10n^2+5n-1)x^3 + n^4x^4 \right] (1+x)^{n-5} \\ = \sum_{k=0}^n k^5 \binom{n}{k} x^k$$

Now set  $x = 1$  to obtain

$$\sum_{k=0}^n k^5 \binom{n}{k} = n^2 (n^3 + 10n^2 + 15n - 10) 2^{n-5}.$$

Also solved by Scott H. Brown, Auburn University, Montgomery, Alabama; Pat Costello, Eastern Kentucky University, Richmond, Kentucky; Jose Luis Diaz, Universitat Politecnica de Catalunya, Terrassa, Spain; Russ Euler and Jawad Sadek (jointly), Northwest Missouri State University, Maryville Missouri; Carl Libis, Richard Stockton College of New Jersey, Pomona, New Jersey (second solution); Albert White, St. Bonaventure University, St. Bonaventure New York and the proposer.

**Problem 542.** Proposed by Albert White, St. Bonaventure University, St. Bonaventure, New York.

Instead of using the correct arc length formula, a student used the formula  $\int_b^a \sqrt{1+f''(x)} dx$  on a test and obtained the correct answer. Find all functions  $f(x)$  such that this formula will produce the correct answer for arc length.

**Solution** by Russ Euler and Jawad Sadek (jointly), Northwest Missouri State University, Maryville, Missouri.

First, assuming that  $f''(x)$  is smooth everywhere on  $[b, a]$ , the given formula will produce the correct answer for arc length when

$$f''(x) = [f'(x)]^2 \tag{1}$$

One family of solutions to (1) is  $f(x) = a$  for any constant. So assume that  $f$  is a nonconstant function. Then (1) can be written as

$$[f'(x)]^{-2} f''(x) = 1$$

Then  $-[f'(x)]^{-1} = x + c$  where  $c$  is the constant of integration. Hence,  $f'(x) = \frac{-1}{x+c}$ , and so another family of solutions is given by  $f(x) = -\ln|x+c| + c_1$  for any constant  $c_1$ . It should be noted that if  $f'(x)$  is not smooth, then a linear combination of the above two families of solutions will also be a solution.

Also solved by Ovidiu Furdui, Western Michigan University, Kalamazoo, Michigan and the proposer.

**Problem 543.** Proposed by Carol Browning, Drury College, Springfield, Missouri.

Let  $a_0$  be a given positive integer and define the sequence  $a_k$  by  $a_{k+1} = \frac{a_k}{2}$  if  $2|a_k$  and  $a_{k+1} = 3a_k + 1$  otherwise. For example, if  $a_0$  is 3, the sequence is 3, 10, 5, 16, 8, 4, 2, 1, 4, 2, 1, 4, 2, 1, ... Define the function  $P(n)$  on the positive integers by  $P(n) = j$  if in the sequence originated by  $a_0 = n$ , the first power of 2 to appear is  $2^j$ .

For example,  $P(3) = 4$ . If no power of 2 appears in the sequence, we define  $P(n)$  to be 0. Prove that  $P(n)$  is odd exactly when  $n$  is an odd power of 2.

**Solution** by Russ Euler and Jawad Sadek (jointly), Northwest Missouri State University, Maryville, Missouri.

Suppose that  $n = 2^j$  where  $j$  is odd. [Throughout  $n$ ,  $t$  and  $j$  are positive integers. Ed.] Then  $P(n) = j$  which is odd.

Now to establish the converse, we shall establish the more general result:

**Claim:** If  $3n + 1 = 2^t$ , then  $t$  is even.

To prove this claim, note that  $3n = 2^t - 1 = (2 - 1)(1 + 2 + 2^2 + \dots + 2^{t-1}) = (3 + 2^2 + \dots + 2^{t-1})$ . It follows that  $(2^2 + \dots + 2^{t-1})$  must also be a multiple of 3.

Now  $(2^2 + \dots + 2^{t-1}) = 2^2(1 + 2 + 2^2 + \dots + 2^{t-3}) = 2^2(3 + 2^2 + \dots + 2^{t-3})$ . Again it follows that  $(2^2 + \dots + 2^{t-3})$  must be a multiple of 3. Repeating this process a finite number of times and assuming that  $t$  is odd, at some point, we reach the conclusion that  $2^2 + 2^3 + 2^4 = 28$  is a multiple of 3 which is a contradiction. Hence the claim is true. Since the claim is true, unless  $n = 2^j$ , where  $j$  is odd, to start with, the sequence cannot contain a number of the form  $3a_k + 1 = 2^j$  where  $j$  is odd. This completes the proof of the desired result.

Also solved by the proposer.

**Problem 544.** Proposed by Robert Stump, Richmond, Virginia.

Given triangle  $ABC$  with the lengths of  $AB = c$ ,  $AC = b$ , and  $BC = a$  respectively,

(a) Let  $CM_1 = m_1$  be the median to  $AB$  in triangle  $ABC$ . Let  $M_1M_2 = m_2$  be the median to  $AC$  in  $ACM_1$ . Continuing this process, let  $M_{n-1}M_n = m_n$  be the median to  $AM_{n-2}$  in triangle  $AM_{n-2}M_{n-1}$ . In terms of  $a$ ,  $b$ , and  $c$  find  $\sum_{k=1}^{\infty} m_k$ .

(b) Let  $CH_1 = h_1$  be the altitude to  $AB$  (or  $AB$  extended) in triangle  $ABC$ . Let  $H_1M_2 = h_2$  be the altitude to  $AC$  in triangle  $ACH_1$ . Continuing this process, let  $H_{n-1}H_n = h_n$  be the altitude to  $AH_{n-2}$  in triangle  $DAH_{n-2}H_{n-1}$ . In terms of  $a$ ,  $b$ , and  $c$  find  $\sum_{k=1}^{\infty} h_k$ .

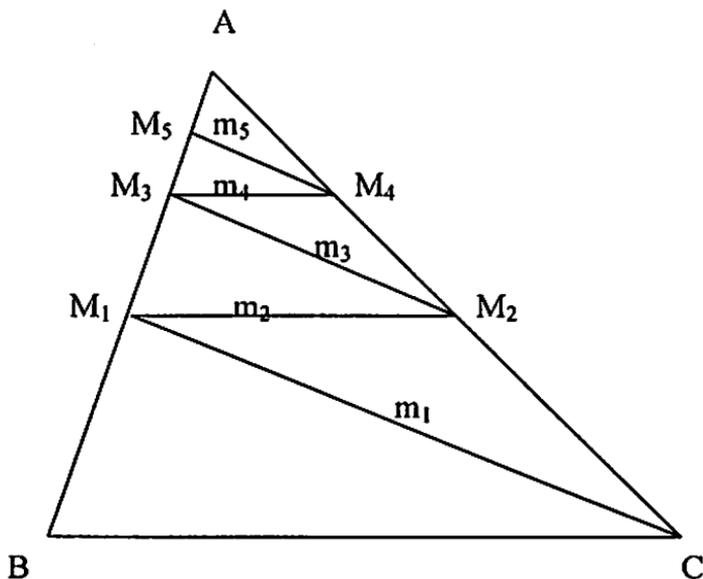


Figure 1

*Solution* by Ovidiu Furdui, Western Michigan University, Kalamazoo, Michigan.

Part (a): Notice that  $M_1M_2 = \frac{a}{2}$ ;  $M_3M_4 = \frac{M_1M_2}{2} = \frac{a}{2^2}$ ;  $M_5M_6 = \frac{a}{2^3}$  and in general  $M_{2k-1}M_{2k} = m_{2k} = \frac{a}{2^k}$ . On the other hand,  $m_3 = M_2M_3 = \frac{M_1C}{2} = \frac{m_1}{2}$ ;  $m_5 = M_4M_5 = \frac{M_2M_3}{2} = \frac{m_1}{2^2}$  and in general  $m_{2k+1} = M_{2k}M_{2k+1} = \frac{m_1}{2^k}$ . Thus  $\sum_{k=1}^{\infty} m_k = \sum_{k=1}^{\infty} m_{2k} + \sum_{k=1}^{\infty} m_{2k-1} =$

$$\sum_{k=1}^{\infty} \frac{a}{2^k} + \left( m_1 + \sum_{k=2}^{\infty} m_{2k-1} \right) = a + 2m_1.$$

But  $2m_1 = \sqrt{2(a^2 + b^2) - c^2}$  so the desired sum is  $2m_1 + a = a + \sqrt{2(a^2 + b^2) - c^2}$ .

Part (b). (Partial Solution) For this case we consider only the case

where  $\angle B < 90^\circ$

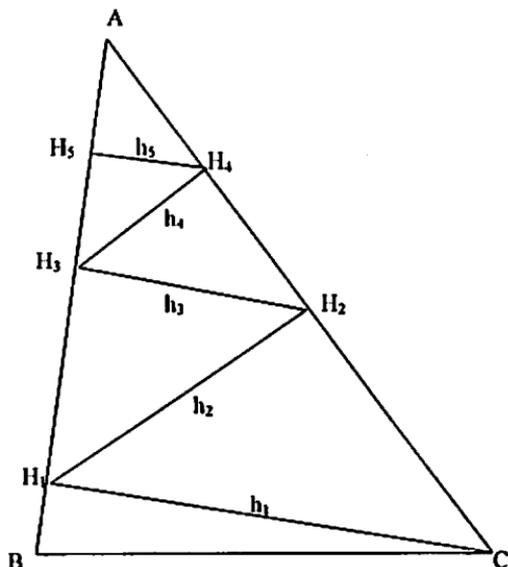


Figure 2

From the diagram, it is easy to show that  $AH_1 = b \cos A$ ;  $AH_2 = AH_1 \cos A = b \cos^2 A$ ; and  $AH_3 = AH_2 \cos A = b \cos^3 A$ . Then  $\frac{H_2 H_3}{CH_1} = \frac{h_3}{h_1} = \frac{AH_2}{AC} = \cos^2 A$  so that  $h_3 = h_1 \cos^2 A$ .  $\frac{H_4 H_5}{H_2 H_3} = \frac{h_5}{h_3} = \frac{AH_3}{AH_1} = \frac{AH_2}{AC} = \cos^2 A$ . Thus  $h_5 = h_3 \cos^2 A = h_1 \cos^4 A$ . By an easy induction, we have  $h_{2n+1} = h_1 \cos^{2n} A$  for  $n > 1$ .

On the other hand,  $\frac{h_4}{h_2} = \frac{H_3 H_4}{H_1 H_2} = \frac{AH_3}{AH_1} = \frac{AH_2}{AC} = \cos^2 A$ . Thus  $h_4 = h_2 \cos^2 A$ . Similarly  $\frac{h_6}{h_4} = \frac{H_5 H_6}{H_3 H_4} = \frac{AH_5}{AH_3} = \frac{AH_4}{AH_2} = \frac{AH_3}{AH_1} = \frac{AH_2}{AC} = \cos^2 A$ . Thus  $h_6 = h_4 \cos^2 A = h_2 \cos^4 A$ . By an easy induction  $h_{2n} = h_2 \cos^{2n-2} A$  for  $n > 1$ . Then  $h_2 = H_1 H_2 = AH_1 \sin A = b \sin A \cos A$ .

$$\begin{aligned} \sum_{k=1}^{\infty} h_k &= \sum_{k=1}^{\infty} h_{2k} + \sum_{k=1}^{\infty} h_{2k-1} = \sum_{k=1}^{\infty} (h_2 \cos^{2k-2} A) + \sum_{k=1}^{\infty} (h_1 \cos^{2k} A) \\ &= h_2 \sum_{k=1}^{\infty} (\cos^2 A)^{k-1} + h_1 \sum_{k=1}^{\infty} (\cos^2 A)^{k-1} \\ &= (h_1 + h_2) \frac{1}{1 - \cos^2 A} = \frac{h_1 + h_2}{\sin^2 A} = \frac{b \cos A \sin A + b \sin A}{\sin^2 A} = \frac{b(1 + \cos A)}{\sin A} \\ &= \frac{b \cos(\frac{A}{2})}{\sin(\frac{A}{2})} = b \cot(\frac{A}{2}). \end{aligned}$$

Also solved by the proposer (Part (a)).

*Editor's Comment:* The result found by our featured solver also holds when  $\angle B = 90^\circ$ . To show this, consider the following figure. In a right triangle, the length of the altitude to the hypotenuse is given by the product of the legs of the right triangle divided by the hypotenuse. Thus from

figure 3, it is clear that the problem leads to an infinite set of similar right triangles having altitudes related by the relation  $h_k = h_{k-1} \frac{c}{b}$  with  $h_0 = a$  or  $h_k = h_0 \left(\frac{c}{b}\right)^k = a \left(\frac{c}{b}\right)^k$  where  $k$  is an integer  $> 0$ . Thus the desired sum of the altitudes is an infinite geometric series with first term  $a$  and ratio  $\frac{c}{b}$ . The sum of this series is  $\frac{a}{1-\frac{c}{b}} = \frac{ba}{b-c}$  which equals  $b \cot\left(\frac{A}{2}\right)$  since  $\cot\left(\frac{A}{2}\right) = \frac{\sin A}{1-\cos A} = \frac{\frac{a}{b}}{1-\frac{c}{b}} = \frac{a}{b-c}$  which coincides with the result found by our featured solver. Perhaps some reader can complete the solution of this problem by resolving the case where  $\angle B > 90^\circ$

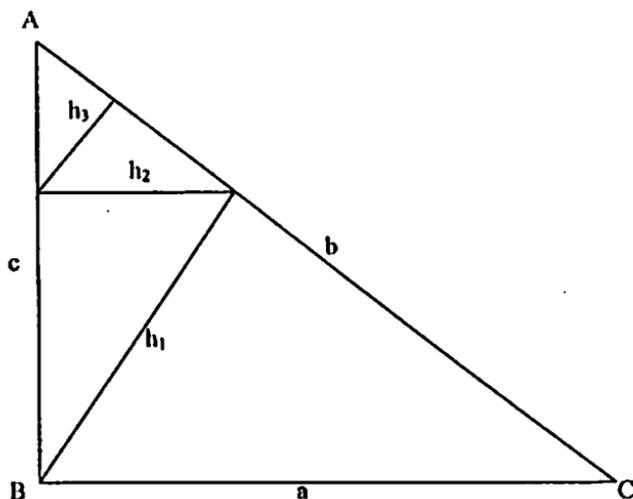


Figure 3

### Magic Magic Squares

8	3	6	4
9	4	7	5
7	2	5	3
11	6	9	7

2	11	3
0	3	1
1	10	2

These squares are not "magic" in the usual sense but have the property that any set of numbers, including one from each row and one from each column, will have the same sum. The sum in the  $4 \times 4$  is 24. If the numbers in the  $3 \times 3$  are in the base 4 system, the sum is 13. You too can make one of these. See *The Pentagon*, Fall 1965, pp. 44-45 to see how.

## *Kappa Mu Epsilon News*

Edited by Connie Schrock, Historian

News of chapter activities and other noteworthy KME events should be sent to Connie Schrock, Historian, Kappa Mu Epsilon, Mathematics Department, Emporia State University, 1200 Commercial, Campus Box 4027, Emporia, KS 66801, or to [schrockc@emporia.edu](mailto:schrockc@emporia.edu).

### **Installation of New Chapters**

#### **Louisiana Delta**

University of Louisiana, Monroe

The installation of the Louisiana Delta chapter of Kappa Mu Epsilon was held on Sunday, February 11, 2001. The ceremony was held on the sixth floor of the beautiful ULM Library Conference Center. Don Tosh, National Historian of KME, was the installing officer. There were twenty-six chapter members installed during the ceremony. Officers were also selected. Following the ceremony a banquet was held in honor of the initiates.

#### **Georgia Delta**

Berry College, Mount Berry

The installation of the Georgia Delta chapter of Kappa Mu Epsilon was held on April 21, 2001, in the brand new Science Building (with an amazing 60-foot Foucault pendulum) on the campus of Berry College in Mount Berry, Georgia. Dr. Pat Costello, President of Kappa Mu Epsilon, conducted the installation ceremony. Dr. Ron Taylor, assistant professor of mathematics, arranged the ceremony. Twelve students constituted the founding group of the new chapter at Berry College. Those initiated were:

*Students:* Cheryl Ann Beierschmitt, Ryan David Fox, Grace Elizabeth Hughes, Alfred Chrisman Hart, Jason Robert Buczyna, Danielle Marie Shea, Donald Davin Millholland, Jennifer Leah Watson, Sarah Anderson Tweedie, Elka Lynn Torblaa, Trisha Michele Heller, Jason Daniel Moon.

Also in attendance were administrators from Berry College including Dr. Kathy McKee, Associate Provost and Dean of Academic Services, Dr. Bruce Conn, Dean of the School of Mathematical and Natural Sciences, and the parents and friends of many of the inductees.

Dr. Costello began the evening ceremony with an introduction to the organization and a brief history of KME. Besides presenting the new chapter with a charter and crest, each student received their membership certificate at the installation. Officers installed during the ceremony were Cheryl

Beierschmitt, president; Jason Buczyna, vice-president; Ryan Fox, secretary; and Sarah Tweedie, treasurer.

Faculty member Ron Taylor accepted the responsibilities of corresponding secretary and faculty sponsor. A reception with cookies and punch was held in the lobby after the ceremony. Then several faculty and Dr. Costello went to dinner at "The Country Gentleman".

## Texas Mu

Schreiner University, Kerrville

The installation of the Texas Mu chapter of Kappa Mu Epsilon was held on Saturday, April 28, 2001. The ceremony was held in the Cailloux Campus Activity Center Ballroom. Don Tosh, President-Elect of KME, was the installing officer. Kelly Hildebrand served as the conductor. There were fifteen chapter members installed during the ceremony. Officers were also selected. Following the ceremony light refreshments were served. That evening several newly installed members had dinner with Don Tosh.

## Chapter News

### AL Beta

University of North Alabama, Florence

26 actives

### AL Gamma

University of Montevallo, Montevallo

Chapter President—Chris Harmon

15 actives, 2 associates

Other spring 2001 officers: Tommy Fitts, Vice President; Jared Phillips, Secretary; Don Alexander, Corresponding Secretary.

### CA Gamma

Polytechnic State University, Obispo

Chapter President—Shar McCarrol

13 actives, 6 associates

Other spring 2001 officers: Brian Miceli, Vice President; Sherryl Cruz, Secretary; Donna Kandel, Treasurer; Jonathon Shapiro, Corresponding Secretary.

### CO Delta

Mesa State College, Grand Junction

Chapter President—Raymond Mitchell

12 actives, 14 associates

Thirty-six members, guests, and initiates attended the Twelfth Annual Initiation Ceremony held in the Krey-Zeigel Room of the College Center on April 26, 2001. Fourteen new members were initiated into the CO Delta Chapter. New officers were elected for the 2001-2002 year at a business meeting following the initiation. All present were invited to a joint reception for KME initiates and 2001 departmental graduates in the Science Court Yard. The Chapter was represented at the Thirty-third Biennial Convention by faculty members Kimberly Schneider, who served on the Awards Committee, and Donna Hafner, who chaired the Audit Committee.

- Co Gamma** Chapter President—Bethany Lyles  
 Fort Lewis College, Durango 20 actives, 14 associates  
 Other spring 2001 officers: Jennifer DuBois, Vice President; Chad Lundin, Secretary; Chad Lundin, Treasurer; Deborah Berrier, Corresponding Secretary.
- GA Alpha** Chapter President—Christin Phillips  
 State University of West Georgia 15 actives, 7 associates  
 On April 19, 2001, Georgia Alpha held its 27th Annual Initiation Meeting at which seven new members were initiated and Chapter Officers were elected for 2001-2002. Following the initiation, a reception was held in honor of the new initiates. At the reception, the winners of the UWG mathematics scholarships for 2001-2002 were announced. These winners include: Karen Jones (Sigma Xi Award and Marion Crider Award), Kaitlin Lewis (Burson Calculus Award), Bryan Crawford (Marion Crider Award), Natalie Young (Marion Crider Award), Christian Phillips (Cooley Scholarship), Beth Gibbs (Georgia Math Scholarship), and Jessica Pritchett (Whatley Scholarship).
- GA Gamma** Chapter President—Tony McCullers  
 Piedmont College 3 actives  
 Other spring 2001 officers: Heather Knight, Vice President; Amie Mills, Secretary; Tony McCullers, Treasurer; Shahryar Heydari, Corresponding Secretary.
- IL Delta** Chapter President  
 University of St. Francis 12 associates
- IL Eta** Chapter President—Jessica Holdsworth  
 Western Illinois University 12 actives, 9 associates  
 Other spring 2001 officers: Eric Snodgrass, Vice President; Sandra Jazarb, Secretary; Hong Tran Treasurer; Alan Bishop, Corresponding Secretary
- IL Theta** Chapter President—Steven Lafser  
 Benedictine University 20 actives, 11 associates  
 The Math Club/KME met several times this semester. Featured events included a trip to Navy Pier to see the water garden, a T shirt contest (top 10 reasons to study math at BU) and watching the NOVA video on Fermat's Last Theorem. The KME induction was held at a tea honoring academic achievements in mathematics.
- IL Zeta** Chapter President—Janine Smuda  
 Dominican University 35 actives, 7 associates  
 On April 8, 2001 we inducted seven new members and two associate

members. The ceremony included a lecture by Professor Jill Van Newenhizen (Lake Forest College) entitled "The Possibilities and Impossibilities of Voting."

**IN Delta** Chapter President—Katherine Kratochvil  
University of Evansville 40 actives, 13 associates

Other spring 2001 officers: Kara Leonard, Vice President; Cathleen Morales, Secretary; Mohammad Azarian, Treasurer; Mohammad Azarian, Corresponding Secretary

**IN Gamma** Chapter President—Courtney Taylor  
Anderson University 7 actives, 8 associates

Other spring 2001 officers: Matthew Cherry, Vice President; Sarah Nowlin, Secretary; Sarah Nowlin, Treasurer; Stanley Stephens, Corresponding Secretary

**IA Alpha** Chapter President—Brad Rolling  
University of Northern Iowa 36 actives, 7 associates

Student member Heather Broek presented her paper "Mozart's Waltzes" at our first spring KME meeting in February at Professor Mark Ecker's residence. Our second meeting was held on March 1, 2001 at Professor Cathy Miller's residence where Cassidy Schremser presented her student paper on "The Number Zero." Students Brad Rolling and Erin Carlson attended the KME national convention with faculty advisor Mark Ecker at Washburn University in Topeka, Kansas from April 5-7, 2001. Student member Julia Schreckengast addressed the spring initiation banquet with "History of Pi." Our banquet was held at Grand China Buffet restaurant on April 26, 2001 where seven new members were initiated.

**IA Delta** Chapter President—Matthew Trettin  
Wartburg College 56 actives

The belated Christmas party was held in January at the Other Place II. On February 3rd, the chapter co-sponsored with the Mathematics, Computer Science and Physics Department the Eight Annual Explorations in the Mathematical Sciences event for high school students. The regular meeting in February, centered on, planning the initiation banquet. On March 17th, the chapter elected its new officers prior to the banquet and initiation program for seventeen new members. Laura Arns, a Wartburg Alumnus completing her PhD in computer science at Iowa State University, talked about her use of mathematics in her soon-to-be-completed dissertation about virtual reality as the program for the event. The chapter enjoyed pizza at Cedar Bend Conservation Park as its May meeting.

**IA Gamma**

Morningside College

Chapter President—Mike Husmann

17 actives, 6 associates

Other spring 2001 officers: B.J. Koch, Vice President; Erin Bull, Secretary; Mara Cook, Treasurer; Doug Swan, Corresponding Secretary

**KS Alpha**

Pittsburg State University

Chapter President—Tim Pierce

20 actives, 30 associates

This spring the Kansas Alpha Chapter was very active. We had one of the largest initiations in our history. We initiated 30 new members this spring. Our members also attended and presented at the Kansas Section meeting of the Mathematical Association of America and the KME National Convention. Many of our members were also recognized for outstanding scholarship at the math department's annual awards banquet. Dr. McGrath and his wife Fran hosted the annual KME Ice Cream Social to close out the semester.

**KS Beta**

Emporia State University

Chapter President—Leah McBride

14 actives, 5 associates

The Kansas Beta chapter hosted a service project in Spring 2001. The project consisted of calculator training sessions. Two sessions were held to help students become better acquainted with their graphing calculators. The mathematics faculty publicized the events and the sessions were well attended by general education students as well as new majors. Members attended the Biennial KME convention. Leah McBride represented our chapter by presenting a paper, "Under construction: Foci of Conic Sections."

**KS Delta**

Washburn University

Chapter President—Mark Allen Smith

26 actives

The Kansas Delta chapter met with the Washburn Mathematics Club, Mathematica, for three afternoon lunch meetings. A speaker was present at two of the meetings. On February 12, 2001 the chapter held its annual initiation banquet at which 7 new student members were initiated into KME. During much of the semester, the chapter was preparing for hosting the National 33rd Biennial KME Convention which was held on the Washburn University Campus on April 5-7, 2001. Those most heavily involved with the preparations were Ron Wasserstein, Donna LaLonde, Al Riveland, Sarah Cook and several alumni members, Ann Ukena, Debbie Evans, and Janet Gardiner. There were about 100 KME members representing 20 chapters in attendance for the convention. Ron Wasserstein gave the Keynote address at the Friday evening banquet. KME President Mark Smith gave a welcoming address and led the student section discussion. He also presented a paper at the convention. His presentation was

awarded a Top-4 designation. Ken Wilke was presented with a prestigious Mach Award for his many years of remarkable service as Problem Corner editor of the Pentagon.

### **KS Epsilon**

Chapter President—Wendy Scott

Fort Hays State University

26 actives, 4 associates

Other spring 2001 officers: Lora Clark, Vice President; Adam North, Secretary; Chenglie Hu, Corresponding Secretary

### **KS Gamma**

Chapter President—Angela Shomin

Benedictine College

9 actives, 11 associates

Throughout the spring semester KS Gamma members profited from short talks given by candidates for two faculty positions open in the department for fall 2001. After the initiation of two students on March 14, a short video was shown. At the Honors Banquet on March 20, faculty moderator Sister Jo Ann Fellin, OSB presented Sister Helen Sullivan Scholarship certificates to April Bailey, Brett Herbers, Janelle Kroll, David Livingston, and Angela Shomin. Curtis Sander presented his directed research “Finding Optimal Piano Fingerings” at the national KME convention April 6. Sister Jo Ann Fellin, OSB served as chair for the Nominating Committee and was a recipient of a George R. Mach Distinguished Service Award. Also attending the convention at Washburn were Janelle Kroll, Angela Shomin, April Bailey, and David Livingston. The Exponent, sent annually to KS Gamma alums, featured the retirement of Richard Farrell who completed 39 years of service to Benedictine College. Farrell has been a consistent supporter of chapter activity on campus. At a reception on May 6 in his honor KS Gamma presented Farrell with a t-shirt which displayed one of his sayings: “Sample the problems .... Mmmmmmmmmmm ... like chocolates!”

### **KY Alpha**

Chapter President—Katy Fritz

Eastern Kentucky University

48 actives, 20 associates

The spring semester began with floppy disk sales (together with the ACM chapter) to students in the computer literacy class and the Mathematica class. The first event was a winter retreat at Maywoods. Katy Fritz brought her karaoke machine and everyone got a chance to show off their spring talent including Dr. C. singing “Long Cool Woman in a Black Dress.” At the January meeting, Dr. Ray Tennant spoke on “Historic Proofs of Euler’s Polyhedral Formula.” After the talk, plans were made for initiation and the trip to the national convention. On March 6, there were twenty students initiated as national members. Dr. Ralph Stinebrickner from Berea College gave an interesting talk entitled “Why Would Cluster Analysis Interest a Mathematician?” He gave a good description of how

to organize observed data into meaningful structures. Seven students went with Dr. C. to the national convention at Washburn University in Topeka, Kansas. We heard a lot of interesting talks and Jenny Philips had a hard time on the Awards Committee deciding who the top four speakers should be. After the convention, we drove to St. Louis (ate at Planet Hollywood) and stayed overnight. This allowed us a chance to go up the Arch on Sunday morning. April is Math Awareness Month and every day lists of several interesting facts about that day's number were placed all over the Wallace Building. For example, 27 is the first odd cube greater than 1 and all positive integers are the sum of at most 27 primes. The number facts were also available on the department web site.

### **KY Beta**

Chapter President—Eric Ritchie

Cumberland College

39 actives

On February 23, 2001, the Kentucky Beta chapter held an initiation and a joint banquet with Sigma Pi Sigma, physics honor society at the Atrium. Kappa Mu Epsilon inducted ten new student members and two new faculty members at the banquet, presided over by outgoing president, Eric Ritchie. As an additional feature, senior awards were given by the department at the banquet.

Jointly with the Mathematics and Physics Club, the Kentucky Beta Chapter hosted Dr. Carroll Wells from David Lipscomb University on April 19. He spoke on "Magic or Mathematics", focusing on the mathematics behind the tricks of David Copperfield and other magicians. On April 20, members also assisted in hosting a regional high school math contest, held annually at Cumberland College. As part of the contest activities, several chapter members gave a multi-media presentation "Newton and Leibniz in Conflict." On April 24, the entire department, including the Math and Physics Club, Sigma Pi Sigma, and the Kentucky Beta Chapter, held the annual spring picnic at Briar Creek Park.

### **LA Delta**

Chapter President—Maggie Durbin

University of Louisiana at Monroe

26 actives

Other spring 2001 officers: Telitha Doke, Vice President; Sam Kwok, Secretary; April Jeffcoat, Treasurer; Maribeth Olberding, Corresponding Secretary

### **MD Alpha**

Chapter President—LaVida Cooper

College of Notre Dame of Maryland

9 actives, 6 associates

On May 6, 2001 the annual Induction of Permanent members was held. At this time 4 students became permanent members and 2 students were admitted to temporary membership. Following the induction ceremony Dr. James H. Stith, Director of Physics Programs for the American Institute of

Physics, spoke on "Living in a Technological Age". After his presentation the members were treated to a picnic held in the Gazebo located on the college campus.

**MD Beta**

Western Maryland College

Chapter President—Paul Ostazeski

27 associates

Other spring 2001 officers: Amy Bittering, Vice President; Teresa Needer Secretary; Susan Miller, Treasurer; Linda Eshleman, Corresponding Secretary

**MD Delta**

Frostburg State University

Chapter President—Erin Resh

40 actives

Maryland Delta Chapter held its induction in March – on the second try, the first having been snowed out. Inductees were: Christopher Bender, Jonathan Brunelle, Sinead Browning, Kesha Frisby, Jennifer Gell, Brendon LaBuz, Andrew Limbaugh, Brian McKinley, Sherri Raley, John Simmons and Kandi Wertz. Prof. Kathleen Elder entertained the inductees and guests with a presentation on the Irish educational system and her activities. The chapter continued its walk-in tutoring service, held a successful fund raiser by selling coupons from a regional convenience store chain, set up a Pi Day booth in the Student Center on Mar. 14, and closed out the semester with a picnic held jointly with the Physics Club.

**MI Epsilon (Section A)**

Kettering University

Chapter President—Drew Spooner

74 actives, 30 associates

During the Winter Term of 2001 the KME Applied Math Noon-Time Movie took place on the 2nd Thursday. The Movie "The Video Math Festival" was selected from a collection of juried mathematical videos presented at the 1998 International Congress of Mathematicians in Berlin. The videos are winners of a world-wide competition, in which the international jury evaluated their mathematical relevance, technical quality, and artistic imagination. The mathematical themes include visualizations of classical mathematical ideas of Archimedes, Eratosthenes, Pythagorean, and Fibonacci, as well as application of modern numerical methods to real world simulations. Pizza and pop was served.

The Initiation Ceremony for the new members of our Michigan Epsilon Chapter was on Friday, March 9, at the University Cafeteria. 30 new, top quality Kettering students become members of the chapter. Our Keynote Speaker, Prof. Charles MacCluer of Michigan State University, a highly regarded Applied Mathematician, spoke about "The Industrial Mathematics." It was an intriguing excursion through the examples and areas of application in his newly published book. There were many family and faculty members coming to join us in honoring our new members.

**MI Epsilon (Section B)**

Kettering University

Chapter President—Justin McCurdy

128 actives, 32 associates

For the Spring Term of 2001 the Second week Math Noon-Time Movie “The Video Math Festival” was repeatedly presented, and pizza & pop was served again.

The Initiation Ceremony for the new members was held on June 1, at the University Cafeteria, and 32 new, top quality Kettering students became members of the Chapter. The Keynote Speaker at this event was Dr. Ronald Mossier who recently retired from Daimler Chrysler after a long and productive career there as an Industrial Mathematician. His talk entitled “Rating Baseball Relief Pitchers” was extremely artistic, interesting, and was extremely well received.

On June 22 Kettering will graduate its next class of future leaders. Nine of those graduates will be honored by receiving the President’s Medal. Of those nine students, precisely six of them are members of our Chapter of Kappa Mu Epsilon. I am confident that our new initiates will carry on this tradition of excellence.

Yet another initiate came to Kettering: The newly hired Prof. Ruben Hayrapetyan and Prof. Joe Salacuse enthusiastically started the Kettering Mathematics Olympiad series for 9-12 Grade high school students. The first year competition was held on Saturday, April 28. This is a great way to advertise the Mathematics Program at Kettering. The University Presidency promised to provide some “prize money” for the top competitors on a regular basis every year.

**MS Alpha**

Mississippi University for Women

Chapter President—Chris Sansing

15 actives, 3 associates

Other spring 2001 officers: Mindy Hill, Vice President; Jennifer Kimble, Secretary; Kent Smith, Treasurer; Shaochen Yang, Corresponding Secretary

**MS Delta**

William Carey College

Chapter President—Kate Reynolds

3 actives, 1 associates

Other spring 2001 officers: Holly Norwood, Vice President; Apryle Larson Secretary; Justin Patterson, Treasurer; Dr. McShea, Corresponding Secretary

**MS Epsilon**

Delta State University

Chapter President—Ashley Burleson

17 actives

Mississippi Epsilon held an initiation ceremony on Sunday, February 25, 2001. The new members were initiated. The three new members are Joe Anderson, Amy Pearson, and Jared Yates.

**MO Alpha**

Chapter President—Sheri Puestow

Southwest Missouri State University

44 actives

During the Spring 2001 Semester the Missouri Alpha Chapter of KME held monthly meetings. Featured speakers at the meetings included four student presentations and a presentation by a University of Missouri-Rolla faculty member. Six students and the faculty sponsor attended the National Convention in Topeka, Ks at which two Missouri Alpha members presented papers. The chapter co-hosted the annual Mathematics-Computer Science banquet at which Sheri Puestow was a recognized recipient of the KME Merit Award.

**MO Beta**

Chapter President—Beth Hilbish

Central Missouri State University

25 actives, 6 associates

At the February meeting, Dr. McLaughlin talked about cryptography. At the March meeting Dr. McKee gave a talk titled, "The Witch of Agnesi, Spirographs and Crazy Curves." At the April meeting Deanna Houts from Platt County High School talked about the mathematics of roller coasters. Jachin Misko and Becky Pollard received the Claude H. Brown Mathematics Achievement Award for Seniors. Several of our members volunteered at Math Relays and for the Math Clinic. We also held a book sale to raise money. Three students and two faculty attended the national convention in Topeka, KS.

**MO Epsilon**

Chapter President—Sarah Moulder

Central Methodist College

7 actives, 5 associates

Other spring 2001 officers: Amy Ketchum, Vice President; Beth Kurtz, Secretary; Beth Kurtz, Treasurer; William McIntosh, Corresponding Secretary

**MO Iota**

Chapter President—Dondi Mitchell, Ted Walker

Missouri Southern State College

16 actives, 8 associates

Other spring 2001 officers: Luke Krudwig, Vice President; Heather Vannaman Secretary; Jeremy Goins, Treasurer; Chip Curtis, Corresponding Secretary

**MO Lambda**

Chapter President—Yevgeniy Kondratenko

Missouri Western State College

20 actives, 17 associates

Kappa Mu Epsilon activities for the 2000-2001 academic year have been minimal due to the lack of student participants. Many of our students transferred to other institutions or graduated last year. However, on March 25, 2001, we initiated 13 students and 4 faculty into our organization. Many of them knew nothing about Kappa Mu Epsilon until they were invited to join. With this new group of initiates, we have the opportunity to make our organization stronger and increase awareness of Kappa

Mu Epsilon and mathematics on our campus. We have a large number of interested and great students in our chapter, and we hope that we can use their enthusiasm to make the organization more active.

**MO Mu**

Chapter President—James Hammond

Harris-Stowe State College

15 actives, 6 associates

Our KME chapter sponsored a series of weekly meetings, called Math Thursdays, which involved a variety of math related activities. These included graphing calculator workshops, presentations by the math history class, presentations by in-service teachers on classroom math activities for middle school and high school students, review sessions for the PRAXIS (licensure exam for teachers), and opportunities to interact with math faculty and students at Harris-Stowe State College. Some of our KME members attended the Second Annual Missouri Pre-Service Mathematics Teacher Conference where they attended workshops presented by Missouri mathematics teachers and had the chance to network with other Missouri pre-service mathematics teachers.

**MO Theta**

Chapter President—Joey Bohanon

Evangel University

8 actives

MO Theta held monthly meetings and a social at the home of Don Tosh. New officers were elected in January. Don Tosh and six student members attended the national convention in Topeka. Joey Bohanon presented a paper on "Quadratic Varieties" at the convention. Don Tosh, the corresponding secretary, was elected to the office of President-Elect. He had been the national historian for the past four years.

**MO Zeta**

Chapter President—Ryan Hatfield

University of Missouri-Rolla

5 actives, 5 associates

Projects included a "Bridging the Gap" activity for Girl Scouts and completion of a chapter banner. Also, at one of our meetings we were pleased to have former astronaut Col. Tom Akers give a talk and slide show. He discussed his work on the space shuttle and the ways that mathematics is used in astronautics.

**NE Alpha**

Chapter President—Brandi Oligmueller

Wayne State College

24 actives, 12 associates

Other spring 2001 officers: Talina Martin, Vice President; Jeffery Kesting, Secretary; Brain Pribnow, Treasurer; John Fuelberth, Corresponding Secretary

- NE Beta** Chapter President—Scott Barber  
University of Nebraska at Kearney 7 actives, 5 associates  
Other spring 2001 officers: Jeny Rutar, Vice President; Lisa Beckenhauer, Secretary; Tom Mezger, Treasurer
- NE Delta** Chapter President—Michael Vech  
Nebraska Wesleyan University 19 actives, 4 associates  
Other spring 2001 officers: Sarah Barnes, Vice President; Brain Danforth, Secretary; Brian Danforth, Treasurer; Kristin Pfabe, Corresponding Secretary
- NH Alpha** Chapter President—Craig Sheil  
Keene State College 10 actives, 14 associates  
Other spring 2001 officers: Peter Perrinez, Vice President; Sarah Nordin, Secretary; David Caplette, Treasurer; Vincent Ferlini, Corresponding Secretary
- NM Alpha** Chapter President—Tony Malerich  
University of New Mexico 130 actives, 20 associates  
Other spring 2001 officers: William Tierney, Vice President; Paloma Wells, Secretary; Paloma Wells, Treasurer; Pedro Embid, Corresponding Secretary
- NY Alpha** Chapter President—Karilyn Machen  
Hofstra University 18 actives, 8 associates  
Other spring 2001 officers: David Raikowski, Vice President; Richard Koenig, Secretary; Tiffany Baron, Treasurer; Aileen Michaels, Corresponding Secretary
- NY Eta** Chapter President—Ryanne Fullerton  
Niagra University 20 actives, 15 associates  
Chapter meetings were open to all interested students, not just KME members. Most of the meetings were held in the new mathematics tutoring room. Students used various materials to decorate this area, including mathematical posters, hanging polyhedra, etc. Several students volunteered their time in serving as tutors in area schools.
- NY Kappa** Chapter President—Janine Pryor  
Pace University 12 actives, 5 associates  
Other spring 2001 officers: Geraldine Taiani, Corresponding Secretary
- NY Lambda** Chapter President—Stephanie Calzetta  
C.W. Post Campus of Long Island University 25 actives  
Eight students were initiated into the New York Lambda Chapter by the chapter officers during our annual banquet at the Greenvale Town House

restaurant on the evening of April 25th, bringing the Chapter membership to 213. After dinner, Thomas M. Drescher spoke on "Mobius Transformations: Automorphisms of the Extended Complex Plane." Mr. Drescher is completing his M.S. in Applied Mathematics at C.W. Post this semester, and is New York Lambda member number 186. The evening concluded with the announcement of the 2000-2001 department awards: the Claire F. Adler Award to Elizabeth M. Keating, the Lena Sharney Memorial Award to Renee T. des Etages, the Joseph Panzeca Memorial Award to Stephanie A. Calzetta, and the Hubert B. Huntley Memorial Award to Andrea M. Lorusso.

**NY Mu** Chapter President  
St Thomas Aquinas College 80 actives, 5 associates  
Other spring 2001 officers: Joseph A. Keane, Corresponding Secretary

**NY Nu** Chapter President—Catherine Paolucci  
Hartwick College 12 actives, 8 associates  
Other spring 2001 officers: Adam Parsells, Vice President; Jaclyn Raffo, Secretary; Barrett Snedaker, Treasurer; Ron Brzenk, Corresponding Secretary

**NC Gamma** Chapter President—Sharon Blatt  
Elon University 25 actives, 16 associates  
Other spring 2001 officers: Kathleen Iwancio, Vice President; Judyth Richardson, Secretary; Katie Park, Treasurer; James Beuerle, Corresponding Secretary

**OH Eta** Chapter President—Laura DiMarco  
Ohio Northern University 33 actives, 18 associates  
Other spring 2001 officers: Stacey Stillion, Vice President; Starli Klobetanz, Secretary; Sarah Miller, Treasurer; Donald Hunt, Corresponding Secretary

**OH Gamma** Chapter President—Jeff Smith  
Baldwin-Wallace College 11 actives, 23 associates  
Other spring 2001 officers: Corina Moise, Vice President; Marianne Fedor, Secretary; Jason Popovic, Treasurer; David Calvis, Corresponding Secretary

**OK Alpha** Chapter President—Aaron Lee  
Northeastern State University 41 actives, 2 associates  
The spring initiation of 7 new members was held at the Grand China Restaurant in Tahlequah. Our chapter was deeply saddened at the sudden loss of Dr. Daniel Hansen in March. He had been a member of the NSU Mathematics Department for 29 years and a member of Kappa Mu Epsilon

since 1973. The chapter made a contribution of \$100 to a scholarship fund in his memory. Tera McGrew was our spring speaker. She is a teacher at the Oklahoma School for the Blind and spoke on "Braille & Nemeth Code and Adaptation for Students with Visual Impairment." Now the fourth year in a row, our chapter designed and sold 48 KME/NSU T-shirts. The logo this year was a geometric design entitled "Mathematics-A World of Creativity." The annual pre-finals ice cream social was held on the grassy knoll outside the Science Building.

**PA Delta**

Chapter President—Susan Careo

Marywood University

3 actives

Other spring 2001 officers: Susan Kulikowski, Secretary; Susan Kulikowski, Treasurer; S. Rohers, Ann Von Ahnen Corresponding Secretary

**PA Iota**

Chapter President—Chad Berry

Shippensburg University

11 actives, 2 associates

Other spring 2001 officers: Brian Keller, Vice President; Nicole Lagreca, Secretary; Brandon McCauslin, Treasurer; Cheryl Olsen, Corresponding Secretary

**PA Mu**

Chapter President—Geri Cooper

Saint Francis

30 actives, 12 associates

Saint Francis College became Saint Francis University on January 1, 2001. The first induction ceremonies at a University were held on Thursday, February 22, 2001 for 12 new members. Dinner followed the ceremony. KME members helped at a Math Day on March 9, 2001 where 60 local high school students attended to hear faculty presentations and play Who Wants to Look Like a Millionaire (in a Math Tee Shirt)? KME members also helped pick track on a two-mile of adopted state highway on Thursday, May 3. Two students, Katie Shaffer and Liz Konopka, joined Dr. Skoner in attending the national convention at Washburn University.

**PA Omicron**

Chapter President—Trisha Aurndt

University of Pittsburgh at Johnstown

18 actives, 13 associates

Other spring 2001 officers: Heather Pritt, Vice President; Josie O'Roark, Secretary; Jeffery Miller, Treasurer; Nina Girard, Corresponding Secretary

**PA Pi**

Chapter President—Crystal Hogue

Slippery Rock University of Pennsylvania

6 actives, 5 associates

Other spring 2001 officers: Carrie Brickbichler, Vice President; David Czapor, Secretary; Carrie Bisher, Treasurer; Elise Grabner, Corresponding Secretary

**SC Delta**

Erskine College

Chapter President—Brandon Martin

7 actives, 2 associates

KME met with the math club three times during the spring semester. We worked math puzzles, learned a little about the history of a few games, and practiced our skills on quickie math problems. Of course, food was an integral part of our meetings. At our last meeting the students presented Susan Patterson, retiring faculty advisor, with a plaque commending her involvement in KME over the years. Plans were made for fall activities.

**SC Gamma**

Winthrop University

Chapter President—Anna Ulrey

7 actives, 11 associates

Other spring 2001 officers: Laura Taylor, Vice President; Angel Rush-ton, Secretary; Mathew Foth, Treasurer; Frank Pullano, Corresponding Secretary

**TN Beta**

East Tennessee State University

Chapter President—Susan Hosler

16 actives, 8 associates

A social and meeting was held at the China Buffet in February. In March, Vice President Jason Lewis, accompanied by KME sponsor Dr. Kerley and math students Michael Polson and Clayton Clark attended the MAA meeting in Montgomery, Alabama. On April 19, the following eight members were initiated into KME at the Math Honors Banquet: David Atkins, Akiko Marsh, Morgan Montagnari, Denise Plante, Shane Ratledge, Amanda Rodefer, Lalita Roy, and Katherine Williams. Dr. Gary Henson, ETSU Physics Department, presented an interesting talk on the "Temperature of a Star" at the banquet. The spring 2001 KME officers are President: Susan Hosler, Vice President: Jason Lewis, Secretary: Scott LaVoie, Treasurer: Jamie Howard. KME members Lalita Roy and Katherine Williams have been awarded mathematics scholarships for 2001- 2002.

**TN Delta**

Carson-Newman College

Chapter President—Kryshelle Smith

20 actives, 6 associates

Dr. Catherine Kong with two KME members, Nicholas D. Stepp and W. Luke Robinson conducted a study of the Morristown-Hamblen Emergency Medical Service, Tennessee, in Spring 2000. The project was recognized as a Meritorious MathServe 2000 project last summer. The report of the project was published in the UMAP winter 2000 issue.

**TN Epsilon**

Bethel College

Chapter President—Belinda Thompson

4 actives, 1 associates

Other spring 2001 officers: Russell Holder, Corresponding Secretary

**TN Gamma**

Union University

Chapter President—Andy Nichols

16 actives, 4 associates

The first KME meeting of the spring semester was held March 21. Dr. Bryan Dawson spoke on the topic of Fair Division. Students Andy Nichols, Breanne Oldham, and Nicki McDowell, along with Dr. Dawson, attended the national convention in Topeka. Andy Nichols' talk, "In Defense of Euclid," was awarded one of the "top four" prizes at the convention. The annual initiation banquet was held April 19 at Barnhill's Country Buffet; former chapter president Lori Davis Evans spoke of her experiences during her first two years as a high school teacher. A semester's end cookout was held May 10 at the home of Dr. Jan Wilms. Several honors were announced at the cookout, including the recipients of the first annual Dr. Joe Tucker Scholarship (Patricia Rush) and the second annual Wolfram Award (Nikki Vassar).

**TX Alpha**

Texas Tech University

Chapter President—Jenifer Blasingame

32 actives, 14 associates

Other spring 2001 officers: Paul Brock, Vice President; April Glenn, Secretary; Matt Grisham, Treasurer; Dr. Victor Shubov, Corresponding Secretary

**TX Eta**

Hardin-Simmons University

Chapter President—Crystal Cookey

9 actives, 11 associates

The 26th annual induction ceremony for the Texas Eta Chapter at Hardin-Simmons University in Abilene, Texas, was held March 22, 2001. There were eleven new members: Emily Barrow, Melissa Easley, Kurt Evans, Amber Holloway, Wen Kuo, Shannon McLaughlin, Andy Nelson, Michael Piland, Stephanie Stephenson, Kane Swetnam, and Andrea Wooley. With the induction of these members, membership in the local chapter stands at 213. Leading the induction ceremonies were President Crystal Cooksey, Vice-President Brooke Motheral, and Secretary-Treasurer Katie Smith. Following the induction ceremony, membership shingles and pins were presented to the 2000 inductees. New officers and club pictures were taken. KME then adjourned, and the members, inductees, and chapter sponsors enjoyed pizza and cold drinks.

Crystal Cooksey was reelected President and Brooke Motheral was reelected Vice-President for the 2001-2002 year. In addition Amber Holloway was elected secretary-treasurer for the 2001-2002 year. Dr. Ed Hewett, Dr. James Ochoa, and Dr. Andrew Potter are chapter sponsors. Frances Renfro is the corresponding secretary of the chapter.

**TX Iota** Chapter President—Benjamin Martin  
 McMurry 35 actives, 11 associates  
 Other spring 2001 officers: Doug McEwen, Vice President; Alex Reyes, Secretary; Geoffrey Whitley, Treasurer;

**TX Mu** Chapter President—Leigh Owens  
 Schreiner University 15 actives  
 Other spring 2001 officers: Geneva Conner, Vice President; Jeremy Gutierrez, Secretary; Jeremy Gutierrez, Treasurer; William Silva, Corresponding Secretary

**VA Gamma** Chapter President—Derek Culp  
 Liberty University 25 actives, 11 associates  
 Other spring 2001 officers: Christine Gagnon, Vice President; Courtney Eshbaugh, Secretary; Jennifer Shea, Treasurer; Dr. Glyn Wooldridge, Corresponding Secretary

**WV Alpha** Chapter President—Adam Fletcher  
 Bethany College 35 actives, 21 associates  
 On March 21, 2001, the chapter initiated 21 new members. This was in preparation for the Spring Honor's Day Convocation at Bethany College.  
 On March 31, 2001, honorary officers organized three teams of students to compete in the East Central College Consortium Mathematics Competition held at Marietta College. In Spring, 2002, the chapter will host the competition along with members from the Bethany College Mathematics and Computer Science Club.

The elections of new officers for the 2001-2002 academic year were held in May, 2001. The election was conducted by online balloting.

### *We've Moved!*

The national *Kappa Mu Epsilon* homepage has moved. It is now housed at Eastern Kentucky University. The new URL is [www.kme.eku.edu](http://www.kme.eku.edu)

Our thanks go to the folks at Eastern Kentucky University for hosting our website and Kirk Jones - the new webmaster. Also we want to thank Richard Lamb, Arnie Hammel and Carey Hammel for all the work they put into creating and maintaining our website up to now.

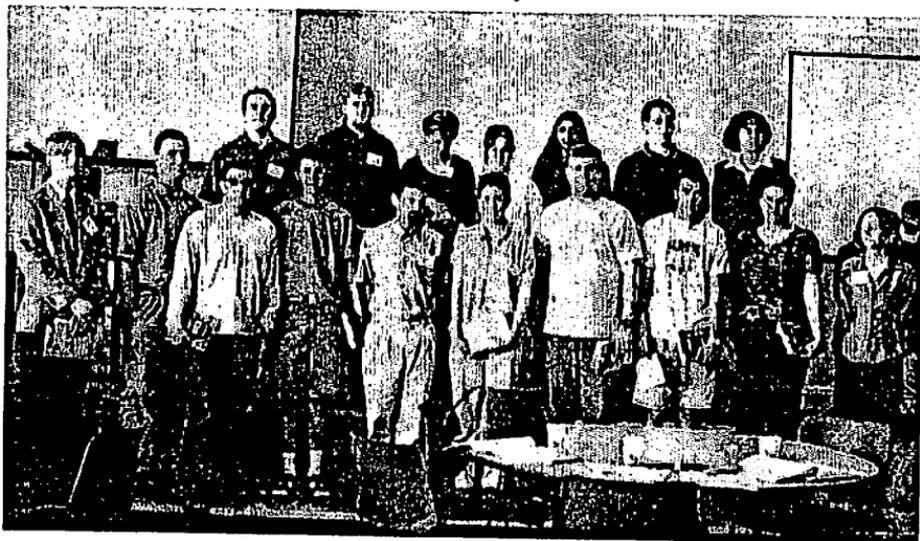
## *Report of the 33<sup>rd</sup> National Biennial Convention*

The Thirty-Third National Biennial Convention of the *Kappa Mu Epsilon*, Mathematics Honor Society, was hosted by the Kansas Delta Chapter at Washburn University, April 5–7, 2001, in Topeka, Kansas. Topeka was founded in 1854 and developed rapidly as the Kansas Territory became a state in 1861. Soon thereafter, a Congregational College was started in 1865. Since proprieties were strictly enforced during those early days, one story tells of a rule forbidding students of the opposite sex from walking together without a chaperone. The only exception allowed men to escort women home from evening visits to the library. On nice evenings dozens of couples could be seen taking several laps around the library before finding their way home. Thus, the path completely encircled the library, and eventually it became so well worn that it needed to be paved. Though geometry doesn't change much, people do. Topeka has grown since those early days, and this Congregational College eventually became a dynamic metropolitan institution, called Washburn University in honor of Ichabod Washburn — certainly an appropriate location for the Thirty-Third National Biennial Convention of the *Kappa Mu Epsilon*, Mathematics Honor Society.

We were welcomed by Jerry Farley, the President of Washburn University, as soon as Patrick Costello, our National President, called the first general session to order. Dr. Farley observed that many organizations, such as the *Kappa Mu Epsilon*, Mathematics Honor Society, were specially welcome, since they helped students to get involved by promoting cooperation and teamwork — also to develop and share ideas by communicating formally and informally. He hoped that we would have a good convention, and we did. Robert Bailey, our President Elect, provided the response, and then Mark Smith, the President of the Kansas Delta Chapter, greeted us on behalf of the students. The roll call was conducted by Waldemar Weber, the National Secretary, and Rhonda McKee, the Secretary Designate. Since Waldemar retired from teaching, the National Council appointed Rhonda to complete the remainder of the unexpired term, effective January 1, 2001. Rhonda was formally installed as the National Secretary during the closing session of this convention on April 7, 2001.

Twenty chapters from ten states were recognized and participated in this convention. They were Colorado Delta from Mesa State University, Iowa Alpha from the University of Northern Iowa, Iowa Gamma from Morningside College, Kansas Alpha from Pittsburg State University, Kansas Beta from Emporia State University, Kansas Gamma from Benedictine College, Kansas Delta from Washburn University, Kansas Epsilon from

Fort Hays State University, Kentucky Alpha from Eastern Kentucky University, Missouri Alpha from Southwest Missouri State University, Missouri Beta from Central Missouri State University, Missouri Theta from Evangel University, Missouri Iota from Missouri Southern State College, Missouri Kappa from Drury University, New York Eta from Niagara University, Ohio Alpha from Bowling Green State University, Oklahoma Gamma from Southwestern Oklahoma State University, Oklahoma Delta from Oral Roberts University, Pennsylvania Mu from Saint Francis University, and Tennessee Gamma from Union University.



Seventeen students (pictured above) from ten of these chapters presented fifteen papers later in the program. Casio <[www.casio.com/calculators](http://www.casio.com/calculators)> provided a Graphing Calculator, Model FX2.0, for each one of these presenters, and the authors were invited to submit their papers for publication in *The Pentagon*, which is our official journal. Listed in order of presentation, they were:

*"Math Magic"*

Jack D. McCush, Missouri Alpha  
Southwest Missouri State University

*"An Analysis of Heinlein's Fourth Dimensional Crooked House"*

Michael Victorine, Missouri Kappa  
Drury University

*"Under Construction, Foci of Conic Sections"*

Leah McBride, Kansas Beta  
Emporia State University

*"Finding Optimal Piano Fingerings"*

Curtis M. Sander, Kansas Gamma  
Benedictine College

*"Symmetry to Infinity"*

Jennifer Bower, Kansas Alpha  
Pittsburg State University

*"Division of a Number by Multiple Divisors and the Underlying Geometry"*

Blake Boldon, Missouri Alpha  
Southwest Missouri State University

*"At Point of Explosion"*

Michelle Wilson, Aparna Tripathy, and Molly Van Gorp, Oklahoma  
Delta

Oral Roberts University

*"Peck, Peck, Peck"*

Mark Allen Smith, Kansas Delta  
Washburn University

*"Generalized Pascal Triangles of Finite Groups with Two Generators"*

Tim Pierce, Kansas Alpha  
Pittsburg State University

*"In Defense of Euclid"*

Andy Nichols, Tennessee Gamma  
Union University

*"Quadratic Varieties"*

Joseph Bohanon, Missouri Theta  
Evangel University

*"Investigation into Buffon's Needle"*

Brad Rolling, Iowa Alpha  
University of Northern Iowa

*"A Statistical Analysis of Disease Rates in the Counties of Missouri"*

Heather Vannaman, Missouri Iota  
Missouri Southern State College

*"Transformations of the Unit Circle"*

Dondi Mitchell, Missouri Iota  
Missouri Southern State College

*"On Iterative Maps"*

Ted Walker, Missouri Iota  
Missouri Southern State College

These papers were judged on the relative merits of their presentation as well as content, and the top four received additional awards of \$100.00 from the National Council and \$25.00 from Brooks/Cole Publishing <[www.brookscole.com](http://www.brookscole.com)>. Listed in order of presentation, the top four papers were (a) Mark Allen Smith from the Kansas Delta Chapter at Washburn University, (b) Andy Nichols from the Tennessee Gamma Chapter at Union University, (c) Heather Vannaman from the Missouri Iota Chapter at Missouri Southern State College, and (d) Ted Walker from the Missouri Iota Chapter at Missouri Southern State College.



Top Four Paper Presenters. From left to right: Heather Vannaman, Andy Nichols, Ted Walker, and Mark Smith

The Paper Selection Committee consisted of Peter Chen of the Texas Kappa Chapter at the University of Mary Hardin-Baylor, Thomas Sharp of the Georgia Alpha Chapter at the State University of West Georgia, and Connie Schrock of the Kansas Beta Chapter at Emporia State University. Meanwhile, Chenglie Hu of the Kansas Epsilon Chapter at Fort Hays State University on the Awards Committee was assisted by Kimberly Schneider of the Colorado Delta Chapter at Mesa State University, Gery East of the Oklahoma Gamma Chapter at Southwestern Oklahoma State University, Virginia Phillips of the Kentucky Alpha Chapter at Eastern Kentucky University, and Trisha White of the Missouri Beta Chapter at Central Missouri State University.

Ronald Wasserstein, who is a Professor in the Department of Mathematics and Statistics as well as the Vice President for Academic Affairs at

Washburn University, presented the keynote address on "*Lotto Luck: What Lotteries (and Forrest Gump) Teach Us about Probability and About Ourselves*" during the banquet on April 6, 2001. He convinced all of us that state lotteries represent a voluntary tax, since a typical commission only returns a portion of each dollar to the players. Indeed, with the aid of a computerized simulation, we played the lottery about two thousand times to compare our earnings and the two of us with the greatest and least success received a graphing calculator, Model TI-89, from Texas Instruments <education.ti.com>, since the same amount of skill was exercised in either case. Thus, we enjoyed food for the mind as well as the body. Two people then were recognized for distinguished service, as follows.

### **Citation for Jo Ann Fellin, OSB**

#### **The George R. Mach Distinguished Service Award, April 6, 2001**

Most of us know Jo Ann Fellin for her highly effective service to the Kappa Mu Epsilon, Mathematics Honor Society, as National Treasurer from 1987–1995 and National Historian from 1975–1979. Serving twelve years as a national officer should alone qualify one for consideration for the George R. Mach Distinguished Service Award. However, Jo Ann's impressive contributions reach far beyond her well-known and significant service as a national officer.

Jo Ann Fellin has been associated with Kappa Mu Epsilon over a span of almost fifty years. She was initiated into Kappa Mu Epsilon in 1952 by the Kansas Gamma Chapter at Mount St. Scholastica College. She became president of that chapter and attended her first national convention in 1955. After receiving a master's and doctorate degree from the University of Illinois, she returned to her alma mater (now called Benedictine College) where she remains on the faculty today. At Kansas Gamma, she served as Corresponding Secretary and Faculty Sponsor at least for twenty years, and as a faculty member, she regularly attended national conventions since 1971 and has directed at least ten student papers presented at these conventions.

Additionally, Jo Ann with Kansas Gamma hosted three regional conferences. She has served on several national committees, some as chair person, and also served as an Installation Officer for three new chapters. Not directly related to Kappa Mu Epsilon, she recently earned the "Distinguished Educator Award" from Benedictine College, which is not a surprise to those of us who have known her through Kappa Mu Epsilon.

In recognition of Jo Ann Fellin's outstanding dedication and ser-

vice to Kappa Mu Epsilon, we take great pleasure in presenting to her the George R. Mach Distinguished Service Award.

### **Citation for Kenneth M. Wilke**

#### **The George R. Mach Distinguished Service Award, April 6, 2001**

During the last twenty-seven years, Kenneth M. Wilke faithfully served as editor of *The Problem Corner* for our Society's journal, *The Pentagon*. He assumed these duties after the death of Robert L. Poe in May of 1973. Since then, Ken has served under five distinguished editors of *The Pentagon* and has edited *The Problem Corner* longer than his six predecessors combined.

What makes this service to the *Kappa Mu Epsilon*, National Mathematics Honor Society, so meritorious is that Ken is a lawyer for the State of Kansas by profession and consequently that it is unlikely he receives any recognition for his service to us from his employer. For most of us, the service that we render is also part of our profession, but for Ken, it's simply his love for mathematics and his interest in sharing this love.

In recognition of Kenneth M. Wilke's outstanding dedication and service to *Kappa Mu Epsilon*, we take great pleasure in presenting to him the George R. Mach Distinguished Service Award.

The convention closed with a business session that began by receiving reports from the Staff of *The Pentagon* and the Officers of the National Council. The reports of the Audit and Resolutions Committees are also included as part of this convention report so that they may be reviewed separately.

The Nominating Committee then submitted the names of Rhonda McKee of the Missouri Beta Chapter at Central Missouri State University and Donald Tosh of the Missouri Theta Chapter at Evangel University as candidates for the next President Elect as well as the names of Connie Schrock of the Kansas Beta Chapter at Emporia State University and Cynthia Woodburn of the Kansas Alpha Chapter at Pittsburg State University as candidates for the next National Historian. After the convention thanked all four nominees for their interest in the society and its mission, Donald Tosh was elected as the next President Elect and Connie Schrock was elected as the next National Historian. According to the rules of succession, Robert Bailey of the New York Eta Chapter at Niagara University was installed as the next National President, and as explained earlier in this report, Rhonda McKee was also installed as the next National Secretary.

According to the results of the mail ballot that closed January 22, 2001, the Louisiana Delta Chapter was installed at the University of Louisiana

at Monroe on February 11, 2001. Thus, the last biennium ended with 130 active chapters. Since then Berry College in Mount Berry completed its petition for the Georgia Delta Charter and Schreiner University in Kerrville completed its petition for the Texas Mu Charter. The National Council decided to bring these requests to a floor vote as part of the new business. It was moved and seconded that both petitions be approved. Though this motion was split, both petitions were accepted unanimously and a hearty welcome was extended to Berry College and Schreiner University by the convention. The installation of the Georgia Delta and Texas Mu Chapters at these two institutions is expected to occur later this spring.

From my retirement, I'd like to close with a personal word of encouragement. Indeed, during the last three decades I've enjoyed working with many people, both students and faculty alike, as the *Kappa Mu Epsilon*, National Mathematics Honor Society, grew not only in numbers but more importantly in its dedication to the five-fold mission of promoting mathematics in every way.

Waldemar Weber  
National Secretary

### Report of the National President

The past two years as President have been busy, but exciting. Within the biennium, there were five new chapters installed. They were North Carolina Delta at High Point University (installed by Donald Aplin on March 24, 1999), Pennsylvania Pi at Slippery Rock University (installed by Peter Skoner on April 19, 1999), Texas Lambda at Trinity University (installed by Donna Hafner on November 22, 1999), Georgia Gamma at Piedmont College (installed by Joe Sharp on April 7, 2000). The most recent addition, installed during this academic year, was Louisiana Delta at the University of Louisiana in Monroe (installed by Don Tosh on February 11, 2001). That brings the current number of active chapters to 130. I hope that we can make these new chapters feel welcome and an integral part of our organization. In addition, two petitions have been received recently. Because of the timing, we will act on these petitions at this convention in accordance with provisions in the Constitution. These petitions are from Berry College in Mount Berry, Georgia, and Schreiner University in Kerrville, Texas. I have also corresponded with nineteen other colleges and universities that have indicated interest in establishing a chapter, including the University of Michigan at Flint, Salem International, the University of Wisconsin at La Crosse, the University of Texas at San Antonio, Bowie State University, the University of Idaho, and South Florida. Several of these have petitions. If you have friends and colleagues at schools that do

not have a chapter of the *Kappa Mu Epsilon*, National Mathematics Honor Society, but are interested, please have them contact President-Elect Bob Bailey.

The National Council continues to support the regional structure. Please refer to the report of Bob Bailey for a discussion of our regional conferences. With much gratitude, we recognize the work and efforts of our Regional Directors, who have served our Society well and deserve the thanks of each of us. I would especially like to thank all of the following, who will be leaving their positions: Carol Harrison, Gayle Kent, Rhonda McKee, and Donna Hafner. Carol Harrison has been the New England Director and her term expires with this convention. Carol has been diligently looking after the Society's interests in her region since 1993. Gayle Kent has been the Southeastern Regional Director, but is planning on retiring soon. Gayle has been diligently looking after the Society's interests in her region since 1997. Rhonda McKee has been the North Central Regional Director and recently moved into a position on the National Council. Rhonda has been diligently looking after the Society's interests in her region since 1999. Donna Hafner has been the South Central Regional Director, but is also planning on retiring soon. Donna has been diligently looking after the Society's interests in her region since 1997. Each one of these Regional directors deserves our thanks for a job well done. I hope that we can find new people to fill their shoes, who are as dedicated.

A special thanks is extended to each of the faculty members who serve as the Corresponding Secretaries and Faculty Sponsors of our active chapters. I know many of you played an important role in assisting your students in the preparation of the excellent papers we have on the convention program this year. Furthermore, we all express our gratitude to each of the students who did the research, writing, editing, and practicing for the presentations we will hear. Without these student contributions, the principle purpose of the convention would not exist. Since we are also indebted to all the individuals who did the work in preparation for this convention, we are anticipating a great time together. The faculty and students alike, who agreed to serve on convention committees, deserve our thanks as well. Without exception, everyone who was asked to serve on a committee willingly agreed to do so. To all of you at Kansas Delta and to every committee member, please accept my most sincere thanks for making the privilege of being your President much easier.

During the past biennium, I represented the *Kappa Mu Epsilon*, National Mathematics Honor Society, at one meeting of the Association of College Honor Societies. It has been very helpful to meet with officers of the other honor societies that are members of this Association for the

purpose of exchanging ideas and acquiring suggestions as to how we can possibly improve on the programs that we currently have in place. Last spring, the Association celebrated its seventy-fifth anniversary, and each member society was asked to recognize five individuals for outstanding service. Thus, special certificates for distinguished service were presented to Arnold D. Hammel, Harold D. Larsen, George R. Mach, Harold L. Thomas, and Emily K. Wyant, all of whom helped our Society to obtain national recognition in significant and pioneering ways. I also submitted two annual reports to the Association, giving specific details regarding the activities of our Society during the last two academic years.

I also want to recognize the fantastic job that is being done by those who work with, manage, write for, and produce our official journal, *The Pentagon*. We are most appreciative of the editorial leadership of Steve Nimmo and the sound business management of Larry Scott. Please feel free to give them any suggestions that you may have for the continued excellence of our journal, which continues to receive national recognition for its emphasis on student contributions. For example, as a new feature, *The Pentagon* will begin listing the names of recent initiates to emphasize that their membership creates a permanent record. Indeed, their names should appear in the first issue that they receive from their complimentary two-year subscription.

Finally, I want to recognize and applaud the outstanding and sometime gargantuan efforts put forth by the other members of the National Council in their respective areas of responsibility. Our national Treasurer, Al Riveland, continued to supply detailed financial reports so that even someone like me could easily see where we are and where trends are going. Our National Secretary, Waldemar Weber, worked diligently to keep records current and professional looking, and as Secretary Designate, Rhonda McKee has admirably taken over those responsibilities. Our National Historian, Don Tosh, has taken on the task of producing certificates in order to reduce expenses and allow us to keep low registration fees. With the bankruptcy of our national jewelers, the timing of his initiative was quite fortuitous. Meanwhile, our President Elect, Bob Bailey, handled the publicity, collection, selection, and scheduling of papers at this convention as well as the scheduling and publicity of regional conferences last year. I can honestly say that I have thoroughly enjoyed the privilege of serving as your National president the last four years. Best wishes to everyone involved, as you continue to support the mission and work for the improvement of the Society.

Patrick Costello  
National President

## Report of the President Elect

By working with Regional Directors, the President Elect is responsible for serving as the coordinator of regional activities. Three regional conferences were held last year spring. They were hosted by (a) the Ohio Zeta Chapter at Muskingum College, March 24-25, 2000, for the Great Lakes and New England Regions with Peter Skoner of the Pennsylvania Mu Chapter at Saint Francis University and Carol Harrison of the Pennsylvania Theta Chapter at Susquehanna University presiding, (b) the Kansas Gamma Chapter at Benedictine College, April 7-8, 2000, with Rhonda McKee of the North Central Region presiding, and (c) the Oklahoma Delta Chapter at Oral Roberts University, April 24, 2000, with Donna Hafner of the South Central Region presiding. Programs at these conferences included student papers, guest speakers, and related social events. Our sincere thanks are extended to the host chapters, their regional directors, and all those who participated in these important activities. We also appreciate the efforts of the remaining directors to have conferences in their regions.

A second duty of the President Elect is to seek nominations for the George R. Mach Service Award, and consequently, a letter seeking such nominations was sent to all of the active chapters last spring. Five persons were nominated, two of which will receive the award at an appropriate time during this convention (their citations appear in the convention report).

Since the President Elect also arranges the presentation of student papers at the National Convention, I am pleased to report that seventeen undergraduate students, representing eleven chapters and five states, submitted fifteen papers for this convention. The Paper Selection Committee, which read all of the manuscripts that were submitted, experienced some difficulty with their ranking because of the high quality that they found. Likewise, the Awards Committee will be judging the oral presentations of these papers on the program, and their results will be combined with those of the Paper Selection Committee to determine a final ranking. The top four papers will each receive \$100 from the National Council and \$25 from Brooks/Cole Publishing. On behalf of the Society, I want to express our sincere thanks to the members of the paper Selection and Awards Committees as well as all the students who prepared and submitted papers. It is this work that makes for a truly successful convention.

As I close my four-year term of President Elect, I would like to express my thanks to present and past members of the National Council, Regional Directors, Corresponding Secretaries and Faculty Sponsors without whose dedicated assistance our organization could not survive, to active members of local chapters, and finally to George Mach, who patiently in-

roduced me to the duties of a national officer when I became the National Secretary in 1987.

Robert L. Bailey  
President Elect

### Report of the National Historian

One of the main roles of the National Historian is to solicit and edit chapter reports. Near the end of each semester a request for chapter news is sent to each chapter. These reports along with installation reports are edited and forwarded to the editor of *The Pentagon* for inclusion in the *Kappa Mu Epsilon News* section. I would especially like to thank Steve Nimmo, the editor of *The Pentagon*, for the pleasant working relationship we have had. The unedited original reports are placed in the chapter folder and become a permanent record of each chapter's activities. I encourage each chapter to take advantage of this opportunity to remember their officers and activities.

Beginning this semester, the historian will also have the responsibility of collecting and editing the names of all initiates for inclusion in a new section of *The Pentagon*. The first issue of *The Pentagon* that a new member receives should have his or her name in it.

Since becoming historian four years ago, I began soliciting and accepting chapter reports using electronic as well as surface mail. I print hard copies of all electronic responses for inclusion in the chapter files. At the last national convention, I reported that 60% of the chapters were reporting electronically. That proportion appears to be increasing, and in the last year, 80% of the responses were electronic. If a chapter is not receiving a request electronically, it is probably because I do not have a correct address for the Corresponding Secretary.

In this biennium, eighty-eight chapters reported at least once. On the average, over fifty-nine reported each semester. To be commended are thirty chapters which reported all four times. Those schools are (listed by state): Birmingham-Southern College, Mesa State College, State University of West Georgia, Benedictine University, University of Northern Iowa, Wartburg College, Benedictine College, Washburn University, Fort Hays State University, Eastern Kentucky University, Cumberland College, Kettering University, Southwest Missouri State University, William Jewel College, Evangel University, Missouri Southern State College, Missouri Western State College, Mississippi University for Women, Delta state University, Wayne State College, University of New Mexico, Pace University, C. W. Post Campus of Long Island University, Baldwin-Wallace College, Northwestern State University, Southwestern Oklahoma State University,

St. Francis University, University of Pittsburgh at Johnstown, Union University, and Bethel College. If you are from one of these schools, please let your Corresponding Secretary know that you appreciate his or her efforts.

I would also like to thank the Corresponding Secretaries for their active participation in maintaining the history and tradition of the *Kappa Mu Epsilon*, Mathematics Honor Society. We have tried to find ways, however small, to show our appreciation to these people who have such a crucial role in our organization. One semester, we sent KME Mugs to all Corresponding Secretaries, who sent in that semester's news. We also brought mugs to this convention to distribute to all Corresponding Secretaries, who are present. We want you to know that you are appreciated. I have enjoyed tremendously the time I have spent as historian. I appreciate the many times several of you have just added a note of appreciation or greeting to the bottom of a chapter report. You have been wonderful to work with, and I have become convinced that KME's strength lies in its people. I wish my successor as much enjoyment as I have had during these past four years.

Don Tosh  
National Historian

### Report of the National Secretary

The *Kappa Mu Epsilon*, Mathematics Honor Society, initiated 2,090 new members in 108 chapters during the biennium that ended March 15, 2001. That brings the cumulative total to 59,030 members in active chapters with another 6,159 members in 28 inactive chapters for a grand total of 65,189 members since the founding on April 18, 1931.

As National Secretary, I receive all initiation reports from the chapters, make a record of them, update mailing list information for Corresponding Secretaries, and forward copies of the reports to other officers. In addition, I receive orders for key pins and replacement certificates.

I assumed the responsibilities of national Secretary in January of this year after the retirement of the previous secretary, Waldemar Weber. The transition has not been without challenges, but taking into consideration the fact that I was in Warrensburg, Missouri, Professor Weber was in Colorado Springs, Colorado, and the past records were in Bowling Green, Ohio, things proceeded relatively smoothly. I am especially grateful to Professor Weber, Al Riveland, and the other national officers for their help and encouragement.

Rhonda McKee  
Secretary Designate

## Report of the National Treasurer

The Biennium Asset and Cashflow Reports are included in this report for the biennium that began on April 1, 1999, and ended on March 15, 2001. These reports show assets of \$37,924.96 at the end of the biennium and an asset reduction of \$6,440.57 during the biennium. A well-established goal of the National Council to maintain an asset base of at least \$30,000.00 has been met.

By way of comparison, there was a positive cashflow of \$9,771.41 for the previous biennium, but that included a special gift of about \$5,000.00 that was given to the *Kappa Mu Epsilon*, Mathematics Honor Society, by the Dorothy Horn Estate. This biennium we initiated 173 fewer members, and so the income from the registration fees dropped significantly. Additionally, increases in convention expenses, membership certificates, official jewelry, shipping charges, and publication expenses, each contributed to the biennium loss.

Production of the certificates for all initiates is now being done at Evangel University in Springfield, Missouri, thanks to the ambitious initiative of Don Tosh and the Missouri Theta chapter. The National Council expects that this move will result in a helpful decrease in our expenses in the next biennium. However, we will continue to watch our cashflow, maintaining an asset base of \$30,000.00 if possible. If it appears that we will fall significantly below that base in future years, an increase in the registration fee will be considered.

Allan Riveland  
National Treasurer

### Biennium Asset Report

<b>Total Assets (April 1, 1999)</b>	<b>\$44,365.53</b>
<b>Current Assets</b>	
Mercantile Bank	\$8,772.62
Educational Credit Union	
Savings Account	3,331.42
Certificates of Deposit	25,820.92
<b>Total Assets, (March 15, 2001)</b>	<b>\$37,924.96</b>
<b>Biennium Asset reduction</b>	<b>6,440.57</b>

### Biennium Cashflow Report

#### Receipts

Registration Fees	\$41,960.00	
Installation Fees	93.00	
Interest Income	3,221.52	
Inventory Income	650.05	
Overpayments Received	135.00	
<b>Total Biennium Receipts</b>		<b>\$46,059.57</b>

#### Expenditures

Association of College Honor Societies	\$1,330.17	
Administrative Expenses	4,768.20	
National Convention	10,692.58	
Regional Conferences	831.28	
Travel Expenses for Council Meetings	1,979.23	
Jewelry and Certificates	18,543.76	
Installation Expenses	492.66	
Inventory Expenses	56.40	
Overpayments Returned	135.00	
Pentagon Expenses	13,441.47	
Miscellaneous Expenses	229.39	
<b>Total Biennium Expenses</b>		<b>\$52,500.14</b>
<b>Biennium Cashflow</b>		<b>(6,440.57)</b>

### Report of The Pentagon Editor

Nineteen papers comprise volumes 59 and 60 of *The Pentagon*. Of these, eighteen are student papers and one is a faculty paper. All eighteen of the student papers were presented at national conventions or regional conferences of the *Kappa Mu Epsilon*, National Mathematics Honor Society, and the faculty paper was a banquet address at one of these conferences. *The Problem Corner*, edited by Kenneth M. Wilke, and the *Kappa Mu Epsilon News*, edited by Don Tosh, as well as various reports, which continue to make up a large portion of this journal, are essential to its success. The efforts of these associate editors are greatly appreciated. *The Pentagon* would not be the quality journal that many look forward to receiving without all of their hard work.

Manuscripts received by *The Pentagon* other than those presented at our conventions are still refereed by faculty volunteers. The efforts of twenty-three such referees were acknowledged in the Spring 2000 issue and several have already provided additional service since that time. Nearly all referees teach at institutions, where KME Chapters have been installed.

These referees have been a great help to the editor. Likewise, Larry Scott, the Business Manager, continues to keep the delivery of each issue running smoothly. The members of the National Council and the Regional Directors have also been very helpful with answering questions, preparing official reports, forwarding papers to the Editor, and dealing with many additional details. Continue to keep those manuscripts coming!

A special thanks goes to Bryan Dawson, the preceding Editor, for helping me to become familiar with the position and answering all of my questions.

Steven Nimmo  
Editor

### Report of The Pentagon Business Manager

It is a pleasure to make my third report as the Business Manager of *The Pentagon* at this 33rd National Biennial Convention of the *Kappa Mu Epsilon*, National Mathematics Honor Society. As many already know, it is my primary responsibility to maintain a current list of subscribers, to oversee our mailings, and to assist the editor of our official journal.

All new members receive a two-year complimentary subscription to *The Pentagon* and are encouraged to continue their subscriptions for a modest fee of \$5.00 per year. The library rate is \$10.00 per year and international subscriptions are \$7.00 per year. Issues are prepared and distributed in December and May of each academic year. Our mailing list includes subscribers in this country, South America, Asia, Africa, and Europe. Approximately 3,000 subscribers received each issue during the last biennium, and approximately 500 renewal notices are mailed to subscribers each semester.

Complimentary copies of *The Pentagon* are sent to the library of each college or university with an active chapter. Anyone contributing an article for an issue will receive two free copies, and speakers at this national convention will have their subscriptions extended two years.

Please watch the expiration date for your subscription and renew early, since there is only one renewal reminder. Postal regulations for bulk mailings require that each address be validated at least once per year and that each address contain either a physical street number or postal box number. Please check your address and make corrections as necessary, since their penalties for returning undeliverable issues. Corrections may be made by sending a note to me at <scottlar@emporia.edu>.

I am appreciative of the support and assistance given by the National Council. In particular, I would like to thank Steven D. Nimmo, Editor of *The Pentagon*, Patrick J. Costello, our National President, A. Allan Rive-

land, our National Treasurer, Waldemar C. Weber, our National Secretary, and Rhonda L. McKee, the Secretary Designate. Their cooperation and assistance have made things move smoothly. I also gratefully acknowledge the assistance of my secretary, Teresa Rios.

Larry Scott  
Business Manager

**The Pentagon Financial Report**

<b>Balance as of April 1, 1999</b>		<b>\$1,463.63</b>
<b>Total Receipts from Subscriptions</b>		<b>4,272.99</b>
<b>Total Expenditures</b>		<b>(5087.42)</b>
<b>Postal Costs &amp; Return Charges</b>	<b>\$140.60</b>	
(8/31/97 to 4/30/99)		
<b>Volume 58, Issue 2</b>		
Renewal Notices	135.20	
Bulk Mailing	646.35	
Late Mailing	129.05	
Editor Expenses	67.88	
<b>Volume 59, Issue 1</b>		
Renewal Notices	36.17	
Bulk Mailing	744.87	
Bulk Rate Permit	100.00	
Return Charges	510.25	
<b>Volume 59, Issue 2</b>		
Renewal Notices	152.95	
Bulk Mailing	884.92	
Return Charges	48.38	
Extra Stamps and Postage	27.41	
<b>Volume 60, Issue 1</b>		
Renewal Notices	280.28	
Bulk Mailing	746.78	
Bulk Rate Permit	100.00	
Extra Stamps and Postage	24.35	
Copyright Registration	210.00	
Other Expenses	57.00	
<b>Volume 60, Issue 2</b>		
Renewal Notices	44.98	
<b>Balance as of April 5, 2001</b>		<b>\$649.20</b>

### **Report of the Resolutions Committee**

The Resolutions Committee consisted of Bryan Dawson, faculty member from the Tennessee Gamma Chapter, Gayla Hobbs, student member from the Missouri Alpha Chapter, Carter McDaniel, student member from the Missouri Theta Chapter, and Jamie Smith, student member from the Oklahoma Gamma Chapter. The Committee proposed the following resolutions.

“Whereas Washburn University and the surrounding community of Topeka has provided this convention with gracious hospitality, be it resolved:

1. that this Thirty-third Biennial Convention express its gratitude to the Kansas Delta Chapter for the thorough arrangements they have planned and carried out so successfully, and
2. that this convention recognize and thank Jerry Farley, President of Washburn University, as well as Donna LaLonde, Ron Wasserstein, Al Riveland and Sarah Cook, together with all the other members of Kansas Delta, who devoted countless hours to ensure the success of this meeting.

“Whereas the success of any undertaking is directly proportional to the dedication and ability of its leaders, be it further resolved:

1. that this Thirty-third Biennial National Convention express its gratitude to (a) Donna Hafner, Gayle Kent, Rhonda McKee, and Carol Harrison, outgoing regional directors for their years of faithful service; (b) outgoing officers Don Tosh, Waldemar Weber, and Robert Bailey, for their faithful service to the offices of national historian, national secretary, and national president-elect, respectively; and especially to (c) Pat Costello, for his faithful service guiding Kappa Mu Epsilon as its president.
2. that this Convention acknowledge the participation of the students and faculty who served on the Auditing, Awards, Local Arrangements, Nominating, Paper Selection, Resolutions, and Banquet Committees, which is so essential for the success of this meeting.

“Finally, whereas the primary purpose of Kappa Mu Epsilon is to encourage participation in mathematics and the development of a deeper understanding of its beauty, be it further resolved:

1. that the students who prepared, submitted, and then presented their papers be given special commendation by this Thirty-third Biennial Convention for their enthusiasm and dedication,
2. that this convention express thanks to David Thornburg for entertaining us with magic Thursday evening and to Ron Wasserstein for his wonderfully insightful presentation about lotteries at the Friday night banquet, and
3. that this convention recognize the contributions of Brooks/Cole/Thomson Learning Publishers, Casio, Texas Instruments, and the local food service, to the success of this meeting.”

Bryan Dawson  
Chair Person

### **Report of the Audit Committee**

The Audit Committee consisted of Donna Hafner, faculty member from the Colorado Delta Chapter, Thomas Jang, student member from the Missouri Alpha Chapter, Jenny McGavock, student member from the Oklahoma Gamma Chapter, and Mike Richardson, student member from the Missouri Theta Chapter. The National Treasurer, Allan Riveland from the Kansas Delta Chapter, provided all of the relevant reports and documents for review and attended the initial meeting of the Audit Committee. Since this information was well organized, the review process was expedited. The committee members compared the Biennium Asset and Cashflow Reports with the receipts, transactions, and pay orders, as well as the account statements from the Educational Credit Union and Mercantile/Firststar Bank. After verifying the balances of these accounts over the telephone, they also interviewed the National President and National Secretary regarding the receipt and disbursement of funds.

The Audit Committee found that the internal checks, namely the double signatures on outflow and double recording of inflow, provide important safeguards for the National Treasurer. Since time constraints at a national convention prevent a complete review of the records, the Council may wish to consider periodically whether a professional audit is warranted. However, the Committee sees no reason to recommend such an audit at this time. Besides noting the favorable asset reserves in excess of the \$30,000 minimum recommended by the National Council, the Committee finds that inefficiency and unnecessary cost result from the receipt of multiple checks in payment of registration fees from single chapters. Therefore, each chapter is encouraged to remit a single check in payment of registration fees.

The Audit Committee commends Professor Riveland for his excellent

management and presentation of the financial records and for his dedication and generous donation of time through six years of service as the National Treasurer. Likewise, the Committee commends the national President, Secretary and Treasurer, for the manner in which they communicate and cooperate to maintain the internal checks that preserve the integrity of the Office of the Treasurer. The Committee also commends the National Council for its judicious and conservative oversight. For these reasons, the Audit Committee recommends the acceptance of the financial records and reports for the 1999–2001 Biennium, as prepared and presented by the National Treasurer, A. Allan Riveland.

Donna Hafner  
Chair Person

---

### ***Subscription Renewals and Change of Address***

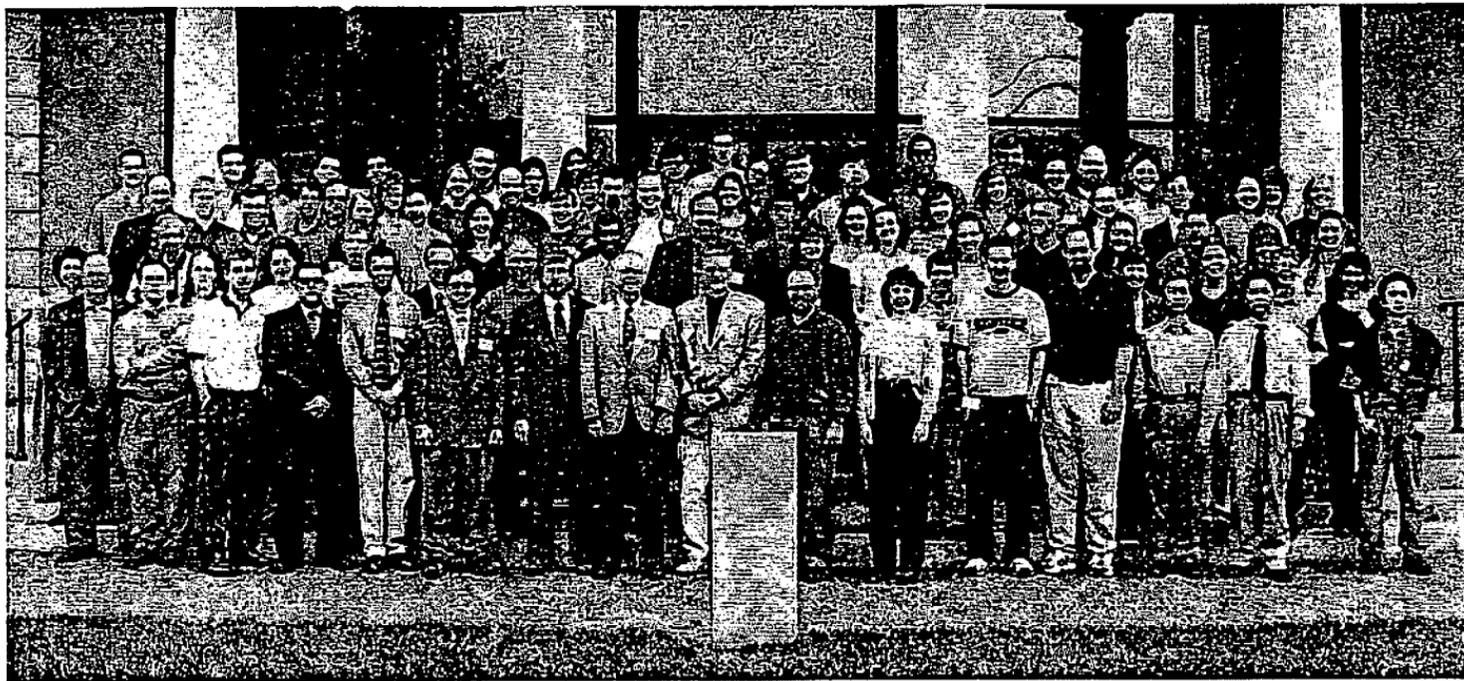
Your *Pentagon* subscription expires with the volume and number that appears in the upper right corner of your address label (see back cover). Since this issue is volume 60 Number 2, if the code 60-2 appears on your label then this is your last issue!

**Early renewals save us the cost of mailing out renewal notices.** To renew, please send your check - just \$10 for four more issues (domestic individuals only; see page 2 for rates for libraries and foreign subscriptions) - together with your name and address and a copy of your old address label to:

*The Pentagon* Business manager  
Division of Mathematics and Computer Science  
Emporia State University  
Emporia, KS 66801 USA

Please renew promptly to avoid gaps in your journal collection.

**If you have a change of address, please don't forget *The Pentagon!*** Send your change of address to the business manager at the above address. Failure to do so may cost KME additional postage fees!



Thirty-Third Biennial convention of *Kappa Mu Epsilon*, April 5-7, 2001,  
Topeka, Kansas, hosted by Kansas Delta Chapter at Washburn University

## *Kappa Mu Epsilon National Officers*

Robert Bailey

*President*

Mathematics Department  
Niagara University, Niagara, NY 14109  
rbl@niagara.edu

Don Tosh

*President-Elect*

Department of Science and Technology  
Evangel College, 1111 N. Glenstone Avenue, Springfield, MO 65802  
toshd@evangel.edu

Rhonda McKee

*Secretary*

Department of Mathematics  
Central Missouri State University, Warrensburg, MO 64093-5045  
mckee@cmsu1.cmsu.edu

Al Allan Riveland

*Treasurer*

Department of Mathematics and Statistics  
Washburn University, Topeka, KS 66621  
zzrive@acc.wuacc.edu

Connie Schrock

*Historian*

Department of Mathematics  
Emporia State University  
Emporia, KS 66801-5087  
schrockc@emporia.edu

---

### *Back Issues*

Is your journal collection complete? Copies of most back issues of *The Pentagon* are still available for \$5.00 per copy. Please send inquiries to:

*The Pentagon* Business manager  
Division of Mathematics and Computer Science  
Emporia State University  
Emporia, KS 66801 USA

## *Active Chapters of Kappa Mu Epsilon*

*Listed by date of installation*

Chapter	Location	Installation Date
OK Alpha	Northeastern State University, Tahlequah	18 April 1931
IA Alpha	University of Northern Iowa, Cedar Falls	27 May 1931
KS Alpha	Pittsburg State University, Pittsburg	30 Jan 1932
MO Alpha	Southwest Missouri State University, Springfield	20 May 1932
MS Alpha	Mississippi University for Women, Columbus	30 May 1932
MS Beta	Mississippi State University, Mississippi State	14 Dec 1932
NE Alpha	Wayne State College, Wayne	17 Jan 1933
KS Beta	Emporia State University, Emporia	12 May 1934
NM Alpha	University of New Mexico, Albuquerque	28 March 1935
IL Beta	Eastern Illinois University, Charleston	11 April 1935
AL Beta	University of North Alabama, Florence	20 May 1935
AL Gamma	University of Montevallo, Montevallo	24 April 1937
OH Alpha	Bowling Green State University, Bowling Green	24 April 1937
MI Alpha	Albion College, Albion	29 May 1937
MO Beta	Central Missouri State University, Warrensburg	10 June 1938
TX Alpha	Texas Tech University, Lubbock	10 May 1940
TX Beta	Southern Methodist University, Dallas	15 May 1940
KS Gamma	Benedictine College, Atchison	26 May 1940
IA Beta	Drake University, Des Moines	27 May 1940
TN Alpha	Tennessee Technological University, Cookeville	5 June 1941
NY Alpha	Hofstra University, Hempstead	4 April 1942
MI Beta	Central Michigan University, Mount Pleasant	25 April 1942
NJ Beta	Montclair State University, Upper Montclair	21 April 1944
IL Delta	University of St. Francis, Joliet	21 May 1945
KS Delta	Washburn University, Topeka	29 March 1947
MO Gamma	William Jewell College, Liberty	7 May 1947
TX Gamma	Texas Woman's University, Denton	7 May 1947
WI Alpha	Mount Mary College, Milwaukee	11 May 1947
OH Gamma	Baldwin-Wallace College, Berea	6 June 1947
CO Alpha	Colorado State University, Fort Collins	16 May 1948
MO Epsilon	Central Methodist College, Fayette	18 May 1949
MS Gamma	University of Southern Mississippi, Hattiesburg	21 May 1949
IN Alpha	Manchester College, North Manchester	16 May 1950
PA Alpha	Westminster College, New Wilmington	17 May 1950
IN Beta	Butler University, Indianapolis	16 May 1952
KS Epsilon	Fort Hays State University, Hays	6 Dec 1952
PA Beta	LaSalle University, Philadelphia	19 May 1953
VA Alpha	Virginia State University, Petersburg	29 Jan 1955
IN Gamma	Anderson University, Anderson	5 April 1957
CA Gamma	California Polytechnic State University, San Luis Obispo	23 May 1958
TN Beta	East Tennessee State University, Johnson City	22 May 1959
PA Gamma	Waynesburg College, Waynesburg	23 May 1959
VA Beta	Radford University, Radford	12 Nov 1959
NE Beta	University of Nebraska—Kearney, Kearney	11 Dec 1959
IN Delta	University of Evansville, Evansville	27 May 1960

OH Epsilon	Marietta College, Marietta	29 Oct 1960
MO Zeta	University of Missouri—Rolla, Rolla	19 May 1961
NE Gamma	Chadron State College, Chadron	19 May 1962
MD Alpha	College of Notre Dame of Maryland, Baltimore	22 May 1963
IL Epsilon	North Park College, Chicago	22 May 1963
OK Beta	University of Tulsa, Tulsa	3 May 1964
CA Delta	California State Polytechnic University, Pomona	5 Nov 1964
PA Delta	Marywood University, Scranton	8 Nov 1964
PA Epsilon	Kutztown University of Pennsylvania, Kutztown	3 April 1965
AL Epsilon	Huntingdon College, Montgomery	15 April 1965
PA Zeta	Indiana University of Pennsylvania, Indiana	6 May 1965
AR Alpha	Arkansas State University, State University	21 May 1965
TN Gamma	Union University, Jackson	24 May 1965
WI Beta	University of Wisconsin—River Falls, River Falls	25 May 1965
IA Gamma	Morningside College, Sioux City	25 May 1965
MD Beta	Western Maryland College, Westminster	30 May 1965
IL Zeta	Rosary College, River Forest	26 Feb 1967
SC Beta	South Carolina State College, Orangeburg	6 May 1967
PA Eta	Grove City College, Grove City	13 May 1967
NY Eta	Niagara University, Niagara University	18 May 1968
MA Alpha	Assumption College, Worcester	19 Nov 1968
MO Eta	Truman State University, Kirksville	7 Dec 1968
IL Eta	Western Illinois University, Macomb	9 May 1969
OH Zeta	Muskingum College, New Concord	17 May 1969
PA Theta	Susquehanna University, Selinsgrove	26 May 1969
PA Iota	Shippensburg University of Pennsylvania, Shippensburg	1 Nov 1969
MS Delta	William Carey College, Hattiesburg	17 Dec 1970
MO Theta	Evangel University, Springfield	12 Jan 1971
PA Kappa	Holy Family College, Philadelphia	23 Jan 1971
CO Beta	Colorado School of Mines, Golden	4 March 1971
KY Alpha	Eastern Kentucky University, Richmond	27 March 1971
TN Delta	Carson-Newman College, Jefferson City	15 May 1971
NY Iota	Wagner College, Staten Island	19 May 1971
SC Gamma	Winthrop University, Rock Hill	3 Nov 1972
IA Delta	Wartburg College, Waverly	6 April 1973
PA Lambda	Bloomsburg University of Pennsylvania, Bloomsburg	17 Oct 1973
OK Gamma	Southwestern Oklahoma State University, Weatherford	1 May 1973
NY Kappa	Pace University, New York	24 April 1974
TX Eta	Hardin-Simmons University, Abilene	3 May 1975
MO Iota	Missouri Southern State College, Joplin	8 May 1975
GA Alpha	State University of West Georgia, Carrollton	21 May 1975
WV Alpha	Bethany College, Bethany	21 May 1975
FL Beta	Florida Southern College, Lakeland	31 Oct 1976
WI Gamma	University of Wisconsin—Eau Claire, Eau Claire	4 Feb 1978
MD Delta	Frostburg State University, Frostburg	17 Sept 1978
IL Theta	Benedictine University, Lisle	18 May 1979
PA Mu	St. Francis College, Loretto	14 Sept 1979
AL Zeta	Birmingham-Southern College, Birmingham	18 Feb 1981
CT Beta	Eastern Connecticut State University, Willimantic	2 May 1981
NY Lambda	C.W. Post Campus of Long Island University, Brookville	2 May 1983

MO Kappa	Drury College, Springfield	30 Nov 1984
CO Gamma	Fort Lewis College, Durango	29 March 1985
NE Delta	Nebraska Wesleyan University, Lincoln	18 April 1986
TX Iota	McMurry University, Abilene	25 April 1987
PA Nu	Ursinus College, Collegeville	28 April 1987
VA Gamma	Liberty University, Lynchburg	30 April 1987
NY Mu	St. Thomas Aquinas College, Sparkill	14 May 1987
OH Eta	Ohio Northern University, Ada	15 Dec 1987
OK Delta	Oral Roberts University, Tulsa	10 April 1990
CO Delta	Mesa State College, Grand Junction	27 April 1990
NC Gamma	Elon College, Elon College	3 May 1990
PA Xi	Cedar Crest College, Allentown	30 Oct 1990
MO Lambda	Missouri Western State College, St. Joseph	10 Feb 1991
TX Kappa	University of Mary Hardin-Baylor, Belton	21 Feb 1991
SC Delta	Erskine College, Due West	28 April 1991
SD Alpha	Northern State University, Aberdeen	3 May 1992
NY Nu	Hartwick College, Oneonta	14 May 1992
NH Alpha	Keene State College, Keene	16 Feb 1993
LA Gamma	Northwestern State University, Natchitoches	24 March 1993
KY Beta	Cumberland College, Williamsburg	3 May 1993
MS Epsilon	Delta State University, Cleveland	19 Nov 1994
PA Omicron	University of Pittsburgh at Johnstown, Johnstown	10 April 1997
MI Delta	Hillsdale College, Hillsdale	30 April 1997
MI Epsilon	Kettering University, Flint	28 March 1998
KS Zeta	Southwestern College, Winfield	14 April 1998
TN Epsilon	Bethel College, McKenzie	16 April 1998
MO Mu	Harris-Stowe College, St. Louis	25 April 1998
GA Beta	Georgia College and State University, Milledgeville	25 April 1998
AL Eta	University of West Alabama, Livingston	4 May 1998
NY Xi	Buffalo State College, Buffalo	12 May 1998
NC Delta	High Point University, High Point	24 March 1999
PA Pi	Slippery Rock University, Slippery Rock	19 April 1999
TX Lambda	Trinity University, San Antonio	22 November 1999
GA Gamma	Piedmont College, Demorest	7 April 2000
LA Delta	University of Louisiana, Monroe	11 February, 2001
GA Delta	Berry College, Mount Berry	21 April, 2001
TX Mu	Schreiner University, Kerrville	28 April, 2001

### *Cumulative Subject Index*

Have you visited the Cumulative Subject Index for *The Pentagon* lately? Check it out at [www.kme.eku.edu/indpent.htm](http://www.kme.eku.edu/indpent.htm).

What does it have, you ask? Mostly organized by standard course titles, there are 25 topics to choose from. This can be a great resource for your courses, whether you are a student or a faculty member! The titles of hundreds of articles are listed on an incredible variety of fascinating topics. Check it out today!