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Switching Lattice Path Segments to Solve Ballot Problems with Ties

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1. Introduction

The Ballot Problem. *Suppose that Al gets A-votes in an election, Betty gets B-votes. Let p be the number of A-votes, and q be the number of B-votes, where $p > q$. Votes are tallied in a random order. Find the probability that Al always leads during the process of vote counting.*

According to my proof from "The Ballot Theorem" in the Spring 2011 issue of *The Pentagon*, the probability that Al always leads is $(p-q)/(p+q)$, in other words, Al's margin of victory divided by the total number of votes.

Another Version of The Ballot Problem. Suppose that Al receives p votes in an election, Betty receives q votes, where $p \geq k \cdot q$ for some positive integer k. Compute the number of ways the ballots can be ordered so that Al maintains more than k times as many votes as Betty throughout the counting of the ballots.

In this paper, I will show the probability properties on cases where Al is not being ahead. Instead of counting votes, let us count cards. Assume there are $p + q$ cards, where p cards are "A" cards and q cards are "B" cards.

Case I: $p - q = 1$

As before, p is the number of "A" cards and q is the number of "B" cards. In this case, there is one more "A" card than "B" card since $p - q = 1$. Let T be the total number of positions in a string of $p + q$ cards such that the number of appearances of "A" cards is less or equal to the

number of appearance of "B" cards, and let us call such a position a "tie." Assume cards are tallied in a random order. Then the distribution of T is $P(T = 0) = P(T = 1) = \ldots = P(T = 2 \cdot q) = 1/(p + q)$.

Approach: Let's call a string of cards a sequence and a partial string a subsequence. If in a subsequence the amount of "A" cards is the same as the amount of "B" cards, and the total amount is the minimum it could be, then we call such a subsequence a "tie subsequence." Let us arrange a sequence on a circle and count the sequence clockwise. On a cycle, each card can be the first element of a sequence, and so we have $p + q$ possible sequences (there may be repeating sequences, but this will not affect our answer). Note that we can find a single card "A" or many cards "A" depending on the difference of $p - q$, such that when this card "A" leads as the first element in a sequence, $T = 0$ by [1]. (Note that in Case I, "A" is unique since $p - q = 1$. See [1] for details of the proof.) We then start counting from the "B" card next to that "A" card counterclockwise, and mark the first "A" that satisfies the condition that "A" and "B" cards have the minimum same amount, i.e. a "tie subsequence." This marked "A" card is the leading card which contains $T = 1$ on this cycle. Then we find the next "tie subsequence" with the same procedure counterclockwise, and starting from this "tie subsequence," the sequence contains $T = 2$. We keep doing this procedure until we can no longer find a "tie subsequence," and in such a sequence we reach the peak of T, where $T = 2 \cdot q$.

2. An Example

Let's start with an example as the demonstration of this "counting backwards" method, which we just illustrated above in "Approach." Let $p = 3$ and $q = 2$. One of the possible cycles, sequence of (A, A, A, B, B), can be displayed as in Figure 1 below.

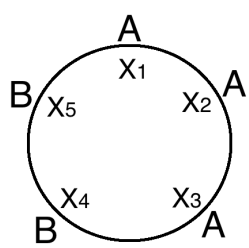


Figure 1

By the "cycling" method of Ballot Theorem [1] in which we cancel

out all the "B"s with previous "A"s on the cycle, we can easily find the leading card "A," which is marked as x_1 with $P(T = 0) = 1/5$. Once we locate x_1, we can locate the position of the leading element of a sequence, such that the sequence has $T = 1$, by finding the first "tie subsequence," counterclockwise. In Figure 1, we locate the position of x_1 and we start from the card right before x_1 clockwise, which is x_5. Then we find the first card "A" counterclockwise from x_5 such that "A" and "B" have the same amount of cards. Here, x_2 is the leading card of the first "tie subsequence," and sequence starting from x_2 has one "tie," i.e. $T = 1$. Then from the next card counterclockwise, x_1, we find the first "B" such that the subsequence between x_1 and some "B," counterclockwise, has the same amount of "A" as "B" cards. It is not hard to see that x_5 is the card "B" we find for the next "tie subsequence," and sequence starting from x_5 has $T = 2$. Similarly, for the rest of the cards, x_3 ends up with $T = 3$ and x_4 ends up with $T =$4. We stop here since we can no longer find a "tie subsequence."

Therefore, in this cycle arrangement, we have

$$P(T = 0) = P(T = 1) = P(T = 2) = P(T = 3) = P(T = 4) = 1/5.$$

On the other possible cycle, a sequence can be arranged as (A, A, B, A, B), shown as in Figure 2 below.

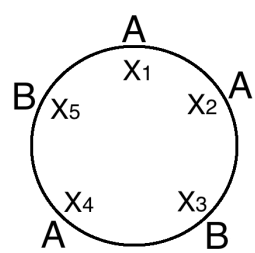


Figure 2

With the same procedure, we find that when we count the sequence clockwise x_1 ends up with $T = 0$, x_4 ends up with $T = 1$, x_2 ends up with $T = 2$, x_5 ends up with $T = 3$, and x_3 ends up with $T = 4$.

As in Figure 1,

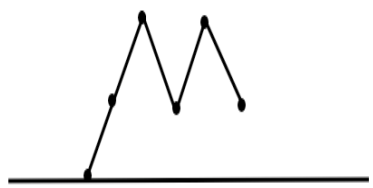
$$P(T = 0) = P(T = 1) = P(T = 2) = P(T = 3) = P(T = 4) = 1/5.$$

Therefore, no matter how we arrange these cards, we have $P(T = 0) = P(T = 1) = P(T = 2) = P(T = 3) = P(T = 4) = 1/5$, for $p = 3$ and $q = 2$.

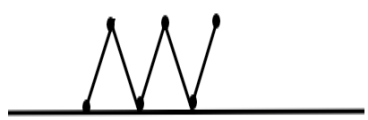
Lattice Paths. Another way of listing the sequences of cards is by using "Lattice Paths," and here, we use "Dyck paths." For example, sequences of (A, A, A, B, B) and (A, A, B, A, B) from the above example

can be displayed, respectively, as shown below, where an upward path stands for an “A” card, while a downward path stands for a “B” card.

Note that rotating sequences on a circle is the same as switching segments on a Lattice path. For example, rotating from



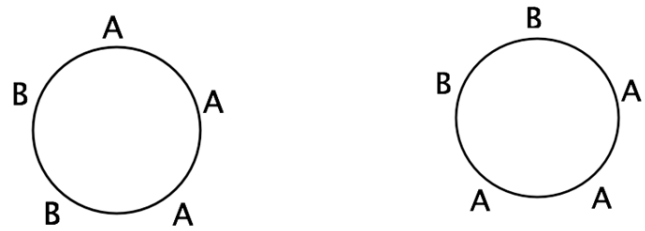
to



is the same as switching the first segment with the rest.

3. Proof of the “backwards counting”

Let p and q be two positive integers, where $p - q = 1$. Assume each possible sequence of $p + q$ cards is equally likely. Arrange a circle with a string of cards around, and allow each card along the cycle to be a possible first element in a sequence, where such a sequence runs clockwise. Define the following equivalence relation on all possible cycles: a cycle is approachable by another cycle by rotating the cards along the circle. For example, (A, A, A, B, B) is approachable by (A, A, B, B, A) simply by rotating the cycle, then these two cycles are equivalent, as shown below.



Therefore, each equivalence class of cycles is equally likely to occur. Note that by the "cycle" proof of the ballot theorem [1], we can always find the starting card "A" such that this "A" card leads to $T = 0$.

Since $p - q = 1$, any given sequence has a fixed starting position and ending position on the horizontal line of Lattice Paths, as shown in Figure 3 below.

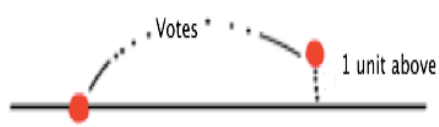


Figure 3

Note that on the graph of Lattice Paths, a "tie" occurs when a card is either on, or below, the horizontal line.

Assume a sequence is given, and mark each card in order as

$$x_1, x_2, \ldots, x_i, \ldots, x_{2n-1},$$

where x_i can be an "A" card or a "B" card, and n is the total number of cards with letter "A". Then the last card, x_{2n-1}, of any sequence can be one of two cases.

Case A: x_{2n-1} is a card "B," where "tie subsequences" are above the horizontal line.

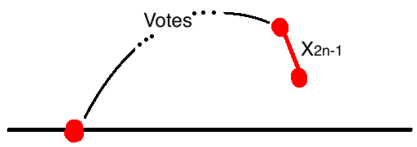


Figure 4

From this given sequence, in order to find the "tie subsequence" we start from the last card of this sequence in the form of Lattice Paths backwards. We end at the rightmost card x_i, such that x_i lands on the same horizontal line, which is 1 unit of a single segment above, as x_{2n-1} shown below. Since x_{2n-1} is a "B" card, x_i has to be an "A" card, shown in Figure 5 below.

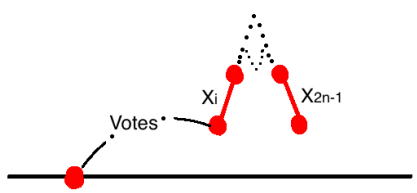


Figure 5

Note that the subsequence of $x_i, \ldots, x_{2n-1}$ is the "tie subsequence." (This is unique by the definition of "tie subsequence" defined in "Approach" above.) Since x_i is the first card that lands on the same level as x_{2n-1}, there is no card in the subsequence that would land on, or below, the horizontal line of the Lattice Paths. By taking the "tie subsequence" ahead of the current sequence, we have a new sequence as shown in Figure 6 below.

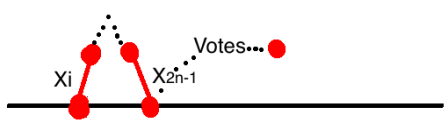


Figure 6

Therefore, if a given sequence falls into Case A, then the sequence starting at the "tie subsequence" would create one more "tie" than the given sequence.

Case B: x_{2n-1} is an "A" card, where "tie subsequences" are on, or below, the horizontal line.

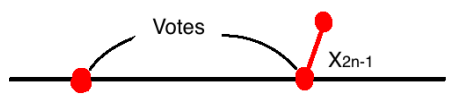


Figure 7

To find the "tie subsequence" from the current sequence, we count backwards from $x_{2n-}1$ until we find the first "B" card that lands on the same horizontal line as x_{2n-1}. The rightmost "B" card touches the horizontal line from above, as shown in Figure 8 below.

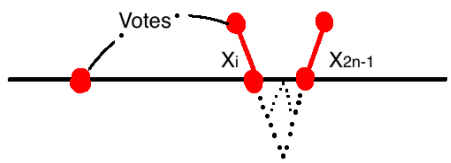


Figure 8

To create a new sequence, we place the "tie subsequence" ahead of the previous sequence. Then the sequence starting at the "tie subsequence" is shown in Figure 9 below.

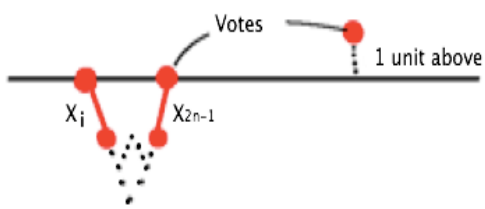


Figure 9

Since the "tie subsequence" back in the previous sequence in Figure 8 is one level up from the "tie subsequence" in Figure 9, the number of "ties" created from the "tie subsequence" in Figure 9 is one more than the "ties" contained in the same subsequence from Figure 8.

Note that the time that we can no longer find a "tie subsequence" is the time that, in a sequence, all cards besides the last card, x_{2n-1}, are either on, or below, the horizontal line of Lattice Paths. Since all cards before x_{2n-1} are not above the horizontal line, the last card, x_{2n-1}, is an "A" card, and there are $(p-1)+q$ cards in such a sequence which create "ties". Therefore, in such a sequence we have $T = 2 \cdot q$. Since we can always find

a sequence that contains $T = 0$ on any given "cycle" by proof of the ballot theorem [1], we can start the procedure of finding the "tie subsequence," previously introduced in "Approach," from the sequence that has $T = 0$ on a "cycle." Therefore, we can easily show that

$$P(T = 0) = P(T = 1) = \ldots = P(T = 2 \cdot q) = 1/(p + q).$$

Case II: $p = k \cdot q + 1$, where k is a positive integer

Let N equal the number of possible "i" where $1 \leq i \leq p + q$, such that the number of "A" cards seen up to the position i is greater than k times the number of "B" cards seen up to the position i. Let such a position i be called a "non-tie." Assume these cards are tallied in a random order. Then the distribution of N is

$$P(N = 1) = P(N = 2) = \ldots = P(N = p + q) = 1/(p + q).$$

Method: On any given sequence, we locate the position in a sequence such that the number of cards of "A" is equal to k times the number of "B" cards. Then we move this subsequence to the end of the sequence to create a new sequence. Let us call such a subsequence a "non-tie subsequence." Then we get a sequence where N is increased by 1. We keep doing the same procedure until there is no proper "non-tie subsequence." Note that at this point, we reach our maximum N, where $N = (p+q)$. If the original sequence which we started with, when we count backwards from the end, has no "non-tie subsequence," then the original sequence has $N = 1$; if not, then we take the minimum subsequence starting from the end of this sequence, which satisfies the condition and places this subsequence ahead of the given sequence to create a new sequence which decreases the value of N by 1.

$2'$. An Example

Let $p = 5$, q =2, and $k = 2$. On one of the possible cycles, sequence of (A, A, A, A, A, B, B) can be displayed as shown in Figure 10 below, where we mark each card from x_1 to x_7.

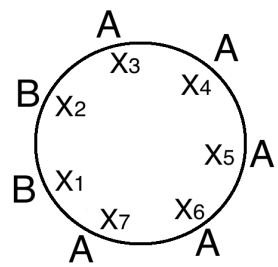


Figure 10

We can randomly pick a sequence, and in this example, let's take the sequence starting from x_1. Note that when sequence starts from x_1, we have $N = 1$. Then, we take the first subsequence from the beginning of the sequence, such that the number of "A" cards appeared equals 2 times the number of "B" cards appeared, to the end of the sequence. In the first subsequence, we have $(x_1, x_2, x_3, x_4, x_5, x_6)$, and after placing it to the end we have a new sequence starting from x_7. In the new sequence, $x_7, x_1, x_2, x_3, x_4, x_5, x_6$, we have $N = 2$. We repeat this procedure, and we can get the following:

$$\begin{aligned}
x_6, x_7, x_1, x_2, x_3, x_4, x_5 &\longrightarrow N = 3 \\
x_2, x_3, x_4, x_5, x_6, x_7, x_1 &\longrightarrow N = 4 \\
x_5, x_6, x_7, x_1, x_2, x_3, x_4 &\longrightarrow N = 5 \\
x_4, x_5, x_6, x_7, x_1, x_2, x_3 &\longrightarrow N = 6 \\
x_3, x_4, x_5, x_6, x_7, x_1, x_2 &\longrightarrow N = 7.
\end{aligned}$$

Why is the value of N increased by 1 by moving the minimum subsequence under such a condition from the beginning of a sequence?

3′. Proof

Assume each possible sequence of $p + q$ cards is equally likely, and arrange a possible sequence, clockwise, around a circle. On the cycle, allow each card along the circle to be a possible first element in a sequence, where such a sequence starts clockwise. Define an equivalence relation on all possible cycles, such that if a cycle is approachable by another cycle by rotating the cards along the circle, then these two cycles are equivalent. Thus, each equivalence class of cycles is equally likely. Let us mark each card of a possible sequence as

$$(x_1, x_2, \ldots, x_i, \ldots, x_{p+q}),$$

where $x_i = 1$ when x_i is an "A" card, and $x_i = -k$ when x_i is a "B" card. Assume there exists a W where W is a subsequence of a sequence

of cards, such that

$$W = (x_1, \ldots, x_n),$$

such that n is the minimum amount of cards such that $x_1 + x_2 + x_3 + \ldots + x_{n-1} + x_n = 0$.Let W^c be the subsequence after W in a sequence, such that $W^c = (x_{n+1}, \ldots, x_{p+q})$. Let $N(X) = \#\{i : x_1 + x_2 + \ldots + x_i > 0\}$, where $X = (x_1, x_2, \ldots, x_t)$ and $1 \leq i \leq t$.

By the definition of W, we have $x_1 + x_2 + \ldots + x_i > 0$ or $x_1 + x_2 + \ldots + x_i < 0$ at some x_i, where $1 \leq i \leq n-1$ in the subsequence W. Note that when we have $x_1 + x_2 + \ldots + x_i > 0$, where $1 \leq i \leq n-1$, then by adding a card "A" ahead of the subsequence W, where we denote xa as the card "A", we will have $x_a + x_1 + x_2 + \ldots + x_i \geq 1$, i.e. $x_a + x_1 + x_2 + \ldots + x_i > 0$. So, if we have a "non-tie" at the position of x_i, then by adding a card "A" ahead we will still have a "non-tie" at x_i. If we have $x_1 + x_2 + \ldots + x_i < 0$, where $1 \leq i \leq n-1$, then by adding a card of "A" ahead of the subsequence W, we will have $x_a + x_1 + x_2 + \ldots + x_i \leq 0$. So, if we don't have a "non-tie" at the position of x_i, then by adding a card "A" ahead will not create a "non-tie" at x_i. Note that $x_1 + x_2 + \ldots + x_n = 0$ and $x_{n+1} + x_{n+2} + \ldots + x_{p+q} = 1$, so we have

$$x_{n+1} + x_{n+2} + \ldots + x_{p+q} + x_1 + x_2 + \ldots + x_n = x_a + x_1 + x_2 + \ldots + x_n > 0,$$

which creates a "non-tie." Thus, $N(W^cW) - N(W^c) = N(W) + 1$.

Note that by switching W and W^c from a given sequence, we have a sequence of $W^cW = (x_{n+1}, \ldots, x_{p+q}, x_1, \ldots, x_n)$. Since

$$W = (x_1, x_2, \ldots, x_n : x_1 + x_2 + \ldots + x_n = 0),$$

the value of N in W^c, from WW^c to the sequence of W^cW stays the same. Since $W = (x_1, x_2, \ldots, x_n)$ and $x_1 + x_2 + \ldots + x_{n-1} + x_n = 0$, we have $N(WW^c) - N(W) = N(W^c)$. Note that $N(W^cW) - N(W^c) = N(W) + 1$ and $N(WW^c) - N(W) = N(W^c)$, and so it follows that $N(W^cW) = N(WW^c) + 1$. Therefore, by this "switching subsequence" method, each sequence with different value of N on a given "cycle" is equally likely.

Now, we prove the existences of $N = (p + q)$ and $N = 1$, i.e. we can reach to the maximum N and the minimum N from any given sequence.

We keep using this method of switching the subsequences of W and W^c in a sequence to increase the value of N by 1 at each time until we can no longer find any proper subsequence of W, and that is the time that we cannot find $x_1 + x_2 + \ldots + x_n = 0$ at any position of a sequence. Note that given $p = k \cdot q + 1$, so $x_1 + x_2 + \ldots + x_{p+q} = 1$ eventually, and if there exists a position x_m such that $x_1 + x_2 + \ldots + x_m < 0$, then there exists a position that $x_1 + x_2 + \ldots + x_n = 0$, where $m < n < p + q$, which is a contradiction. Thus, when we cannot find a subsequence of W, we have

$x_1 + x_2 + \ldots + x_n > 0$, for all n where $1 \leq n \leq p + q$. Since we have $(p+q)$ positions of any given sequence, we have the value of $N = (p+q)$ in a sequence when we can no longer find a subsequence of W.

Since we can increase the value of N by 1 by switching the subsequence W to the end of the sequence, we can also decrease the value of N by 1 by replacing the subsequence W back to the front of the sequence. Assume we are given a sequence as $(x_1, x_2, \ldots, x_{p+q})$, and let

$$W' = (x_i, \ldots, x_{p+q-1}, x_{p+q}),$$

such that i is the largest possible subscript in a sequence such that $x_i + x_{i+1} + \ldots + x_{p+q} = 0$. Note that the time we cannot find a subsequence of W' is the time we cannot find $x_i + x_{i+1} + \ldots + x_{p+q} = 0$, for all i where $1 \leq i \leq p + q$. Thus, from the inverse direction of a sequence, i.e. from x_{p+q} to x_1, we have $x_i + \ldots + x_{p+q} > 0$ throughout all positions. It follows that from the direction of x_1 to x_{p+q}, we have $x_1 + x_2 + \ldots + x_i \leq 0$, where $1 \leq i \leq p + q - 1$. Since $p = k \cdot q + 1$ is given and $x_1 + x_2 + \ldots + x_i \leq 0$, where $1 \leq i \leq p + q - 1$, we have the value of $N = 1$ in such a sequence.

Thus, in ballot problems where $p = k \cdot q + 1$ and k is a positive integer, we have each value of N equally likely, i.e. each value of N has the probability of $1/(p + q)$.

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An Algebraic Construction of the Zero Divisor Lattice of $\mathbb{Z}/n\mathbb{Z}$ and Observations Concerning Other Zero Divisor Lattices

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This paper was presented at the 2014 National Convention
and given one of the "Top 2 Awards" by the Awards Committee.

Abstract

This work outlines an algebraic manner of constructing the zero divisor lattice of $\mathbb{Z}/n\mathbb{Z}$. This construction method is markedly faster than the de nitional construction, and also illuminates the algebraic structure of the zero divisor lattice of $\mathbb{Z}/n\mathbb{Z}$, which suggests several characteristics of these and other zero divisor lattices, which are conjectured in this work.

1. Introduction

Zero divisor graphs of algebraic rings have yielded fruitful results in the past by bridging the two mathematical subfields of algebra and graph theory [1, 2]. Information about a graphical representation of a given ring can provide one with information about the structure of the ring itself, and *vice versa*; much of this information can be found in [3], which reviews much of the literature on these representations. Because $\mathbb{Z}/n\mathbb{Z}$ is a finite commutative ring with identity under residue class addition and multiplication for any fixed natural number n, these rings in particular have been the object of much investigation. One such graphical representation is the zero divisor lattice, which was first defined in [5].

This study investigated the zero divisor lattice of $\mathbb{Z}/n\mathbb{Z}$. Construction of the zero divisor lattice for $\mathbb{Z}/n\mathbb{Z}$ directly from the definition given in

[5] is relatively laborious to carry out in practice even for small n values, as it involves a large number of computations and comparisons, specically looking for set equalities and maximal proper set inclusions. This work presents an alternate, algebraic construction of the zero divisor lattice of $\mathbb{Z}/n\mathbb{Z}$ that is much quicker to carry out manually, and also establishes an algebraic structure on the graph itself. Finally, observational results comparing and contrasting the zero divisor lattices of $\mathbb{Z}/n\mathbb{Z}$ and $(\mathbb{Z}/n\mathbb{Z})\,[X]/P(X)$ [the quotient of the ring of polynomials with coefficients in $\mathbb{Z}/n\mathbb{Z}$ by an individual polynomial in said ring $P(X)$] will be presented, with some conjectures concerning their structure concluding these results.

2. Definitions and Notational Devices

1. Throughout this work, R will denote a finite commutative ring with identity.
2. The set $[a]_n = \{x \in \mathbb{Z} \mid a \equiv x (\bmod n)\} = \{x \in \mathbb{Z} \mid \exists k \in \mathbb{Z} a : a = x + kn\}$ is the **residue class of** x **modulo** n; usually, the subscript n will be understood from context, and hence omitted.
3. The family of sets $\mathbb{Z}/n\mathbb{Z} = \{[a]_n \mid a \in \mathbb{Z}\}$ is the **ring of integers modulo** n or, equivalently, the **quotient ring of** $\mathbb{Z}$ **modulo** $n\mathbb{Z}$.
4. An element $z \in R$ is a **zero divisor of** R if there exists $r \in R$ such that $r \neq 0_R$ and $z \cdot r = 0_R$; the set of all zero divisors of R is denoted $Z(R)$.
5. The **annihilator set of** $z \in R$ is given by ann$(z) = \{r \in R \mid z \cdot r = 0\}$; an element in ann$(z)$may be called an **annihilator of** z.
6. For all $x, y \in R$, we say x and y are **similar**, denoted $x \tilde{} y$, iff ann(x) =ann(y); similarity is clearly seen to be an equivalence relation on R (essentially due to the fact that set equality is reflexive, symmetric, and transitive), and so we will denote the equivalence classes by $\overline{z} = \{a \in R \mid a \tilde{} z\}$.
7. For all equivalence classes $\overline{x}$ and $\overline{y}$ in R modulo similarity, we say $\overline{x}$ is **below** $\overline{y}$, denoted $\overline{x} \preceq \overline{y}$, if and only if ann$(x) \subseteq$ ann(y); below is then clearly seen to be a partial ordering on R modulo similarity (since subset inclusion is a partial ordering on any set, and ann$(z) \subseteq R$ for any $z \in R$). We also define $\overline{x}$ to be **strictly below** $\overline{y}$, denoted $\overline{x} \prec \overline{y}$, if and only if ann$(x) \subsetneq$ ann(y).

8. The **zero divisor lattice of** R, denoted $\Lambda(R)$, is the graph given by $V[\Lambda(R)] = \{\overline{x} \mid \overline{x} \in R/\tilde{}\}$ and $E[\Lambda(R)] = \{\overline{y} \to \overline{x} \mid \overline{x} \prec \overline{y}$ and $\nexists \overline{\imath} : \overline{x} \prec \overline{\imath} \prec \overline{y}\}$. This definition departs slightly from the original definition given in [5], but is a more natural one for this study.

9. An element $u \in R$ is a **unit** of R if there exists $v \in R$ such that $u \cdot v = 1_R$; the set of all units of R is denoted $U(R)$.

10. For all $x, y \in R$, x and y are **associates** if there exists a unit $u \in U(R)$ such that $x = uy$.

3. Conventions

In order to facilitate communication of the results, the following conventions will be adopted unless specifically stated otherwise. Throughout this work, n represents a positive integer greater than 1. In $\mathbb{Z}/n\mathbb{Z}$, each residue class modulo n will usually be represented by its smallest positive member; that is, $\forall [a] \in \mathbb{Z}/n\mathbb{Z}$, we will usually require that $0 < a \leq n$. In particular, we will denote the additive identity by $[60]$ rather than the typical $[0]$, except possibly when computing products of residue classes; the reasoning behind the former will be made clear later in this work, and the latter simply allows for the identi cation of zero divisors.

4. Example: $\mathbb{Z}/60\mathbb{Z}$

Using the given definitions, by "brute force" computation we find the following; note that each bullet point groups together elements within a single unique equivalence class of $\mathbb{Z}/60\mathbb{Z}$ modulo similarity:

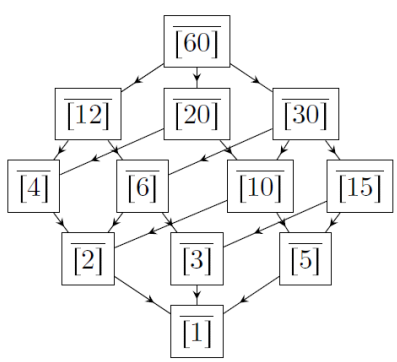
- Each of $[1]$, $[7]$, $[11]$, $[13]$, $[17]$, $[19]$, $[23]$, $[29]$, $[31]$, $[37]$, $[41]$, $[43]$, $[47]$, $[49]$, $[53]$, and $[59]$ has annihilator set equal to $\{[60]\} = \left\langle \left[\frac{60}{1}\right] \right\rangle$.
- Each of $[2]$, $[14]$, $[22]$, $[26]$, $[34]$, $[38]$, $[46]$, and $[58]$ has annihilator set equal to $\{[30], [60]\} = \left\langle \left[\frac{60}{2}\right] \right\rangle$.
- Each of $[3]$, $[9]$, $[21]$, $[27]$, $[33]$, $[39]$, $[51]$, and $[57]$ has annihilator set equal to $\{[20], [40], [60]\} = \left\langle \left[\frac{60}{3}\right] \right\rangle$.
- Each of $[5]$, $[25]$, $[35]$, and $[55]$ has annihilator set equal to

$$\{[12], [24], [36], [48], [60]\} = \left\langle \left[\frac{60}{5}\right] \right\rangle.$$

- Each of $[6]$, $[18]$, $[42]$, and $[54]$ has annihilator set equal to
$$\{[10], [20], [30], ..., [60]\} = \left\langle \left[\frac{60}{6}\right] \right\rangle.$$
- Both $[10]$ and $[50]$ have annihilator set equal to
$$\{[6], [12], [18], ..., [60]\} = \left\langle \left[\frac{60}{10}\right] \right\rangle.$$
- Each of $[12]$, $[24]$, $[36]$, and $[48]$ has annihilator set equal to
$$\{[5], [10], [15], ..., [60]\} = \left\langle \left[\frac{60}{12}\right] \right\rangle.$$
- Both $[15]$ and $[45]$ have annihilator set equal to
$$\{[4], [8], [12], ..., [60]\} = \left\langle \left[\frac{60}{15}\right] \right\rangle.$$
- Both $[20]$ and $[40]$ have annihilator set equal to
$$\{[6], [12], [18], ..., [60]\} = \left\langle \left[\frac{60}{10}\right] \right\rangle.$$
- $\text{ann}([30]) = \{[2], [4], [6], ..., [60]\} = \left\langle \left[\frac{60}{30}\right] \right\rangle.$
- $\text{ann}([60]) = \mathbb{Z}/60\mathbb{Z} = \{[1], [2], ..., [60]\} = \left\langle \left[\frac{60}{60}\right] \right\rangle.$

Then, by direct annihilator set equality and inclusion comparisons, we construct $\Lambda(\mathbb{Z}/60\mathbb{Z})$:

$\Lambda(\mathbb{Z}/60\mathbb{Z})$



Example 1: The Zero Divisor Lattice of $\mathbb{Z}/60\mathbb{Z}$

As has been shown, the computation needed to construct $\Lambda(\mathbb{Z}/n\mathbb{Z})$ directly from its definition can be quite tedious, even for small n. As we present the faster algebraic construction method discovered in this work, we invite the reader to apply each result to the given example of $\mathbb{Z}/60\mathbb{Z}$.

5. Results

The following work will culminate in the main result concerning the construction of $\Lambda(\mathbb{Z}/n\mathbb{Z})$.

Proposition 1 *For all* $[a] \in \mathbb{Z}/n\mathbb{Z}$*, if* $x \in \mathbb{Z}$ *such that* $x \in [a]$*, then* $\gcd(a, n) = \gcd(x, n)$.

Proof. Let $[a] \in \mathbb{Z}/n\mathbb{Z}$. Let $x \in \mathbb{Z}$ such that $x \in [a]$. Since $x \in [a]$, $\exists m \in \mathbb{Z}$ such that $x = a + mn$. By Bezout's Identity, $\exists b, c, y, z \in \mathbb{Z}$ such that $ab + nc = \gcd(a, n)$ and $xy + nz = \gcd(x, n)$. Then, since $x = a + mn$, $(a + mn)y + (nz) = ay + n(my + z) = \gcd(x, n)$. Furthermore, since $\gcd(a, n)$ divides both a and n, $\gcd(x, n) = ay+n(my+z) = \gcd(a, n)\cdot ky+\gcd(a, n)\cdot l(my+z) = \gcd(a, n)\cdot(kny+l(my+z))$ for some $k, l \in \mathbb{Z}$; that is, $\gcd(a, n)$ divides $\gcd(x, n)$. Since $a = x - mn = x + (\text{-}m)n$, $a \in [x]$. Then the previous argument with a and x interchanged shows $\gcd(x, n)$ divides $\gcd(a, n)$. Then $\gcd(a, n)$ and $\gcd(x, n)$ are positive divisors of one another. Therefore, $\gcd(a, n) = \gcd(x, n)$. ■

Proposition 1 ensures that for all $[a] \in \mathbb{Z}/n\mathbb{Z}$, the value of $\gcd(a, n)$ is independent of the chosen representative a.

Proposition 2 *For all* $[a] \in \mathbb{Z}/n\mathbb{Z}$*,* $[a]$ *is a zero divisor if and only if* $\gcd(a, n) > 1$.

Proof.

($\Rightarrow$) Let $[a] \in Z\big(\mathbb{Z}/n\mathbb{Z}\big)$, and suppose to the contrary that $\gcd(a, n) = 1$. Then $\exists[x] \in \mathbb{Z}/n\mathbb{Z}$ such that $[x] \neq [0]$ and $[a][x] = [ax] = [0]$. If $[ax] = [0]$, then $ax \equiv_n 0$. That is, $\exists k \in \mathbb{Z}$ such that $ax = kn$. By Bezout's Identity, $\gcd(a, n) = 1$ implies that $\exists b, p \in \mathbb{Z}$ such that $ab+np = 1$. Then $ab = 1 - pn = 1 + (\text{-}p)n$. By definition of $[ab] \in \mathbb{Z}/n\mathbb{Z}$, $[ab] = [1]$. Then $[x][ab] = [x][1] = [x]$ since $[1]$ is the multiplicative identity of $\mathbb{Z}/n\mathbb{Z}$. Then $[x] = [x][ab] = [ax][b] = [0][b] = [0]$, a contradiction. Therefore, $[a] \in Z\big(\mathbb{Z}/n\mathbb{Z}\big)$ implies $\gcd(a, n) = 1$.

($\Leftarrow$) Let $a \in \mathbb{Z}$ such that $\gcd(a, n) = k > 1$, where $k \in \mathbb{N}$. Since k divides both a and n, $\exists b \in \mathbb{Z}$ and $m \in \mathbb{N}$ such that $bk = a$ and $mk = n$. Since $k > 1$ and $n > 1$, $0 < m < n$. Then, since $m \neq 0$, $bk = a$ implies

$bkm = am$. Then $bmk = am$, and thus $bn = am$; that is, $am \equiv_n 0$. That is, $[a][m] = [am] = [0]$. Since $0 < m < n$, $[m] \neq [0]$. Therefore, $[a] \in Z\big(\mathbb{Z}/n\mathbb{Z}\big)$. ■

From Proposition 2, $Z\big(\mathbb{Z}/n\mathbb{Z}\big)$ can be generated relatively quickly; each integer $0 < m \leq n$ that is not coprime to n determines a unique zero divisor $[m]$. Referring back to our example, we see that this is in fact the case; in particular, if $\gcd(m, 60) = d > 1$, then $[60] \neq \left[\frac{60}{d}\right]$ and $\left[\frac{60}{d}\right] \in \operatorname{ann}([m])$.

Since $\mathbb{Z}/n\mathbb{Z}$ is a finite commutative ring with identity and R, $R = Z(R) \bigsqcup U(R)$ for any finite commutative ring with identity, it is clear that $[a] \in U(\mathbb{Z}/n\mathbb{Z})$ if and only if $\gcd(a, n) = 1$. We include the proof here for both completeness and to emphasize the importance of the fact that $\mathbb{Z}$ is a Euclidean Domain, and in particular the importance of the extended Euclidean Algorithm, Bezout's Identity, and the computatbility of the greatest common divisor of two integers.

Proposition 3 *For all* $[a] \in \mathbb{Z}/n\mathbb{Z}$, $[a]$ *is a unit if and only if* $\gcd(a, n) = 1$.

Proof.

($\Rightarrow$) Let $[a] \in U(\mathbb{Z}/n\mathbb{Z})$. Then $\exists [b] \in \mathbb{Z}/n\mathbb{Z}$ such that $[a][b] = [ab] = [1]$. Then $ab \equiv_n 1$; that is, $\exists k \in \mathbb{Z}$ such that $ab - 1 = kn$. Then $ab +$ (-k)$n = 1$. Therefore, since b, -$k \in \mathbb{Z}$, $\gcd(a, n) = 1$ by Bezout's Identity.

($\Leftarrow$) Let $[a] \in \mathbb{Z}/n\mathbb{Z}$ such that $\gcd(a, n) = 1$. By Bezout's Identity, $\exists b, p \in \mathbb{Z}$ such that $ab + np = 1$. Then $ab - 1 =$ (-p)n; that is, by definition of $[ab] \in \mathbb{Z}/n\mathbb{Z}$, $[ab] = [1]$. Then $[a][b] = [ab] = [1]$.Therefore, $[a] \in U(\mathbb{Z}/n\mathbb{Z})$. ■

Again, this proposition is exhibited in our example since each of the elements $[r]$ such that $\operatorname{ann}([r]) = \{[60]\}$ satisfy $\gcd(r, 60) = 1$, and *vice versa*. In fact, given any ring $\mathcal{R}$, $\operatorname{ann}(u) = \{0_{\mathcal{R}}\}$ for any $u \in U(\mathcal{R})$ since units can never be zero divisors.

Proposition 4 *For all* $[x], [y] \in \mathbb{Z}/n\mathbb{Z}$, $[x]$ *and* $[y]$ *have the same annihilator set* (*and thus* $[x]$ *is similar to* $[y]$) *if and only if* $\gcd(x, n) = \gcd(y, n)$.

Proof.

($\Rightarrow$) Let $[x], [y] \in \mathbb{Z}/n\mathbb{Z}$ such that $\operatorname{ann}([x]) = \operatorname{ann}([y])$. There are then two cases to consider.

Case 1: Assume $\operatorname{ann}([x]) = \{[0]\} = \operatorname{ann}([y])$. By definition, $[x], [y] \notin Z(\mathbb{Z}/n\mathbb{Z})$. Then, by the converse of Proposition 2, $\gcd(x, n) = 1 = \gcd(y, n)$.

Case 2: Assume $\mathrm{ann}([x]) = \mathrm{ann}([y]) \supsetneq \{[0]\}$. Then by definition, $[x], [y] \in Z(\mathbb{Z}/n\mathbb{Z})$. By Proposition 2, $\exists d, f \in \mathbb{N}$ such that $d, f > 1$, $\gcd(x, n) = d$, and $\gcd(y, n) = f$. Since d divides x, $\exists k \in \mathbb{Z}$ such that $x = kd$. Since d divides n, $\frac{n}{d} \in \mathbb{N}$. Furthermore, $\frac{n}{d}x = \frac{n}{d}kd = nk$; that is, $[\frac{n}{d}][x] = [0]$, which implies $[\frac{n}{d}] \in \mathrm{ann}([x])$. Then, since $\mathrm{ann}([x]) = \mathrm{ann}([y])$, $[\frac{n}{d}] \in \mathrm{ann}([y])$. Then $[\frac{n}{d}][y] = [0]$, which implies that $\exists m \in \mathbb{Z}$ such that $\frac{n}{d}y = mn$. Then $y = md$, and thus d divides y. Since d divides both y and n, d is a positive common divisor of y and n, and thus $\gcd(y, n) = f \geq d$. A similar argument shows $\gcd(x, n) = d \geq f$. Then we must have $\gcd(x, n) = \gcd(y, n)$. Therefore, in either case, $\mathrm{ann}([x]) = \mathrm{ann}([y])$ implies $\gcd(x, n) = \gcd(y, n)$.

($\Leftarrow$) Let $x, y \in \mathbb{Z}$ such that $\gcd(x, n) = \gcd(y, n)$. By Proposition 1, we may also assume that $x, y \in \mathbb{Z}$ such that $0 \leq x, y < n$. Let $d \in \mathbb{N}$ such that $\gcd(x, n) = d = \gcd(y, n)$. We will show that $\mathrm{ann}([x]) = \mathrm{ann}([d]) = \mathrm{ann}([y])$. Let $[z] \in \mathrm{ann}([x])$. Then $[z][x] = [zx] = [0]$. Thus, $\exists k \in \mathbb{N}$ such that $zx = kn$. Since $\gcd(x, n) = d$, by Bezout's Identity, $\exists w, m \in \mathbb{Z}$ such that $xw + mn = d$. Then $zxw + zmn = zd$. Since $zx = kn$, $zxw + zmn = zd$ implies $knw + zmn = zd$. Then $n(kw+zm) = zd$, and thus $[z][d] = [0]$; that is, $[z] \in \mathrm{ann}([d])$. Therefore, $\mathrm{ann}([x]) \subseteq \mathrm{ann}([d])$. Let $[a] \in \mathrm{ann}([d])$. Since $d = \gcd(x, n)$, d divides x. Then $\exists c \in \mathbb{Z}$ such that $x = dc$. Then $[a][x] = [a][dc] = [a][d][c] = [0][c] = [0]$; that is, $[a] \in \mathrm{ann}([x])$. Thus, $\mathrm{ann}([d]) \subseteq \mathrm{ann}([x])$. Since $\mathrm{ann}([x]) \subseteq \mathrm{ann}([d])$ and $\mathrm{ann}([d]) \subseteq \mathrm{ann}([x])$, $\mathrm{ann}([x]) = \mathrm{ann}([d])$. By similar argument, $\mathrm{ann}([y]) = \mathrm{ann}([d])$. Therefore, $\mathrm{ann}([x]) = \mathrm{ann}([y])$. ■

Proposition 4 is readily seen in our example; in fact, since the residue classes of each equivalence class of $\mathbb{Z}/60\mathbb{Z}$ are listed together within the same bullet point, it is relatively quick to check that for each $[a], [b]$ such that $\mathrm{ann}([a]) = \mathrm{ann}([b])$, $\gcd(a, 60) = \gcd(b, 60)$, and *vice versa*.

Corollary 5 *The positive divisors of n and the equivalence classes of $\mathbb{Z}/n\mathbb{Z}$ modulo similarity are in bijective correspondence; if d divides n, then $\overline{[d]}$ is an equivalence class modulo similarity, the residue classes modulo n of distinct divisors are in distinct equivalence classes, and each equivalence class contains the residue class modulo n of some divisor of n. Additionally, if d divides n, then d is the minimal positive integer m such that $\gcd(m, n) = d$, and thus $[d]$ is the minimal representative of $\overline{[d]}$ in that for all $[a] \in \overline{[d]}$, $d \leq a$. Furthermore, given a divisor d of n, it is then clear that $\mathrm{ann}([d]) = \left\langle \left[\frac{n}{d}\right] \right\rangle$, and thus $\mathrm{ann}([a]) = \left\langle \left[\frac{n}{\gcd(a,n)}\right] \right\rangle$ for any $[a] \in \mathbb{Z}/n\mathbb{Z}$.*

Corollary 5 enables the expedient generation of $V[\Lambda(\mathbb{Z}/n\mathbb{Z})]$; simply compute the positive divisors of n, and then take the equivalence classes with a divisor as the reprentative of the representative residue class. Moreover, these equivalence classes will be convieniently represented; in following with our convention, Corollary 5 shows that representing $\overline{[a]}$ minimally is equivalent to assuming a divides n. We shall adopt this as a convention for the remainder of the work. In our example, we simply take the first listed residue class of each bullet point (which groups together one equivalence class of $\mathbb{Z}/60\mathbb{Z}$ modulo similarity) as the representative of the equivalence class; we have indicated $\mathrm{ann}([a]) = \left\langle \left[\frac{60}{\gcd(a,60)}\right] \right\rangle$ for all $[a] \in \mathbb{Z}/n\mathbb{Z}$.

Proposition 6 *In $\mathbb{Z}/n\mathbb{Z}$ modulo similarity, $\overline{[x]}$ is below $\overline{[y]}$ if and only if x is a divisor of y.*

Proof.

($\Rightarrow$) Suppose $\overline{[x]}$ is below $\overline{[y]}$. Then $\mathrm{ann}([x]) \subseteq \mathrm{ann}([y])$. By our convention, x is a positive divisor of n. Then $\exists k \in \mathbb{N}$ such that $x = \dfrac{n}{k}$. Then, by definition of congruence modulo n, $[0] = [kx] = [k][x]$; that is, $k \in \mathrm{ann}([x])$. Then, since $\mathrm{ann}([x]) \subseteq \mathrm{ann}([y])$, $[k] \in \mathrm{ann}([y])$, and thus $[k][y] = [ky] = [0]$; that is, $ky = mn$ for some $m \in \mathbb{N}$. Then $y = m\dfrac{n}{k} = mx$. Therefore, x divides y.

($\Leftarrow$) Suppose x divides y. Then $y = dx$ for some $d \in \mathbb{N}$. Since $[0] \in \mathrm{ann}([x])$, $\mathrm{ann}([x])$ is not empty. Suppose $[a] \in \mathrm{ann}([x])$. Then $[0] = [d][0] = [d][ax] = [dax] = [adx] = [a][dx] = [a][y]$, and thus $[a] \in \mathrm{ann}([y])$. Then $\mathrm{ann}([x]) \subseteq \mathrm{ann}([y])$. Therefore, $\overline{[x]}$ is below $\overline{[y]}$. ■

Proposition 6 can be applied to our example, as can be verified by a case by case analysis.

Corollary 7 *In $\mathbb{Z}/n\mathbb{Z}$ modulo similarity, $\overline{[x]} \prec \overline{[y]}$ and there does not exists $\overline{[z]}$ such that $\overline{[x]} \prec \overline{[z]} \prec \overline{[y]}$ if and only if there exists a prime $p \in \mathbb{N}$ such that $y = px$; that is, x must be a maximal proper divisor of y. Graphically, $\overline{[y]} \to \overline{[x]} \in E[\Lambda(\mathbb{Z}/n\mathbb{Z})]$ if and only if there exists a prime $p \in \mathbb{N}$ such that $y = px$.*

By Proposition 6, if $y = px$, then x is a proper divisor of y, and thus $\overline{[x]} \prec \overline{[y]}$. If $y = mx$, where m is composite, then $m = pr$ for some prime p and some $r > 1$, and thus $\overline{[x]} \prec \overline{[px]} \prec \overline{[mx]} = \overline{[y]}$ by Proposition 6; that is, $\overline{[y]} \not\to \overline{[x]}$.

Corollaries 5 and 7 allow one to quickly generate $V[\Lambda(\mathbb{Z}/n\mathbb{Z})]$ and $E[\Lambda(\mathbb{Z}/n\mathbb{Z})]$ respectively, and thus construct $\Lambda(\mathbb{Z}/n\mathbb{Z})$. Referring back to our example, Corollaries 5 and 7 allow one to construct $\Lambda(\mathbb{Z}/60\mathbb{Z})$ as follows.

The equivalence class $\overline{[60]}$ is the maximal element in $\Lambda(\mathbb{Z}/60\mathbb{Z})$ since $\mathrm{ann}([60]) = \{[1],[2],\ldots,[60]\} = \mathbb{Z}/60\mathbb{Z} \supsetneq \mathrm{ann}([d])$ for any proper divisor d of 60. Corollary 7 shows that dividing 60 by each of its distinct prime factors $2, 3$, and 5 yields vertices $\overline{\left[\frac{60}{2}\right]} = \overline{[30]}, \overline{\left[\frac{60}{3}\right]} = \overline{[20]}$, and $\overline{\left[\frac{60}{5}\right]} = \overline{[12]}$ such that $\overline{[30]} \to \overline{[60]}, \overline{[20]} \to \overline{[60]}$, and $\overline{[12]} \to \overline{[60]}$. For the vertex $\overline{[20]}$, say, dividing 20 by each of its distinct prime divisors 2 and 5 similarly gives $\overline{[10]} \to \overline{[20]}$ and $\overline{[4]} \to \overline{[20]}$. Continuing similarly "downwards" (relative to divisibility) through each of the divisors of 60 gives $\Lambda(\mathbb{Z}/60\mathbb{Z})$; in general, Corollary 7 and this construction method are why we choose to represent $[0]$ by $[n]$.

Most importantly, this method is much faster to carry out than the annihilator set calculations and comparisons necessitated by the definition of $\Lambda(\mathbb{Z}/n\mathbb{Z})$ and elucidates some of the properties of $\Lambda(\mathbb{Z}/n\mathbb{Z})$. By Corollaries 5 and 7, $\Lambda(\mathbb{Z}/n\mathbb{Z})$ is completely determined once the prime factorization of n is known; in fact, we have the following result.

Corollary 8 *The zero divisor lattice of $\mathbb{Z}/n\mathbb{Z}$ is (lattice) isomorphic to the divisor lattice of n, the set of all positive divisors of n partially ordered by the relation divides.*

Corollary 8 essentially allows one to replace the computationally intensive task of computing and comparing annihilator sets in $\mathbb{Z}/n\mathbb{Z}$ with the much easier construction of the divisor lattice of n. Direct computation shows that our example illustrates Corollary 8. In fact, Corollary 8 is the motivation behind several of the forthcoming conjectures concerning the general structure of $\Lambda(\mathbb{Z}/n\mathbb{Z})$.

Proposition 9 *For all $[x],[y] \in \mathbb{Z}/n\mathbb{Z}, [x]$ and $[y]$ are associates if and only if $\gcd(x,n) = \gcd(y,n)$.*

Proof.

($\Rightarrow$) Let $[x],[y] \in \mathbb{Z}/n\mathbb{Z}$ such that $[x]$ and $[y]$ are associates. Then $\exists [u] \in U(\mathbb{Z}/n\mathbb{Z})$ such that $[x] = [u][y] = [uy]$. Thus, by Proposition 3, $\gcd(x,n) = \gcd(uy,n)$. Since $[u] \in U(\mathbb{Z}/n\mathbb{Z})$, $\gcd(u,n) = 1$ by Proposition 2. Now, suppose $f \in \mathbb{N}$ such that f divides y. Then, clearly, f divides uy. Next, suppose $d \in \mathbb{N}$ such that d divides uy and d divides n. Then $\exists j,k \in \mathbb{N}$ such that $uy = jd$ and $n = kd$. Since

$\gcd(u,n)=1$, by Bezout's Identity $\exists a,b\in\mathbb{Z}$ such that $1=au+bn$. Then $y=auy+bny=ajd+bkdy=d(aj+bky)$; that is, d divides y. Specifically, we have now shown that $\forall m\in\mathbb{N}$, m is a common divisor of y and n if and only if m is a common divisor of uy and n. Thus, we must have $\gcd(y,n)=\gcd(uy,n)$. Therefore, $\gcd(x,n)=\gcd(uy,n)=\gcd(y,n)$.

($\Leftarrow$) Let $[x],[y]\in\mathbb{Z}/n\mathbb{Z}$ and $d\in\mathbb{N}$ such that $\gcd(x,n)=d=\gcd(y,n)$. Then $\exists u,v\in\mathbb{N}$ such that $x=du$ and $y=dv$. Since $\gcd(x,n)=d=\gcd(y,n)$, we must have $\gcd(u,n)=1=\gcd(v,n)$. Thus, by Proposition 2, $[u],[v]\in U(\mathbb{Z}/n\mathbb{Z})$. Then $[x][v]=[xv]=[duv]=[dvu]=[dv][u]=[y][u]$; since $[v]\in U(\mathbb{Z}/n\mathbb{Z})$, $\exists[b]\in\mathbb{Z}/n\mathbb{Z}$ such that $[v][b]=[v\nu]=[1]$. Thus, $[x]=[1\cdot x]=[1][x]=[b][v][x]=[b][u][y]=[bu][y]$. Since $[u]\in U(\mathbb{Z}/n\mathbb{Z})$, $\exists[a]\in\mathbb{Z}/n\mathbb{Z}$ such that $[u][a]=[ua]=1$. Then, since $[a][v]=[av]\in\mathbb{Z}/n\mathbb{Z}$ satisfies $[av][ub]=[avub]=[auvb]=[ua][vb]=[1]\cdot[1]=[1]$, $[bu]\in U(\mathbb{Z}/n\mathbb{Z})$. Therefore, since $[x]=[y][bu]$ and $[bu]\in U(\mathbb{Z}/n\mathbb{Z})$, $[x]$ and $[y]$ are associates. ■

Corollary 10 *For all $[x],[y]\in\mathbb{Z}/n\mathbb{Z}$, $[x]\sim[y]$ if and only if $[x]$ and $[y]$ are associates.*

Corollary 10 follows directly from Propositions 4, and 9; if $[a]$ is similar to $[b]$, then $\gcd(a,n)=\gcd(b,n)$ by Proposition 4, and thus $[a]$ and $[b]$ are associates by Proposition 9, with the same argument in reverse completing the proof. Essentially, Corollary 10 gives an alternate way in which to generate the equivalence classes of $\mathbb{Z}/n\mathbb{Z}$ modulo similarity; first, compute $U(\mathbb{Z}/n\mathbb{Z})=\overline{[1]}$ using Proposition 3. Set $[a_1]=[1]$. Then, inductively pick $[a_i]\notin\bigsqcup_{j=1}^{i-1}\overline{[a_j]}$ and form the set $[a_i]U(\mathbb{Z}/n\mathbb{Z})=\big\{[a_i][u]\mid[u]\in U(\mathbb{Z}/n\mathbb{Z})\big\}$. Then, by Corollary 10, $[a_i]U(\mathbb{Z}/n\mathbb{Z}=\overline{[a_i]}$. As previously, this can be checked directly in our example of $\mathbb{Z}/60\mathbb{Z}$.

In practice, this is an inefficient method to compute the equivalence classes of $\mathbb{Z}/n\mathbb{Z}$ modulo similarity as opposed to Corollary 5. However, as will be shown later in this work, it is thought that Corollary 10 may extend to certain instances of $\big(\mathbb{Z}/n\mathbb{Z}\big)[X]\big/P(X)$; it is important to note that the computation of $U\Big(\big(\mathbb{Z}/n\mathbb{Z}\big)[X]\big/P(X)\Big)$ is (seemingly) not as simple as in the case of $U(\mathbb{Z}/n\mathbb{Z})$.

6. Conjectures Concerning the Graphical Representations of $\mathbb{Z}/n\mathbb{Z}$, Finite Quotients of Euclidean Domains, and $(\mathbb{Z}/n\mathbb{Z})\,[X]/P(X)$

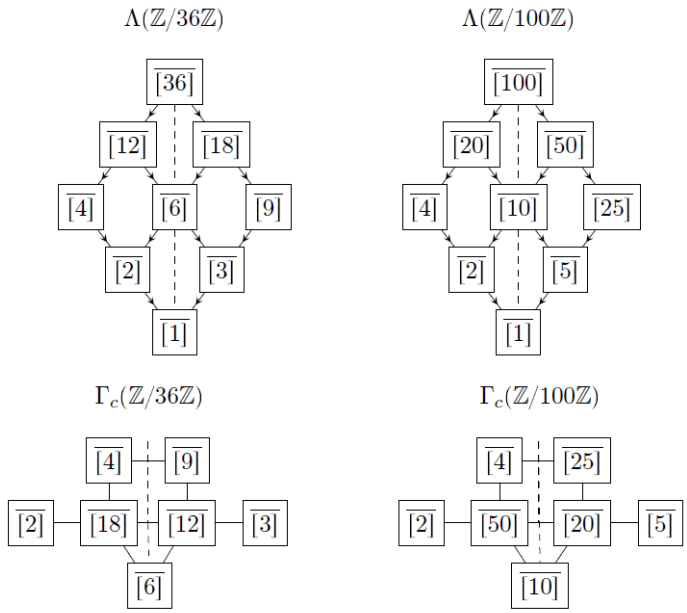
Based on this new construction method, and particularly both the algebraic structure of $\Lambda(\mathbb{Z}/n\mathbb{Z})$ that is illuminated by Corollary 8 and the role of associatedness implied by Corollary 10, the following conjectures seem likely but have not been proven by this work.

Conjecture 1 *Two zero divisor lattices $\Lambda(\mathbb{Z}/m\mathbb{Z})$ and $\Lambda(\mathbb{Z}/n\mathbb{Z})$ are lattice isomorphic if and only if there exists $k \in \mathbb{N}$ such that for each $1 \leq i \leq k$, there exist primes p_i and q_i such that the p_i are distinct from one another and the q_i are distinct from one another, and there exists* ***a single power*** *$e_i \in \mathbb{N}$ for each i such that $m = \prod_{i=1}^{k} p_i^{e_i}$ and $n = \prod_{i=1}^{k} q_i^{e_i}$; moreover, all such isomorphisms $\phi : \Lambda(\mathbb{Z}/m\mathbb{Z}) \to \Lambda(\mathbb{Z}/n\mathbb{Z})$ are of the form $\phi\Big(\overline{\big[\prod_{i=1}^{k} p_i^{a_i}\big]}\Big) = \overline{\big[\prod_{i=1}^{k} q_i^{a_i}\big]}$.*

In short, Conjecture 1 says that $\mathbb{Z}/m\mathbb{Z}$ and $\mathbb{Z}/n\mathbb{Z}$ have isomorphic zero divisor lattices if and only if the prime factorizations of m and n are "isomorphic" in the sense that m and n have the same number of distinct prime factors (corresponding to the $k \in \mathbb{N}$) and the prime factorizations of m and n can be arranged so that for all distinct prime factors of m, the power of a given prime factor of m is equal to the power of the corresponding prime factor of n (that corresponding power being the $e_i \in \mathbb{N}$); moreover, each such possible arrangement yields an isomorphism.

In fact, this same logic is conjectured to apply equally well to another graphical representation of these rings, the **compressed zero divisor graph** of $\mathbb{Z}/n\mathbb{Z}$, denoted $\Gamma_c(\mathbb{Z}/n\mathbb{Z})$, which, for a commutative ring with identity $\mathcal{R}$, is the graph with vertices given by the equivalence classes of $\mathcal{R}$ modulo similarity excluding $\overline{0_{\mathcal{R}}} = 0_{\mathcal{R}}$ and $\overline{1_{\mathcal{R}}} = U(\mathcal{R})$ and an (undirected) edge connecting two vertices $\overline{a}$ and $\overline{b}$ if and only if $ab = 0_{\mathcal{R}}$, as defined in [2].

The following page illustrates Conjecture 1 for both zero divisor lattices and compressed zero divisor graphs.



Note that $36 = 2^2 \cdot 3^2$ and $100 = 2^2 \cdot 5^2$

Figure 1: Graphical Zero Divisor Isomorphism for $\mathbb{Z}/36\mathbb{Z}$ and $\mathbb{Z}/100\mathbb{Z}$

As in Conjecture 1, we identify that $m = 36$ and $n = 100$ have $k = 2$ distinct prime factors; for $m = 36$, $p_1 = 2$ and $p_2 = 3$. For $n = 100$, $q_1 = 2$ and $q_2 = 5$; as required, $e_1 = 2$ and $e_2 = 2$. These identifications lead to an explicit isomorphism between $\Lambda(\mathbb{Z}/36\mathbb{Z})$ and $\Lambda(\mathbb{Z}/100\mathbb{Z})$; the map $\overline{[2^a \cdot 3^b]} \mapsto \overline{[2^a \cdot 5^b]}$ is clearly a lattice isomorphism. The analogous mapping $\overline{[2^a \cdot 3^b]} \mapsto \overline{[2^a \cdot 5^b]}$ between $\Gamma_c(\mathbb{Z}/36\mathbb{Z})$ and $\Gamma_c(\mathbb{Z}/100\mathbb{Z})$ is also clearly a graph isomorphism. In the figure above, these particular isomorphisms map a given vertex in $\Lambda(\mathbb{Z}/36\mathbb{Z})$ [$\Gamma_c(\mathbb{Z}/36\mathbb{Z})$] to the vertex in the same relative location in $\Lambda(\mathbb{Z}/100\mathbb{Z})$ [$\Gamma_c(\mathbb{Z}/100\mathbb{Z})$, *e.g.* $\overline{[2]}$ is mapped to $\overline{[2]}$, $\overline{[6]}$ is mapped to $\overline{[6]}$, *etc*. In this particular instance, because $e_1 = 2 = e_2$, by switching our identifications to $q_1 = 5$ and $q_2 = 2$, Conjecture 1 implies that the mapping $\overline{[2^a \cdot 3^b]} \mapsto \overline{[5^a \cdot 2^b]}$ is also an isomorphism for both the lattices and the compressed zero divisor graphs, which manual computation verifies; in this particular depiction, this isomorphism maps a given vertex in $\Lambda(\mathbb{Z}/36\mathbb{Z})$ [$\Gamma_c(\mathbb{Z}/36\mathbb{Z})$] to the vertex in the same relative location *in the horizontal refection* of $\Lambda(\mathbb{Z}/100\mathbb{Z})$ [$\Gamma_c(\mathbb{Z}/100\mathbb{Z})$]. For example, $\overline{[4]}$ is mapped to $\overline{[25]}$, $\overline{[2]}$ is mapped to $\overline{[5]}$, $\overline{[6]}$ is mapped

to $\overline{[10]}$, *etc*. The symmetric identification $e_1 = 2 = e_2$ is the algebraic symmetry behind the graphically illustrated horizontal symmetry of these particular zero divisor lattices and compressed zero divisor graphs.

Conjecture 2 *For all* $\overline{[d]} \in \Lambda(\mathbb{Z}/n\mathbb{Z})$,

$$\left|\{\ \overline{[D]}\ :\ \overline{[d]} \to \overline{[D]}\ \}\right| = \left|\{\ \overline{[M]}\ :\ \overline{[M]} \to \overline{[\frac{n}{d}]}\ \}\right|.$$

From this conjecture, it is possible to pair up the vertices of $\Lambda(\mathbb{Z}/n\mathbb{Z})$ in an anti-symmetric manner; if vertex $\overline{[x]}$ has i edges directed towards it and j edges directed away from it, then the vertex $\overline{[\frac{n}{x}]}$ has j edges directed towards it and i edges directed away from it. We see in Figure 2A this anti-symmetric pattern. Pair together $\overline{[1]}$ and $\overline{[36]}$, $\overline{[2]}$ and $\overline{[18]}$, $\overline{[3]}$ and $\overline{[12]}$, and $\overline{[4]}$ and $\overline{[9]}$; also, $\overline{[6]}$ paired with itself fits this pattern in that $\overline{[6]}$ has an equal number of incoming and outgoing edges.

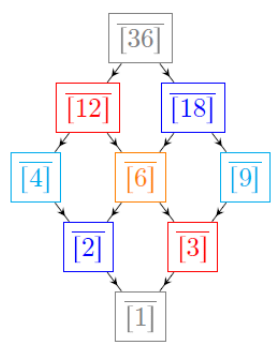


Figure 2A: Anti-Symmetry in $\Lambda(\mathbb{Z}/36\mathbb{Z})$

Intuitively, Conjecture 2 seems likely to be true in that if a vertex $\overline{[x]}$ has a prime "to give" multiplicatively, then the vertex $\overline{[\frac{n}{x}]}$ has that same prime "to take", and *vice versa*. Interestingly, Conjecture 2 does not generally extend to $\Lambda\big[(\mathbb{Z}/n\mathbb{Z})[X]/P(X)\big]$, as evidenced in Figure 2B. Since there are 3 edges directed away from the vertex $\overline{[2X]}$, and the other vertices each have at most 1 edge directed towards them, no such pairing can occur. However, if we were to compress the vertices $\overline{[2]}$, $\overline{[X]}$, and $\overline{[X+2]}$ (which all have the same neighborhood in this zero divisor lattice, namely, $\overline{[2X]}$ above and $\overline{[1]}$ below) into one vertex C, we could then antisymmetrically pair $\overline{[0]}$ and $\overline{[1]}$, and $\overline{[2X]}$ and C.

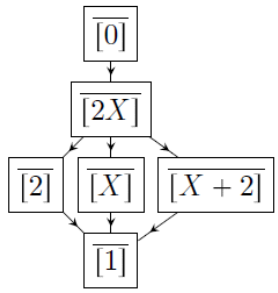


Figure 2B: Lack of Anti-Symmetry in $\Lambda\big[\big(\mathbb{Z}/4\mathbb{Z}\big)[X]/(X^2+2X)\big]$

It seems that Conjecture 2 does not generally extend to

$$\Lambda\big[\big(\mathbb{Z}/n\mathbb{Z}\big)[X]/P(X)\big]$$

because of the different “routes" in which 2 elements in these rings can multiply to give zero; in particular, there is no natural minimal nonzero additive identity due to the difficulty of ordering the elements of these rings. Contrastingly, in $\mathbb{Z}/n\mathbb{Z}$, n is the minimal positive nonzero additive identity. This difference in structure is thought to be the fundamental reason as to why Conjecture 2 must be restricted to $\Lambda(\mathbb{Z}/n\mathbb{Z})$. As in Figure 2A, the paired vertices multiply to give the maximal element in an appropriately minimal way; 36 is the minimal positive number m such that $1 \cdot m \equiv_{36} 0$, 18 is the minimal positive number m such that $2 \cdot m \equiv_{36} 0$, and so on for each individual vertex. This pattern is not so clearly present in the case of $\big(\mathbb{Z}/n\mathbb{Z}\big)\big[X\big]/P(X)$.

Before continuing, we first define some graph theoretic terms, following those given in **[5]**, along with some further terminology.

For a graph G, we will continue to denote the set of vertices of G by $V(G)$, and the set of edges of G by $E(G)$. For $a_1, a_n \in V(G)$, a **path from** a_1 **to** a_n is an ordered(finite) sequence of distinct vertices and edges $\{a_1, e_1, a_2, e_2, \ldots, e_{n-1}, a_n\}$ such that each e_i connectes the vertices a_i and a_{i+1}, if such a sequence exists. A **cut set of a graph** G is a set $C \subset V(G)$ minimal among all subsets of $V(G)$ such that there exist distinct vertices $x, y \in V(G)$ such that every path from x to y contains at least one vertex in C; we then say that C **isolates** x **from** y. If C isolates x from every $y \in V(G) \setminus (C \cup \{x\})$, then we say that C **completely isolates** x. If C consists of a single point $C = \{v\}$, then we say that $v \in V(G)$ is a **cut vertex of** G. Since $\mathrm{ann}(0_{\mathcal{R}}) = \mathcal{R} \supseteq \mathrm{ann}(r)$ and $\mathrm{ann}(1_{\mathcal{R}}) = \{0_{\mathcal{R}}\} \subseteq \mathrm{ann}(r)$ for any commutative ring with identity $\mathcal{R}$ and any $r \in \mathcal{R}$, $\overline{0_{\mathcal{R}}}$ is always the maximal element of $\Lambda(\mathcal{R})$ and, similarly, $\overline{1_{\mathcal{R}}}$ is always the minimal element of $\Lambda(\mathcal{R})$. If $\overline{M} \in \mathcal{R}/\sim$ such that $\overline{M} \prec \overline{A}$

implies $\overline{A} = \overline{0_{\mathcal{R}}}$, then we say that $\overline{M}$ is a **maximal proper element**, or **root**, **of** $\Lambda(\mathcal{R})$. Similarly, if $\overline{m} \in \mathcal{R}/\sim$ such that $\overline{B} \prec \overline{m}$ implies $\overline{B} = \overline{0_{\mathcal{R}}}$, then we say that $\overline{m}$ is a **minimal proper element of** $\Lambda(\mathcal{R})$.

With these definitions in mind, we present the following conjecture.

Conjecture 3 *In $\mathbb{Z}/n\mathbb{Z}$ under $\sim$, $\overline{[x]}$ is a maximal proper element (or root) of $\Lambda(\mathbb{Z}/n\mathbb{Z})$ if and only if $\overline{[x]}$ is a cut vertex of $\Gamma_c(\mathbb{Z}/n\mathbb{Z})$; furthermore, each maximal proper element is of the form $\overline{[\frac{n}{p}]}$, where p is a prime divisor of n, and the cut vertex $\overline{[\frac{n}{p}]}$ completely isolates the vertex $\overline{[p]}$.*

Conjecture 3 follows intuitively from the algebraic construction of $\Lambda(\mathbb{Z}/n\mathbb{Z})$. By Corollary 5, the maximal proper elements, or roots, of $\Lambda(\mathbb{Z}/n\mathbb{Z})$ are precisely the vertices of the form $\overline{[\frac{n}{p}]}$, where p is a prime factor of n. In $\Gamma_c(\mathbb{Z}/n\mathbb{Z})$, every path from $\overline{[p]}$ to any other vertex must pass through $\overline{[\frac{n}{p}]}$ since $\overline{[\frac{n}{p}]}$ is the unique vertex adjacent to $\overline{[p]}$; hence, all vertices of the form $\overline{[\frac{n}{p}]}$ are cut vertices that isloate $\overline{[p]}$ from every other vertex in $\Gamma_c(\mathbb{Z}/n\mathbb{Z})$. The “if" (reverse implication) portion of this conjecture has been confirmed in every instance examined in this study, but has yet to be proven. Interestingly, while the maximal proper elements of $\Lambda(\mathbb{Z}/n\mathbb{Z})$ are cut vertices of $\Gamma_c(\mathbb{Z}/n\mathbb{Z})$, the minimal proper elements of $\Lambda(\mathbb{Z}/n\mathbb{Z})$ are precisely the vertices of $\Gamma_c(\mathbb{Z}/n\mathbb{Z})$ that are isolated upon the removal of the cut vertices. This pattern may perhaps extend to other finite commutative unitary rings, and this connection at least gives a starting point to check if a vertex is in fact a cut vertex.

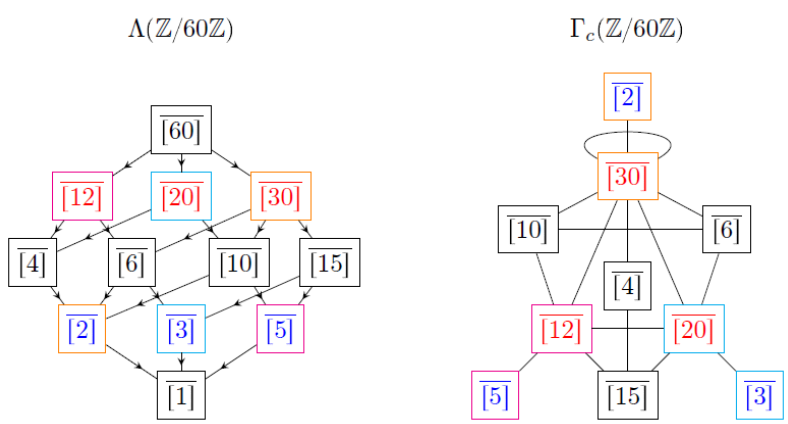


Figure 3: Maximal and Minimal Proper Vertices of $\Lambda(\mathbb{Z}/60\mathbb{Z})$ and Cut and Isolated Vertices of $\Gamma_c(\mathbb{Z}/60\mathbb{Z})$

Figure 3 illustrates this connection. The maximal proper vertices of $\Lambda(\mathbb{Z}/60\mathbb{Z})$ are $\overline{[30]} = \overline{[\frac{60}{2}]}$, $\overline{[20]} = \overline{[\frac{60}{3}]}$, and $\overline{[12]} = \overline{[\frac{60}{5}]}$, shown here with the inner equivalence class colored red. The minimal proper vertices are $\overline{2}$, $\overline{3}$, and $\overline{5}$, with blue equivalence class. The outer box coloring groups a cut vertex with the vertex that it completely isolates, *e.g.*, when the vertex $\overline{[30]}$ (boxed in green) is removed form $\Gamma_c(\mathbb{Z}/60\mathbb{Z})$, $\overline{[2]}$ (also boxed in green) is completely isolated. The colorings confirm that Conjecture 3 is true in this particular example.

Conjecture 4 *Each Proposition, Corollary, and Conjecture concerning $\mathbb{Z}/n\mathbb{Z}$ and its associated graphical representations has an analogous result that holds for any finite $\mathcal{E}/I$, where $\mathcal{E}$ is a Euclidean domain and I is an ideal of $\mathcal{E}$.*

Clearly, each of the Propositions and Corollaries concerning $\mathbb{Z}/n\mathbb{Z}$ relies heavily on the Extended Euclidean Algorithm, Bezout's Identity, and the concept of greatest common divisor, each of which holds (or has an analogue) in a Euclidan domain. Since any Euclidean domain is a principle ideal domain, $I = \langle i \rangle$ for some $i \in \mathcal{E}$. It is thought that this element $i \in \mathcal{E}$ would then take the place of n in the arguments given; in particular, for any $xI \in \mathcal{E}/I$, it is thought that $\gcd(x, i)$ will play the same pivotal role that $\gcd(a, n)$ does for $[a] \in \mathbb{Z}/n\mathbb{Z}$. However, it is unclear whether these results hold in general in any finite Euclidean domain; that is, if there is no particularly special element $i \in \mathcal{E}$ to use in computing greatest common divisors (as in the case of $\mathcal{E}/I$), then it seems that the arguments given in the Propositions and Corollaries will not necessarily hold.

Conjecture 5 *In $\big(\mathbb{Z}/n\mathbb{Z}\big)[X]\big/P(X)$, $[AX+B]$ and $[CX+D]$ have the same annihilator set, and thus $[AX+B] \sim [CX+D]$, if and only if $[AX+B]$ and $[CX+D]$ are associates.*

In short, Conjecture 5 is analogous to Corollary 7, but in the rings of the form $\big(\mathbb{Z}/n\mathbb{Z}\big)[X]\big/P(X)$ rather than $\mathbb{Z}/n\mathbb{Z}$. Conjecture 5 was motivated mainly by manual computation of $\Lambda\big[\big(\mathbb{Z}/n\mathbb{Z}\big)[X]\big/P(X)\big]$ for several natural numbers n and associated polynomials $P(X)$ in $\big(\mathbb{Z}/n\mathbb{Z}\big)X$. Conjecture 5 is not thought to necessarily extend to other similar rings that are not of this specific form: for instance, Conjecture 5 does not extend to the finite commutative ring with identity $\big(\mathbb{Z}/2\mathbb{Z}\big)[X,Y]\big/\big(X^2, Y^2, XY\big)$, because $\mathrm{ann}([X]) = \{[0], [X], [Y], [X+Y]\} = \mathrm{ann}([Y])$, but $[X]$ and $[Y]$ are not associates in this ring.

If Conjecture 5 is in fact true, then the computation of $\big(\mathbb{Z}/n\mathbb{Z}\big)X\big/P(X)$ modulo similarity would be somewhat simplified, as the method outlined in Corollary 10 would generate the equivalence classes.

7. Conclusions

The construction method for $\Lambda(\mathbb{Z}/n\mathbb{Z})$ outlined in this work provides two important tools for future analysis of these objects. First, it allows for the expedient construction of the graphs themselves. Second, it brings to light the algebraic structure of the zero divisor lattice of $\mathbb{Z}/n\mathbb{Z}$. It is hoped that this structure can potentially be used to prove Claims 1 and 2, and could perhaps be extended in some way so as to allow the construction of the zero divisor lattice for some larger or different class of rings. Claims 4 and 5 could potentially be the first step of such a generalization. Additionally, Conjectures 1, 3, 4, and 5 (and perhaps some extension of these conjectures) could potentially further the work done by [5], as was the original motivation of this study.

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The Problem Corner

Edited by Pat Costello

The Problem Corner invites questions of interest to undergraduate students. As a rule, the solution should not demand any tools beyond calculus and linear algebra. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following new problems should be submitted on separate sheets before October 1, 2015. Solutions received after this will be considered up to the time when copy is prepared for publication. The solutions received will be published in the Fall 2015 issue of *The Pentagon*. Preference will be given to correct student solutions. Affirmation of student status and school should be included with solutions. New problems and solutions to problems in this issue should be sent to Pat Costello, Department of Mathematics and Statistics, Eastern Kentucky University, 521 Lancaster Avenue, Richmond, KY 40475-3102 (e-mail: pat.costello@eku.edu, fax: (859) 622-3051).

NEW PROBLEMS 749-759

Problem 749. *Proposed by Tom Moore, Professor Emeritus, Bridgewater State University, Bridgewater, MA.*

The pentagonal numbers are $1, 5, 12, 22, \ldots$ and are given by

$$P_n = \frac{n(3n-1)}{2}, \forall n \geq 1.$$

Prove that every positive even power of 2 is expressible as the difference of two pentagonal numbers.

Problem 750. *Proposed by Tom Moore, Professor Emeritus, Bridgewater State University, Bridgewater, MA.*

It is fairly well known that if (a, b, c) is a primitive Pythagorean triple (PPT), then the product abc is divisible by 60. Find infinitely many PPTs (a, b, c) such that abc is divisible by 120.

Problem 751. *Proposed by Iuliana Trasca, Olt, Romania.*

Prove that if $a, b, c > 0$, then

$$a^{11} + b^{11} + c^{11} \geq a^4 b^4 c^4 \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right).$$

Problem 752. *Proposed by Iuliana Trasca, Olt, Romania.*

Prove that if $x, y, z > 0$, then

$$\frac{2x+2y+4z}{4x+4y+3z}+\frac{2x+4y+2z}{4x+3y+4z}+\frac{4x+2y+2z}{3x+4y+4z}\geq\frac{24}{11}.$$

Problem 753. *Proposed by D.M. Batinetu-Giurgiu, "Matie Basarab" National College, Bucharest and Neculai Stanciu, "George Emil Palade" School, Buzau, Romania.*

If $a, b, c > 0$ and $m \geq 0$, then prove that

$$(a+b+c)^{2m}\left(\frac{1}{(ab)^m}+\frac{1}{(bc)^m}+\frac{1}{(ca)^m}\right)\geq 3^{2m+1}.$$

Problem 754. *Proposed by D.M. Batinetu-Giurgiu, "Matie Basarab" National College, Bucharest and Neculai Stanciu, "George Emil Palade" School, Buzau, Romania.*

Prove that if $a, b > 0$, then

$$\frac{4}{\sqrt{ab(a+b)}}\leq\frac{1}{a}+\frac{1}{b}+\frac{1}{ab}.$$

Problem 755. *Proposed by Mohammad K. Azarian, University of Evansville, Evansville, Indiana.*

Sara has a total number of 26 paper bills in denominations of $\$1$, $\$5$, $\$10$ and $\$20$ in her purse. The number of $\$5$ bills is 4 times the number of $\$10$ bills, and the number of $\$1$ bills is 1 less than twice the number of $\$5$ bills. She remembers that the total amount is less than $\$100$ and more than $\$90$. Furthermore, she remembers that the total amount is an odd number, but it is not 91 or 99. How much money does she have and what is the number of each denomination?

Problem 756. *Proposed by Jose Luis Diaz-Barrero, Technical University of Catalonia (BARCELONA TECH), Barcelona, Spain.*

Find all triples of nonzero real numbers x, y, z such that

$$36\left(\frac{1}{xy}+\frac{1}{yz}+\frac{1}{zx}\right)=1,\quad 36\left(\frac{x}{yz}+\frac{y}{zx}+\frac{z}{xy}\right)=49,$$
$$36^2\left(\frac{1}{x^2y^2}+\frac{1}{y^2z^2}+\frac{1}{z^2x^2}\right)=49.$$

Problem 757. *Proposed by Jose Luis Diaz-Barrero, Technical University of Catalonia (BARCELONA TECH), Barcelona, Spain.*

Let a, b be two complex numbers lying on the circle $|z| = 1$. Prove that

$$\left(\frac{a+b}{1+ab}\right)^2 + \left(\frac{a-b}{1-ab}\right)^2 \geq 1.$$

Problem 758. *Proposed by the editor.*

Prove that the sequence $11231, 1012301, 100123001, 10001230001...$ (i.e., each number starts and ends with 1 and has $k \geq 0$ zeroes on either side of 123) has an infinite subsequence of all composite numbers.

Problem 759. *Proposed by Marcel Chirita, Bucharest, Romania.*

Show how to evaluate the definite integral

$$\int_0^1 \frac{(x-x^2)\arctan x}{(1+x)(1+x^2)}dx.$$

Editor's Note: Tom Moore was a frequent contributor to this column. He passed away in June 2014 after a battle with cancer. He was just 70. Tom had an intense passion for creating and solving problems especially ones that were accessible to students. This editor was fortunate enough to be invited by Tom to be part of a panel discussion on problem solving at the Northeastern Section meeting of the MAA in Fall 2012. Tom was a most gracious host and treated all three of us on the panel in royal fashion. He will be dearly missed.

SOLUTIONS TO PROBLEMS 730-739

Problem 730. *Proposed by Tom Moore, Professor Emeritus, Bridgewater State University, Bridgewater, MA.*

Find two primes such that neither divides any of the numbers 2^a+2^b+1 where a and b are positive integers.

Solution *by the proposer.*

$31 = 2^5 - 1$ and $127 = 2^7 - 1$ are two such primes. If $a = 5q + r$

and $b = 5t + s$ where $0 \leq r, s \leq 4$, then $2^a + 2^b + 1 \equiv 2^r + 2^s + 1 \pmod{31}$, but the range of values for r and s never produce a least residue of $0 \pmod{31}$. So 31 never divides $2^a + 2^b + 1$.

If $a = 7q + r$ and $b = 7t + s$ where $0 \leq r, s \leq 6$, then $2^a + 2^b + 1 \equiv 2^r + 2^s + 1 \pmod{127}$, but the range of values for r and s never produce a least residue of $0 \pmod{127}$. So 127 never divides $2^a + 2^b + 1$.

Also solved by Titu Zvonaru, Comanesti, Romania and Neculai Stanciu, "George Emil Palade" School, Buzau, Romania; the Missouri State University Problem Solving Group, Missouri State University, Springfield, MO; and Ioan Viorel Codreanu, Satulung, Maramures, Romania.

Problem 731. *Proposed by Tom Moore, Professor Emeritus, Bridgewater State University, Bridgewater, MA.*

1. Solve the Diophantine equation $1+2p = y^3$ where p may be any prime.
2. Solve the Diophantine equation $1+4p = y^3$ where p may be any prime.

Solution *Ioan Viorel Codreanu, Satulung, Maramures, Romania.*

1. We have $1 + 2p = y^3 \Leftrightarrow (y-1)(y^2+y+1) = 2p$. Then $y - 1 = 2$ and $y^2 + y + 1 = p$. This means $y = 3$ and $p = 13$.
2. We have $1 + 4p = y^3 \Leftrightarrow (y-1)(y^2+y+1) = 4p$. Then $y - 1 = 4$ and $y^2 + y + 1 = p$. This means $y = 5$ and $p = 31$.

Also solved by Titu Zvonaru, Comanesti, Romania and Neculai Stanciu, "George Emil Palade" School, Buzau, Romania; and the Missouri State University Problem Solving Group, Missouri State University, Springfield, MO; and the proposer.

Problem 732. *Proposed by Tom Moore, Professor Emeritus, Bridgewater State University, Bridgewater, MA.*

By PPT we mean a primitive Pythagorean triple (a, b, c) such that $a^2 + b^2 = c^2$ and $\gcd(a, b) = 1$. Prove the following three things.

1. There are infinitely many multiples of 5 that occur as the hypotenuse (the third value) of a PPT.
2. There are infinitely many multiples of 5 that occur as the odd leg of a PPT.
3. Every odd multiple of 5 occurs as either the hypotenuse or odd leg of a PPT.

Solution *by Titu Zvonaru, Comanesti, Romania and Neculai Stanciu, "George Emil Palade" School, Buzau, Romania.*

A PPT has the hypotenuse $m^2 + n^2$, the odd leg is $m^2 - n^2$, and the even leg is $2mn$ where $m > n$ with different parities and $\gcd(m, n) = 1$.

1. Take $m = 5k + 1$, $n = 5k + 2$ with k any positive integer. Then the numbers m and n are consecutive and have different parities and $\gcd(m, n) = 1$. The hypotenuse is then $m^2 + n^2 = (5k + 1)^2 + (5k + 2)^2 = 5(10k^2 + 6k + 1)$ which is a multiple of 5.

2. Take $m = 5k + 3$, $n = 5k + 2$ with k any positive integer. Then the numbers m and n are consecutive and have different parities and $\gcd(m, n) = 1$. The odd leg is then $m^2 - n^2 = (5k+3)^2 + (5k+2)^2 = 5(2k + 1)$ which is a multiple of 5.

3. Part 2. shows that for any odd multiple of 5 we can choose m and n such that the odd leg is an odd multiple of 5.

Also solved by the Missouri State University Problem Solving Group, Missouri State University, Springfield, MO; and the proposer.

Problem 733. *Proposed by Tom Moore, Professor Emeritus, Bridgewater State University, Bridgewater, MA.*

Prove that every square number $4, 9, 16, \ldots$ is the leg of some PPT by providing an explicit construction.

Solution *by the Missouri State University Problem Solving Group, Missouri State University, Springfield, MO.*

When the three sides are (a, b, c) such that $a^2 + b^2 = c^2$ and $\gcd(a, b) = 1$, we note that the condition that $\gcd(a, b) = 1$ is equivalent to $\gcd(a, b, c) = 1$. If n is odd, then $a = n^2$, $b = (n^4 - 1)/2$, and $c = (n^4 + 1)/2$ gives a Pythagorean triple (note that we must have $n > 1$ for b to be nonzero). Since $c - b = 1$, any common factor of b and c must divide 1, so $\gcd(a, b, c) = 1$. If n is even, then $a = n^2$, $b = (n^4 - 2)/2$, and $c = (n^4 + 2)/2$ gives a Pythagorean triple. Since n is even, $c = n^4/2 + 1$ is odd and $\gcd(a, b, c) = 1$ so we have a PPT.

Also solved by Titu Zvonaru, Comanesti, Romania and Neculai Stanciu, "George Emil Palade" School, Buzau, Romania; Carl Libis, University of Tennessee at Martin, Martin, TN; and the proposer.

Problem 734. *Proposed by Tom Moore, Professor Emeritus, Bridgewater State University, Bridgewater, MA.*

Let $T_n = \frac{n(n+1)}{2}$ and $P_n = \frac{n(3n-1)}{2}$ be the nth triangular and pentagonal numbers, respectively. Prove that there are infinitely many positive integers a, b, c such that $P_a P_b = 2T_c$.

Solution *by the Missouri State University Problem Solving Group, Missouri State University, Springfield, MO.*

Solution 1: One can immediately verify that $b = a + 2$ and $c = (3a^2 + 5a - 2)/2 = P_{a+2} - 1$ gives an infinite family of solutions.

Solution 2: Let $b_1 = 3$ and $c_1 =$3 and recursively define

$$\begin{aligned} b_{n+1} &= 49b_n + 40c_n + 12 \\ c_{n+1} &= 60b_n + 49c_n + 14 \end{aligned}$$

By induction we can show that $(1, b_n, c_n)$ gives an infinite family of solutions. We must show that $3b_n^2 - b_n - 2c_n(c_n + 1) = 0$. If $n = 1$, then $3 * 3^2 - 3 - 2 * 3 * 4 = 0$. Suppose the result holds for $n = k$. Then

$$\begin{aligned} &3b_{k+1}^2 - b_{k+1} - 2c_{k+1}(c_{k+1} + 1) \\ &\quad = 3(49b_k + 40c_k + 12)^2 - (49b_k + 40c_k + 12) \\ &\qquad\qquad -2(60b_k + 49c_k + 14)(60b_k + 49c_k + 15) \\ &\quad = 3b_k^2 - b_k - 2c_k(c_k + 1) = 0, \end{aligned}$$

by the inductive hypothesis. So we have an infinite family of solutions.

Also solved by Titu Zvonaru, Comanesti, Romania and Neculai Stanciu, "George Emil Palade" School, Buzau, Romania; and the proposer.

Problem 735. *Proposed by Ovidiu Furdiu, Technical University of Cluj-Napoca, Cluj, Romania.*

Let $n \geq 1$ be an integer. Calculate the integral

$$\int_0^{2\pi} \sin^2(x) \sin^2(2x) \cdots \sin^2(nx)\, dx.$$

Solution *by the proposer.*

The integral equals $\pi/2^{n-1}$. We have

$$\begin{aligned} &\int_0^{2\pi} \sin^2(x) \sin^2(2x) \cdots \sin^2(nx)\, dx \\ &\quad = \int_0^{2\pi} \frac{1 - \cos(2x)}{2} \cdot \frac{1 - \cos(4x)}{2} \cdot \cdots \cdot \frac{1 - \cos(2nx)}{2}\, dx \end{aligned}$$

$$\begin{aligned}
&= \frac{1}{2^n}\int_0^{2\pi}(1-\cos(2x))\,(1-\cos(4x))\cdots(1-\cos(2nx))\;dx\\
&= \frac{1}{2^n}\int_0^{2\pi}\left(1+\sum_j \varepsilon_j\cos(jx)\right)\;dx\\
&= \frac{1}{2^n}\left(\int_0^{2\pi}dx+\varepsilon_j\int_0^{2\pi}\cos(jx)\;dx\right)\\
&= \pi/2^{n-1},
\end{aligned}$$

where ε_j is either 1 or -1.

Problem 736. *Proposed by D.M. Batinetu-Siurgiu, "Matie Basarab" National College, Bucharest and Neculai Stanciu, "George Emil Palade" School, Buzau, Romania.*

Let $a, b \in \mathbb{R}$ with $a < b$ and continuous functions $f, h : \mathbb{R} \longrightarrow \mathbb{R}$ such that $f(a+b-x) = -f(x), h(a+b-x) = -h(x), \forall x \in \mathbb{R}$. Prove that

$$\int_a^b f(x)\arctan(x)\ln(1+e^{h(x)})\;dx = \frac{1}{2}\int_a^b f(x)h(x)\arctan(x)\;dx.$$

Solution *by the proposers.*

Let $I = \int_a^b f(x)\arctan(x)\ln(1+e^{h(x)})\;dx$, where we make the changes: $x = u(t) = a+b-t$, $u\prime(t) = -1$, $u(a) = b$, $u(b) = a$, and we get

$$\begin{aligned}
I &= -\int_b^a f(a+b-t)\arctan(a+b-t)\ln(1+e^{h(a+b-t)})\;dt\\
&= -\int_a^b f(x)\arctan(x)\ln(1+e^{-h(x)})\;dx\\
&= -\int_a^b f(x)\arctan(x)\ln\left(\frac{1+e^{h(x)}}{e^{h(x)}}\right)\;dx\\
&= -\int_a^b f(x)\arctan(x)\ln\left(1+e^{h(x)}\right)\;dx+\int_a^b f(x)\arctan(x)h(x)\;dx.
\end{aligned}$$

So adding an integral to both sides,

$$2I = \int_a^b f(x)\arctan(x)h(x)\;dx.$$

Dividing by 2 gives the desired result.

Problem 737. *Proposed by Jose Luis Diaz-Barrero, Technical University of Catalonia (BARCELONA TECH), Barcelona, Spain.*

Let α, β, γ be the measures of the angles of an acute triangle ABC. Prove that

$$\cos^2\alpha\sin\beta + \cos^2\beta\sin\gamma + \cos^2\gamma\sin\alpha < \frac{5}{4}.$$

Solution *by the proposer.*

Since ABC is acute, $0 < \alpha, \beta, \gamma < \pi/2$ and also

$$0 < \sin\alpha, \sin\beta, \sin\gamma < 1.$$

Now we write the inequality claimed in the most convenient form

$$\left(1-\sin^2\alpha\right)\sin\beta + \left(1-\sin^2\beta\right)\sin\gamma + \left(1-\sin^2\gamma\right)\sin\alpha < \frac{5}{4}.$$

Putting $\sin\alpha = 1-x$, $\sin\beta = 1-y$, $\sin\gamma = 1-z$ yields

$$(1-(1-x)^2)(1-y)+(1-(1-y)^2)(1-z)+(1-(1-z)^2)(1-x) < \frac{5}{4}.$$

Now, we have

$$\begin{aligned}&(1-(1-x)^2)(1-y)+(1-(1-y)^2)(1-z)+(1-(1-z)^2)(1-x)\\ &\quad = 2(x+y+z)–(x^2+y^2+z^2)–2(xy+yz+zx)+(x^2y+y^2z+z^2x).\end{aligned}$$

Since $0 < x, y, z < 1$, then

$$\left(\frac{x+y+z}{3}\right)^3 > \frac{x^2y+y^2z+z^2x}{4}$$

and

$$(x+y+z)^3 > \frac{27}{4}(x^2y+y^2z+z^2x),$$

or

$$x^2y+y^2z+z^2x < \frac{4}{27}(x+y+z)^3.$$

On account of the proceeding, we have

$$\begin{aligned}&(1-(1-x)^2)(1-y)+(1-(1-y)^2)(1-z)+(1-(1-z)^2)(1-x)\\ &\quad < 2(x+y+z)-(x^2+y^2+z^2)+\frac{4}{27}(x+y+z)^3.\end{aligned}$$

Setting $t = x+y+z$, it will suffice to prove that

$$\frac{4}{27}t^3 - t^2 + 2t - \frac{5}{4} \le 0 \Longleftrightarrow (15-4t)(2t-3)^2 \ge 0.$$

The last expression holds because $t = x+y+z < 3$.

Also solved by Titu Zvonaru, Comanesti, Romania and Neculai Stanciu, "George Emil Palade" School, Buzau, Romania.

Problem 738. *Proposed by Jose Luis Diaz-Barrero, Technical University of Catalonia (BARCELONA TECH), Barcelona, Spain.*

Let x, y, z be positive real numbers such that $xyz = 1$. Prove that

$$\sqrt[4]{x\left(\frac{x+y}{x^2+y^2}\right)^2}+\sqrt[4]{y\left(\frac{y+z}{y^2+z^2}\right)^2}+\sqrt[4]{z\left(\frac{z+x}{z^2+x^2}\right)^2} \leq \sqrt{3(x+y+z)}.$$

Solution *by Ioan Viorel Codreanu, Satulung, Maramures, Romania.*

We have

$$\left(\frac{x+y}{x^2+y^2}\right)^2 \leq \frac{2\left(x^2+y^2\right)}{\left(x^2+y^2\right)^2} = \frac{2}{x^2+y^2} \leq \frac{2}{2xy} = \frac{1}{xy},$$

and then

$$x\left(\frac{x+y}{x^2+y^2}\right)^2 \leq \frac{1}{y}.$$

Using this observation, we get

$$\sum \sqrt[4]{x\left(\frac{x+y}{x^2+y^2}\right)^2} \leq \sum \sqrt[4]{\frac{1}{y}} = \sum \sqrt[4]{xz},$$

since $xyz = 1$. Using the Cauchy-Schwarz Inequality, we obtain

$$\left(\sum \sqrt[4]{xz}\right)^2 \leq 3\left(\sum \sqrt{xz}\right)$$

and

$$\left(\sum \sqrt[4]{xz}\right)^2 \leq \left(\sum x\right)^2 \Longleftrightarrow \sum \sqrt{xz} \leq \sum x.$$

Combining these inequalities gives the desired result.

Also solved by Titu Zvonaru, Comanesti, Romania and Neculai Stanciu, "George Emil Palade" School, Buzau, Romania; and the proposer.

Problem 739. *Proposed by Ed Wilson, Eastern Kentucky University, Richmond, KY.*

A set of seven positive integers form a geometric sequence. The median of the set is 168 and the mean is 381. What is the first term of the sequence and what is the common ratio?

Solution *by Erinn Lawson, Frances Millspaugh and Cally Bond (students), Northeastern State University, Tahlequah,OK.*

Seven positive integers form a geometric sequence and therefore $x_n = ar^{n-1}$. The median is 168, so $ar^3 = 168$. The only perfect cube that is a factor of 168 is 2^3. Therefore $ar^3 = 21 * 2^3$ which makes the sequence 21,

$42, 84, 168, 336, 672, 1344$ and the mean of the sequence is 381. The first term of the sequence is 21 and the common ratio is 2.

Also solved by the Missouri State University Problem Solving Group, Missouri State University, Springfield, MO; Jim Taylor (student), Eastern Kentucky University, Richmond, KY; Titu Zvonaru, Comanesti, Romania and Neculai Stanciu, "George Emil Palade" School, Buzau, Romania; and the proposer.

Report of the 2014 National Convention

Kappa Mu Epsilon
April 3-5, 2014
Jacksonville State University, Alabama Theta
Jacksonville, Alabama

This inaugural National Convention of Kappa Mu Epsilon, sponsored in part by the American Mathematical Society and American Statistical Association, was held during the first half of the 40th Biennium on April 3-5, 2014 at Jacksonville State University in Jacksonville, Alabama, Host Chapter Alabama Theta.

Thursday, April 3, 2014

On Thursday evening, April 3rd from 7-9 p.m. an Ice Cream Social, Trivia, and Photo Scavenger Hunt were held in the Conference Center on the 11th Floor of Houston Cole Library. Participants were able to pick up registration packets at this event. At 8:00 p.m., the National Council held a business meeting in the Houston Cole Library Room 1102.

Friday, April 4, 2014

Friday April 4th's activities began at 8 a.m. to 9 a.m. with breakfast and continued registration outside Houston Cole Library Room 1103B. At 8:30 a.m., the Awards Committee held a meeting in the Houston Cole Library Room 1102. At 9 a.m. in the Houston Cole Library Room 1103A the first general session began, with KME President Rhonda McKee presiding.

Dr. William A. Meehan, President of Jacksonville State University, welcomed participants. Mark S. Hamner, KME Secretary, then called the roll. There were 17 chapters in attendance from nine states (Alabama, Kansas, Michigan, Missouri, Pennsylvania, Rhode Island, South Carolina, Texas, and West Virginia). Nineteen talks were given. With the omission of participants from Alabama Zeta, Birmingham-Southern College, who were not present at the Roll-Call, 28 faculty and 52 students for a total of eighty registrants were officially counted as participating in the convention. Eighty-nine people registered for the convention.

Chapters represented were Alabama Alpha, Athens State University; Alabama Beta, University of North Alabama; Alabama Epsilon, Huntingdon College; Alabama Zeta, Birmingham-Southern College; Alabama Theta, Jacksonville State University; Kansas Alpha, Pittsburg State University; Kansas Beta, Emporia State University; Kansas Delta, Washburn University; Michigan Beta, Central Michigan University; Missouri Beta,

University of Central Missouri; Missouri Theta, Evangel University; Pennsylvania Mu, Saint Francis University; Pennsylvania Tau, DeSales University; Rhode Island Beta, Bryant University; South Carolina Delta, Erskine College; Texas Gamma, Texas Woman's University; and West Virginia Alpha, Bethany College.

Eight talks, a panel discussion, and three workshops were presented. The workshops were a new feature of the KME convention.

Brian Hollenbeck, KME President-Elect of Kansas Beta, recognized the new chapters and then conducted the filing of delegates.

There was no old business.

Introduction of new business began with an update from the KME President, Rhonda McKee. After the update, there was no new business.

Brian Hollenbeck, KME President-Elect, presided over the student paper presentations, held in the Houston Cole Library Room 1103.

The following papers were presented during the morning Session #1, 9:30 a.m.-10:40 a.m.:

- 1-1: *The Rubik's Cube in the Mathematics Classroom*, by Christopher Vaughn, Alabama Alpha, Athens State University
- 1-2: *Benefit Reserves in Actuarial Mathematics*, by Summer Lyons & James Wood, Rhode Island Beta, Bryant University
- 1-3: *First-Order Sigma-Delta Algorithm*, by Alexander Dunkel, Alabama Epsilon, Huntingdon College

At 10:40 a.m. to 11 a.m. there was a 20-minute Refreshment Break in the hallway outside the Houston Cole Library Room 1103.

From 11 a.m. to 12 noon participants attended one of three workshops:

- *Tips for Applying for Graduate School*: Houston Cole Library Room 1103
  - Larry Scott, Moderator, Kansas Beta, Emporia State University
  - Falynn Cartmill Turley, Graduate & Instructor, Jacksonville State University
  - Amy Bretches, Kansas Beta, Emporia State University, and
  - Phuong Minh N. Do, Pennsylvania Mu, St. Francis University
- *Project Math*: Houston Cole Library 1101 C
  - Dr. Tom Leathrum, Alabama Theta, Jacksonville State University
- *Coding Theory in your Mailbox*: Houston Cole Library Room 1101 B
  - Dale Bachman, Missouri Beta, University of Central Missouri

After the workshop sessions at 12 noon a Group Picture was taken in Houston Cole Library Room 1103B.

Lunch was held at 12:15 p.m. in the Jack Hopper Dining Hall followed by a meeting of the National KME Resolutions Committee in the President's Dining Room.

At 1:15 p.m. to 2:15 p.m. participants attended one of two workshops:

- *Project Math*: Houston Cole Library 1101 C
  - Dr. Tom Leathrum, Alabama Theta, Jacksonville State University
- *Coding Theory in your Mailbox*: Houston Cole Library Room 1101 B
  - Dale Bachman, Missouri Beta, University of Central Missouri

Session #2 of the Student Presentations commenced at 2:15 p.m. in the Houston Cole Library Room 1103. The following two papers were presented during this session:

- 2-1: *Lambda Calculus and Its Application to Functional Programming in Haskell*, by Adam Petz, Kansas Beta, Emporia State University
- 2-2: *Colorfully Complex; Visualizing Complex Numbers using Domain Coloring*, by Brandon Marshall, Kansas Delta, Washburn University

At 3 p.m. there was a 15-minute Refreshment Break in the hallway outside the Houston Cole Library Room 1103.

At 3:15 p.m., after the paper session, section meetings were held: the Student Section in the Houston Cole Library Room 1103, and the Faculty Section in the Houston Cole Library Room 1101 C.

The convention banquet was held in the Leone Cole Auditorium at 7:00 p.m. Brian Hollenbeck, KME President-Elect, served as emcee. Following dinner, Arthur Benjamin of Harvey Mudd College in Claremont, California, gave the keynote address. His topic was *Secrets of Mental Math*.

Saturday, April 5, 2014

Saturday April 5th's activities began between 7:30 a.m. and 8:15 a.m. with Registration and Breakfast in the hallway outside Houston Cole Library Room 1103.

Session #3 of the Student Presentations commenced at 8:30 a.m. in the Houston Cole Library Room 1103. The following papers were presented during this session:

- 3-1: *Calculating Population Centers*, by Allison Russell & Kaitlyn Flagg, Alabama Zeta, Birmingham-Southern College

- 3-2: *An Algebraic Method of Constructing the Zero Divisor Lattice of* $\mathbb{Z}/n\mathbb{Z}$, by Jim Crowder, Alabama Zeta, Birmingham-Southern College
- 3-3: *Analysis of a Novel Numerical Scheme for GR-type Non-linear Wave Equations*, by Huda Qureshi, Alabama Zeta, Birmingham-Southern College

At 9:40 a.m. a Refreshment Break was held in the hallway outside Houston Cole Library Room 1103.

The Awards Committee met in the Houston Cole Library Room 1102.

At 10 a.m. in the Houston Cole Library Room 1103 there was a Career Panel Discussion. Panelists included:

- Mary Simmons, Protective Life Insurance Co.
- Falynn Cartmill Turley, Jacksonville State University Faculty and University of Alabama-Birmingham PhD Candidate in Biostatistics
- Jim Roebuck, Weaver High School
- Dale Bachman, Institute for Defense Analysis/ Center for Communications Research

Following at 10:45 a.m., also in the Houston Cole Library Room 1103, the Second General Session began with President Rhonda McKee presiding. Convention Evaluation Forms were distributed and collected.

For the Continuation of New Business, the following national officers made reports:

- Dan Wisniewski, Editor, *The Pentagon*
- Peter Skoner, Historian
- Cynthia Huffman, Treasurer
- Mark S. Hamner, Secretary
- Brian Hollenbeck, President-Elect
- Rhonda McKee, President

Following the national officer reports were reports from the Section Meetings and the Resolutions Committee. All reports are given below.

The report of the Awards Committee and presentation of awards were made. The top two papers were:

- 2-2: *Colorfully Complex; Visualizing Complex Numbers using Domain Coloring*, by Brandon Marshall, Kansas Delta, Washburn University

- 3-2: *An Algebraic Method of Constructing the Zero Divisor Lattice of* $\mathbb{Z}/n\mathbb{Z}$, by Jim Crowder, Alabama Zeta, Birmingham-Southern College

These students were awarded a $100 check for their respective winning paper. The "People's Choice Award," which is selected by submitted votes of the attending KME members, was presented to Christopher Vaughn from Alabama Alpha, Athens State University. The "People's Choice Award" recipient received $50.

The convention concluded with the National Treasurer Cynthia Huffman presenting checks for travel allowances to each chapter present. In addition to the standard travel allowances specified in the KME Constitution, each attending chapter received an extra $200.

Mark Hamner
National Secretary

Report of the National President

This convention marks the end of my first year as president of Kappa Mu Epsilon. It's been a fun and exciting year. Here are highlights of activities since the convention last year at Washburn University in Topeka, Kansas.

- The National Council has continued to meet regularly via teleconference, a tradition begun by Past President Ron Wasserstein.
- In an email vote that ended on June 1, 2013, Brian Hollenbeck was unanimously elected to the position of President Elect.
- Data from the evaluation forms distributed and collected at last year's convention was analyzed and many resulting suggestions were implemented at this convention.
- The national council, along with help from regional director Pedro Muiño, submitted an NSF grant proposal for funding for this year's convention. Unfortunately the proposal was not funded.
- David Gardner, from Texas Woman's University, was appointed by the National Council as our new Web Editor. David has already done several updates to the web page and will oversee a "remodel" of the page later this year.
- Letters were sent to several chapters who have not submitted initiation reports in the last four to six years. Five chapters responded with interest or intent to hold an initiation this year.
- Installations of new chapters (installing officer in parentheses):
  - South Dakota Beta at Black Hills State University, September 20, 2013 (Rhonda McKee)
  - Florida Delta at Embry-Riddle Aeronautical University, April 22, 2014 (Rhonda McKee)
  - Pending approval by chapters: Iowa Delta, Central College (Cynthia Huffman)
  - Preparing petition: Capital University, Columbus, Ohio
- And, of course, we've spent quite a lot of time, organizing this conference. Brian Hollenbeck, President Elect, and David Dempsey, Director of the South East region and sponsor of the host chapter have done a great deal of work and done a great job of organizing and planning this convention. We are very grateful for their dedication to KME.

As other presidents have expressed before me, one of the most rewarding aspects of serving as President of KME is the opportunity to work with the wonderful people who serve KME. From the national council to the local chapter, these folks serve tirelessly in their positions and do so in a fun and pleasant way. I am grateful to Past President Ron Wasserstein, who happily shares documents and insights with me.

Huge thank yous to:

- National Council: Brian Hollenbeck, President Elect; Mark Hamner, Secretary; Cynthia Huffman, Treasurer; Pete Skoner, Historian; David Gardner, Web Master. Because of you, I look forward to our monthly conference calls!
- *Pentagon* staff: Bro. Dan Wisniewski, Editor; Don Tosh, Business Manager; and Patrick Costello, Problems Corner Editor. You all make publishing a journal look easy!
- Regional Directors: Beth Mauch, New England Region; Pedro Muiño, Great Lakes Region; David Dempsey, South Eastern Region; and Vince Dimicelli, South Central Region. Thanks for supporting the chapters in your region!
- (Last but certainly not least!) The corresponding secretaries and faculty sponsors who really are the heart of this organization. We couldn't do it without you!

Rhonda McKee
National President

Report of the National President-Elect

This is my first year as President-Elect. One of the main reasons I agreed to serve was because I knew some of the members of the Council, and saw their passion and effectiveness in serving KME. Without a doubt, it has been a pleasure joining them and being a part of the National Council.

Following the tradition of recent President-Elects, let me mention some statistics about recent conferences.

2014 National Convention

Kappa Mu Epsilon's 2014 Convention is being held this weekend, April 3-5 in Jacksonville, Alabama. Our host chapter is Alabama Theta at Jacksonville State University. There are 17 chapters in attendance from nine states (Alabama, Kansas, Michigan, Missouri, Pennsylvania, Rhode Island, South Carolina, Texas, and West Virginia). Eight talks and a panel discussion will be given over the course of the next two days. We will also have three workshops, which is a new feature of the KME convention. Eighty-nine people are registered for the convention.

By way of comparison, in 2013, the host chapter was Kansas Delta at Washburn University and there were 17 chapters in attendance from eight states (Alabama, Kansas, Louisiana, Michigan, Missouri, New York, Pennsylvania, and Texas). Nineteen talks and a panel discussion were given over the course of two days. Ninety-nine people attended.

In 2011, 16 chapters from nine states (Indiana, Kansas, Kentucky, Michigan, Missouri, New York, Oklahoma, Pennsylvania, and Texas) participated in the convention in St. Louis, Missouri. Eighteen papers were presented. Eighty-seven people attended.

In 2009, 16 chapters from nine states (Georgia, Kansas, Maryland, Michigan, Missouri, New York, Oklahoma, Pennsylvania, and Texas) participated in Philadelphia, PA. Sixteen students presented papers. Seventy-five people attended.

In 2007, 14 chapters from five states (Kansas, Missouri, New York, Oklahoma, and Tennessee) participated in Springfield, Missouri.

In 2005 (Schreiner U., Kerrville, TX), there were 17 chapters from nine states (California, Kansas, Missouri, Michigan, New York, Oklahoma, Pennsylvania, Tennessee, and Texas). There were 15 student presentations.

In 2003 (ORU, Tulsa, OK), there were 19 chapters from 9 states (Iowa, Kansas, Michigan, Missouri, New York, Oklahoma, Pennsylvania, Tennessee, and Texas). Thirteen student papers were presented.

In 2001 (Washburn U., Topeka, KS), there were 20 chapters from 10

states (Colorado, Iowa, Kansas, Kentucky, Missouri, New York, Oklahoma, Ohio, Pennsylvania, and Tennessee)

The following chapters have participated in at least one of the last eight conventions:

Alabama Alpha (2014)
Alabama Beta (2014)
Alabama Epsilon (2014)
Alabama Zeta (2014)
Alabama Theta (2013, 2014)
California Epsilon (2005)
Colorado Delta (2001)
Georgia Alpha (2009)
Indiana Delta (2011)
Iowa Alpha (2001, 2003)
Iowa Gamma (2001)
Kansas Alpha (2001, 2003, 2005, 2007, 2009, 2011, 2013, 2014)
Kansas Beta (2001, 2003, 2005, 2007, 2009, 2011, 2013, 2014)
Kansas Gamma (2001, 2003, 2007)
Kansas Delta (2001, 2003, 2005, 2007, 2009, 2011, 2013, 2014)
Kansas Epsilon (2001)
Kentucky Alpha (2001, 2011)
Louisiana Delta (2013)
Maryland Beta (2009)
Maryland Epsilon (2009)
Michigan Beta (2003, 2005, 2009, 2011, 2013, 2014)
Missouri Alpha (2001, 2003, 2007, 2013)
Missouri Beta (2001, 2003, 2005, 2007, 2009, 2011, 2013, 2014)
Missouri Iota (2001, 2003, 2005, 2007, 2009, 2011, 2013)
Missouri Kappa (2001, 2003, 2005, 2007)
Missouri Lambda (2013)
Missouri Mu (2011)
Missouri Theta (2001, 2003, 2005, 2007, 2009, 2011, 2013, 2014)
New York Eta (2001, 2003, 2005, 2007, 2013)
New York Lambda (2003, 2005)
New York Omicron (2005, 2009, 2011, 2013)
New York Rho (2011, 2013)
Ohio Alpha (2001)
Oklahoma Alpha (2003, 2007)
Oklahoma Gamma (2001, 2003, 2007)
Oklahoma Delta (2001, 2003, 2005, 2007, 2009, 2011)
Pennsylvania Lambda (2003, 2009)

Pennsylvania Mu (2005, 2009, 2011, 2013, 2014)
Pennsylvania Theta (2001)
Pennsylvania Tau (2014)
Rhode Island Beta (2014)
South Carolina Delta (2014)
Tennessee Gamma (2001, 2003, 2005, 2007)
Texas Alpha (2013)
Texas Gamma (2003, 2005, 2009, 2011, 2013, 2014)
Texas Mu (2003, 2005, 2009, 2011)
West Virginia Alpha (2014)

Thus, in the 2000's, 47 different chapters have participated. Five have participated in all eight conventions. Eight chapters are participating for the first time (since 2001) this year.

Again this year the AMS and the ASA both contributed $500 each, which will be used to help defray the cost of student travel to the convention. We are certainly grateful for their support.

Finally, I would like to extend special thanks to David Dempsey and the Alabama Theta chapter for hosting this convention. I know David spent many hours organizing our meeting, and his careful attention to so many details has helped the convention be a success.

Brian Hollenbeck
President-Elect

Report of the National Secretary

Kappa Mu Epsilon, National Mathematics Honor Society initiated 1,292 new members in 95 chapters as of March 15, 2014 during the 40th Biennium. Given that we are in the middle of the 40th Biennium, our current pace of initiation activity indicates we can expect approximately 131 more initiates compared to the last biennium. The 39th Biennium ended March 15, 2013 with a total of 2,453 new members in 122 chapters. So far, forty-eight active chapters have not reported any initiates during the 40th biennium. The total membership of KME is 82,166.

As National Secretary, I receive all initiation reports from chapters, make a record of those reports, up-date mailing list information for corresponding secretaries and forward copies of the reports to other officers. At the beginning of each new biennium, I prepare a new KME brochure. During an academic year, I send out supplies to each chapter. The supplies include information brochures and membership cards. When a college or university petitions for a new chapter of KME, I send out a summary of the petition, prepared by the president, to each chapter and receive the chapter ballots.

Mark Hamner
National Secretary

Report of the National Historian

It continues to be a pleasure to serve the Kappa Mu Epsilon National Mathematics Honor Society as the National Historian. The primary function of the national historian continues to be soliciting, collecting, maintaining and compiling records of chapter activities, installation of new chapters, and other society activities that have historical significance. Most of these records are gathered from individual chapters, who receive several electronic mail requests beginning in January and May of each year asking for a report of chapter activities from the previous semester. The work of the National Historian is impossible without the aid of the corresponding secretaries for each chapter. Thank you for all that you do in serving the students at your institution, your local Kappa Mu Epsilon chapter, and the national organization.

During the past year beginning April 2013, 53 of the active chapters responded at least once to the chapter news request. Special mention goes to the following 19 chapters for their cooperation in responding to both inquiries during the most recent year: AL Zeta, HI Alpha, IA Alpha, MA Beta, MD Delta, MD Epsilon, MI Delta, MO Alpha, MO Theta, NE Delta, NJ Delta, NY Nu, NY Omicron, OH Alpha, OH Epsilon, OK Alpha, PA Mu, RI Beta, and TX Alpha.

A special thank you also goes to the editor of *The Pentagon*, Brother Dan Wisniewski of Pennsylvania Tau chapter. The edited Chapter News section is sent to him each semester, and he has been great to work with.

All of the paper historical records of Kappa Mu Epsilon, which includes a file for each chapter, are now in the office of the National Historian at Pennsylvania Mu. Thank you to the previous Historian Connie Schrock from Kansas Beta, immediate past president Ron Wasserstein, and to Ron's wife Sherry Wasserstein, who drove the last 7 boxes of files from Emporia, Kansas to Bedford, Pennsylvania on June 16, 2013 to meet the Historian's brother with the files. The files have taken a circuitous route, probably not the first time and probably not the last time, and are in safe storage. While not quite the Pony Express, the mail did get through! The process of making electronic copies of all the records is underway and approximately 30% complete.

Peter R. Skoner
National Historian

Report of the National Treasurer

Mid-40th Biennium (March 16, 2013 – March 15, 2014)

A Biennium Asset Report and Biennium Cash Flow Report are given below. The Asset Report shows mid-biennium assets of $90,743.36. The Cash Flow Report shows that we have an asset gain of $1,742.15 so far this biennium. A National Council goal to maintain an asset base of at least $40,000 has been met.

BIENNIUM ASSET REPORT

Total Assets (March 16, 2013)		$88,900.25
Current Assets		
Kansas Teachers Community Credit Union		
Checking	37,368.09	
Share Account	3,375.27	
CD15229	10,000.00	
CD15261	10,000.00	
CD15288	10,000.00	
CD17014	10,000.00	
CD17015	10,000.00	
Total Current Assets		$90,743.36

BIENNIUM CASH FLOW REPORT

Receipts		
Initiation fees received	26,300.00	
Installation fees received	95.00	
Interest income	395.36	
Gifts & misc. income	1000.00	
Total Biennium Receipts		$28,830.36
Expenditures		
Association of College Honor Soc	471.00	
Administrative expenses	2,456.01	
National Convention expenses	16,712.95	
Regional Convention expenses	0.00	
Council Meetings travel	0.00	
Certificates, jewelry & shipping	6,193.43	
Bank charges	0.00	
Pentagon expenses	762.25	
Total Biennium Expenses		$27,088.21
Biennium Cash Flow		$1,742.15

The cash flow last biennium (09-11) was $18,533.69. This was the maximum cash flow for a biennium, partially due to *The Pentagon* moving to an electronic format. The National Council passed on part of this increase to the students and chapters by adding extra to travel allocations and paying for the registration for the speakers at the 2013 National Convention. As a result, national convention expenses were higher in 2013 than in 2011. Each chapter is also receiving an extra $200 in their travel allocation for the 2014 convention.

We have maintained our goal of maintaining assets of at least $40,000. The financial condition of Kappa Mu Epsilon is sound.

I want to offer sincere thanks to the dedicated, talented, hard-working professionals of the National Council (Rhonda McKee, Brian Hollenbeck, Mark Hamner, Pete Skoner, and David Gardner). In addition, a big thanks to the work of the corresponding secretaries who maintain such a vital role in Kappa Mu Epsilon.

Cynthia Huffman
National Treasurer

Report of *The Pentagon* Editor

Introduced in 1941, *The Pentagon* is the official publication of Kappa Mu Epsilon. Publication of student papers continues to be the focus of *The Pentagon*. Following tradition, papers given "top" status and other recognition by the Awards Committee at the KME National Convention are guaranteed an opportunity to be published. Beginning with the Fall 2013 issue, *The Pentagon* is no longer available in a hard copy print format; a small number of subscriptions were fully satisfied recently with the Spring 2013 issue. *The Pentagon* is now completely all-electronic and available for free online via the KME website.

In August, after assisting Chip Curtis with the preparation of the Spring 2013 issue of *The Pentagon*, I officially became the Editor. Since that time, I have been cultivating a renewed list of referees, corresponding with authors of potential articles for submission, and facilitating referee feedback and author corrections for upcoming issues. In particular:

- There is a current list of 17 referees (some new and some returning), 8 of whom have submitted reports for potential articles (5 total articles) during the last six months.
- There have been a total of 6 articles submitted for review and potential publication since August 2013; three have been deemed suitable for publication and are in the final stages of preparation for the Fall 2013 issue (overdue).
- In addition to communicating with the authors of the above six papers, I have corresponded with a total of 17 other potential authors, with varying degrees of response and eagerness to submit articles for submission to *The Pentagon*.

I am grateful to the Associate Editors: Pat Costello, who organizes the Problem Corner for each issue; and, Pete Skoner, who collects and prepares the KME News Items. Also, I am grateful for the support of Don Tosh, who serves as Business Manager of *The Pentagon*. Most especially, I would like to offer my appreciation to my predecessor, Chip Curtis, who served as Editor of *The Pentagon* for almost a decade. During a visit in Joplin last June, Chip shared with me his expertise and experience from working on this publication. I pray that my work will maintain the quality that is evident in the fine journal that Chip has produced since 2004.

Daniel P. Wisniewski, O.S.F.S.
Editor, *The Pentagon*

Report of *The Pentagon* Business Manager

I took over as business manager of *The Pentagon* in December, 2006. My first issue was the Fall, 2006 issue (Volume 66, Number 1). During the 2009-2011 biennium we went to an electronic only edition of *The Pentagon*. We now provide electronic copies of *The Pentagon* for every issue of *The Pentagon* ever printed. The location is kappamuepsilon.org and you can follow the link to find the issues of *The Pentagon*. The site is updated as new issues come out. This past year saw the end of printed copies. The Spring, 2013 issue (Vol 72 Num 2) was the last hardcopy issue.

Because of this, we have gone into a zero balance mode. While we were still printing hard copies of *The Pentagon*, we had to pay for printing and postage of issues. We billed for those expenses as they came in. Now that the balance is $0.00 and there are no more expenses which will be incurred in the printing and postage of hardcopy issues, the business manager is no longer in business. For most of our history, publishing *The Pentagon* was a major expense of KME. That financial obligation is now at $0, and should remain so, since posting new issues to the web site does not cost anything and the fee for the website is not charged to *The Pentagon*.

As you can see from the table, the beginning balance at the start of the biennium was $0. The costs for the last two hardcopy issues of *The Pentagon* are listed, and the final balance is, and will remain to be, $0.

The transition over to the new editor, Bro. Daniel P. Wisniewski, has been smooth and I would like to thank him for the time and effort he has put into *The Pentagon*.

Don Tosh
Business Manager, *The Pentagon*

Pentagon receipts/Expenses 4/13/13 - 4/03/14				
Beginning Balance 4/13/13	**$0.00**		**Ending balance 4/03/14**	**$0.00**
Receipts			**Expenses**	
Vol 72 Num 1 F12 printing	$362.70		Vol 72 Num 1 F12 printing (80 copies)	$362.70
Vol 72 Num 1 F12 postage	$129.03		Vol 72 Num 1 F12 postage	$129.03
Vol 72 Num 2 S13 printing	$226.29		Vol 72 Num 2 S13 printing (60 copies)	$226.29
Vol 72 Num 2 S13 postage	$44.23		Vol 72 Num 2 S13 postage	$44.23
Total Receipts	**$762.25**		**Total expenses**	**$762.25**

Report of the Resolutions Committee

The Resolutions Committee consisted of Derek Newland, faculty member from the Alabama Alpha chapter, Chris Vaughn, student member from the Alabama Alpha chapter, Ethan Francis, student member from the Kansas Beta chapter, Dan Wisniewski, faculty member from the Pennsylvania Tau chapter, Art Gorka, faculty member from the South Carolina Delta chapter, and Richard Smith, from the Rhode Island Beta chapter. The committee proposed the following resolutions.

"Whereas the success of any undertaking relies heavily upon the dedication and ability of its leaders, be it resolved:

1. that this 2014 National Convention express its gratitude to (a) Cynthia Huffman for her faithful service as national treasurer; (b) Mark Hamner for his service as national secretary; (c) Pete Skoner, for his service as national historian; and to (d) Rhonda McKee and Brian Hollenbeck for their efforts in guiding Kappa Mu Epsilon as its president and president-elect,

2. that this Convention acknowledge the participation of the students and faculty who served on the Awards and Resolutions committees, which is so essential for the success of the meeting.

"Whereas the primary purpose of Kappa Mu Epsilon is to encourage participation in mathematics and the development of a deeper understanding of its beauty, be it further resolved:

1. that students Chris Vaughn, Summer Lyons, James Wood, Alexander Dunkel, Adam Petz, Brandon Marshall, Allison Russell, Kaitlyn Flagg, Jim Crowder, and Huda Qureshi, who prepared, submitted, and then presented their papers be given special commendation by this 2014 National Convention for their enthusiasm and dedication, and

2. that this Convention express thanks to Larry Scott, Falynn Cartmill Turley, Amy Bretches, and Phuon Minh N. Do for "Tips for Applying for Graduate School," Tom Leathrum for "Project Math," Dale Bachman for "Coding Theory in your Mailbox," and to Arthur Benjamin for his keynote address "Secrets of Mental Math" at the Friday night banquet.

"Finally, whereas Washburn University and the surrounding community of Topeka have provided this Convention with gracious hospitality, be it resolved:

1. that this 2014 National Convention express its heartfelt appreciation to the Alabama Theta chapter for the thorough arrangements they have

planned and carried out so successfully, and

2. that this Convention recognize and thank Dr. William Meehan, president of Jacksonville State University, as well as David Dempsey, together with all the other members of Alabama Theta, who devoted their time and talents to ensure the success of this meeting."

Respectfully submitted,
Derek Newland, Chairperson

Report of the Awards Committee

The Awards Committee met to select the two award winners. These are:

- 2-2: *Colorfully Complex; Visualizing Complex Numbers using Domain Coloring*, by Brandon Marshall, Kansas Delta, Washburn University
- 3-2: *An Algebraic Method of Constructing the Zero Divisor Lattice of* $\mathbb{Z}/n\mathbb{Z}$, by Jim Crowder, Alabama Zeta, Birmingham-Southern College

Respectfully submitted,
Brian Hollenbeck, Chairperson

Kappa Mu Epsilon News

Edited by Peter Skoner, Historian

Updated information as of November 2014

Send news of chapter activities and other noteworthy KME events to

Peter Skoner, KME Historian
Saint Francis University
117 Evergreen Drive, 313 Scotus Hall
Loretto, PA 15940
or to
pskoner@francis.edu

Installation Report

Florida Delta Chapter
Embry-Riddle Aeronautical University

The Florida Delta Chapter of Kappa Mu Epsilon was installed at 5:15 P.M. on Tuesday, April 22, 2014 at a ceremony held in the College of Arts and Sciences building on the campus of Embry-Riddle Aeronautical University in Daytona Beach, Florida. The meeting was conducted by Dr. Stefan Mancas, an ERAU faculty member. KME national president Dr. Rhonda McKee served as the installing officer. Dr. William Grams, Dean of the College of Arts and Sciences, welcomed and congratulated the initi-

ates. The following charter members were initiated during the installation. (Those who were also installed as officers are noted below.)

Yaqoob Alshamshi	Timothy Parr
Philip Alvarez	Benedict Pineyro
Benjamin Dillahunt (Treasurer)	Ajay Raghavendra (President)
Finn Carlsvi	Christopher Rose
Jonathan Hemingway	Heather Wernke
Sara Huey (Secretary)	Tianyuan Zhao (Vice President)
Jonathan Latim	Jayathi Raghavan

Dr. Jayathi Raghaven was installed as the corresponding secretary and Dr. Greg Spradlin as the faculty sponsor. Following the installation ceremony, Dr. McKee presented a talk titled "Proofs Without Words: Old Fashioned and New Fangled." The evening concluded with a buffet dinner and socializing. About 30 people attended the event.

Installation Report

Iowa Epsilon Chapter
Central College

The Iowa Epsilon chapter of Kappa Mu Epsilon was installed in the Sutphen Room of the Graham Annex on the campus of Central College in Pella, Iowa on Wednesday, April 30, 2014 at 7 P.M. Dr. Russell Goodman, Central College, conducted the meeting with National Treasurer Cynthia Huffman as installing officer. The following charter members were initi-

ated during the installation. (Those who were also installed as officers are noted below.)

Ashley Cliff	Tim Kahl
Shannon Coulson	Melissa Ketcham (Vice President)
Hosh Forst	Kathy Manternach
Brian Hadley	Megan Miller
Ashley Hulsing (President)	Hayley Noll
Kayla Johnson	Katie Todd

There were two charter faculty members initiated in the chapter: Dr. Russ Goodman (Corresponding Secretary/Faculty Advisor) and Dr. Wendy Weber. The program began with a welcome by Dr. Goodman. Initiates and guests then enjoyed ice cream and homemade cookies, followed by remarks by Dr. Mary E.M. Strey, Vice President for Academic Affairs and Dean of the Faculty. Dr. Huffman gave a presentation called "2012 and Maya Math" on mathematics she learned about during a MAA study trip to Guatemala and Honduras. Then Dr. Huffman installed the chapter and the charter chapter officers. The chapter was also presented a charter and crest of the Society.

Chapter News

AL Alpha – Athens State University

Chapter President – Kelsey Turner; 15 Current Members; 8 New Members

Other Spring 2014 Officers: Cory Meyer, Vice President; Jacqueline Brown, Secretary; and Patricia Edge Glaze, Corresponding Secretary and Faculty Sponsor

Chris Vaughn, 2013-2014 President of the Alabama Alpha chapter of KME, presented his student paper "The Rubik's Cube in the Mathematics Classroom" at the KME national convention held at Jacksonville State University on April 4, 2014. The paper was written for MA 470 Senior Mathematics Seminar under the advisement of Dr. Derek Newland. Mr. Vaughn was awarded the "People's Choice Award" for his presentation.

AL Gamma – University of Montevallo

Corresponding Secretary – Scott Varagona; 14 New Members

New Initiates - Brandon Easterling, Tyeler Higgins, Brittany Jacobs, Nicholas Kent, Robert Martin, Brandon McMahan, Carrie Narvaez, Maya O'Neal, Anna Quinn, Steven Sartor, Jasmine Thomas, Racheal Ward, Troy Williams, and Will Winslett.

AL Zeta – Birmingham-Southern College

Chapter President – Huda Qureshi; 41 Current Members; 20 New Members

Other Spring 2014 Officers: Andrew Conner, Vice President; Chase Hoffman, Secretary; Allison Russell and Kaitlyn Flagg (Executive Council), Treasurers; and Maria Stadnik, Corresponding Secretary and Faculty Sponsor

This spring we had many exciting events. We held a Birmingham area Math Jeopardy! tournament on March 10 and our annual initiation ceremony was April 1. We initiated 22 new members this academic year. We were delighted to have the "mathemagician" Dr. Arthur Benjamin as our Spring 2014 Colloquium speaker on April 4. He stopped at our college on the way to deliver his keynote address at the National KME convention at nearby Jacksonville State University. KME members and students from all over campus packed our auditorium to see his presentation. Four of our KME members also give presentations at the National KME conference, and senior Jim Crowder won a "Best Talk" award.

AL Theta – Jacksonville State University

Chapter President – Shannon Bolton; 50 Current Members; 33 New Members

Other Spring 2014 Officers: Paitra Onkst, Vice President; James Tucker Davis, Secretary; Jeremy Moses, Treasurer; and Dr. David Dempsey Cor-

responding Secretary and Faculty Sponsor

On February 24, 2014, the Alabama Theta chapter initiated 33 new members – all students. New members received their certificates, pins, and honor cords in a ceremony held on the 11th floor of Houston Cole Library. Spring activities included Tuesday night study help sessions in the student lounge, trivia nights, and a dinner and bowling night. We created new KME T-shirts and, of course, spent a lot of time preparing to host the 2014 National Convention! New officers were elected during the April meeting.

AR Beta – Henderson State University

Chapter President – Katie Roberts; 15 Current Members

Other Spring 2014 Officers: Samantha Lemp, Vice President; James Easterling, Secretary; Erin Yancey, Treasurer; Dr. Fred Worth, Corresponding Secretary; and Carolyn Eoff, Faculty Sponsor

The Arkansas Beta Chapter of Kappa Mu Epsilon had two meetings in the spring and (in tandem with the HSU Math Club) coordinated Pi Day Activities at HSU on March 14, 2014. This included a bake sale and Digits-of-Pi recitation contest. Three members graduated in May 2014 and were honored to wear Kappa Mu Epsilon cords.

CA Epsilon – California Baptist University

Corresponding Secretary – James Buchholz; 22 New Members

New Initiates - Rosa Mystica Akimana, Kevin Cotton, Sonata Dalimot, Linneah Gomez, Kaitlyn Hayner, Sydney Iturraran, Lucero Jardines, Brittany Ketenbrink, Trevor Logan, Sarah Magiera, Emily McGinnis, Alexandra McMath, Shane Morales, Christian Morris, Michael Ngamije, Angela Nguyen, Jacklyn Pangkee, Jonathan Replogle, Nathaniel Reyes, Caleb Trachte, Rebecca Trupp, and Jessica Yegge.

CT Beta – Eastern Connecticut State University

Corresponding Secretary – Christian Yankov; 17 New Members

New Initiates - Samantha Camolli, James Chadic, Elizabeth Gregory, Nicole Gugliotti, Rebecca Keenan, Mehdi Khorami, Amelia Miceli, Jennifer Mouland, Kelly Provo, Jacqueline Slorach, Jordan Somes, Nicholas Squier, Zoraida Villalobos, Stephen White, Schuylar Whiting, Bailey Wilber, and Melissa Williams.

FL Beta – Florida Southern College

Corresponding Secretary and Faculty Sponsor - Aaron Valdivia; 7 New Members

New Initiates - Lisa DeCastro, Jessica Joan Finocchiaro, Alexander Jose Garcia, David Mathias, Peter J. Oddo, Wei Pin Teh, and Carmela B. Triana.

FL Gamma – Southeastern University

Corresponding Secretary – Dr. Berhane Ghaim; 7 New Members

New Initiates - Shannon Everett, Hanna Flores, Jack Guerra, Michael Hirschi, Marlon

Mariani, April Skipper, and Zachary Wolf.

FL Delta – Embry Riddle Aeronautical University

Corresponding Secretary – Dr. Jayathi Raghavan; 12 New Members

New Initiates - Yaqoob A. Alshamsi, Phillip H. Alvarez, Finn Carlsvi, Benjamin D. Dillahunt, Jonathan P. Hemingway, Sara M. Huey, Timothy J. Parr, Jayathi Raghavan, Ajay Raghavendra, Christopher Rose, Heather N. Wernke, and Tianyuan Zhao.

GA Alpha – University of West Georgia

Corresponding Secretary – Scott Sykes; 3 New Members

New Initiates - Daniel Hartman, Adam Pullen, and Alexis Wagner.

GA Epsilon – Wesleyan College

Corresponding Secretary – Dr. Joe Iskra; 7 New Members

New Initiates - Aahana Bajracharya, Paula-Marie Ivey, Regina Martyanova, Ujjar Prodhan, Aastha Sharma, Chau Tran, and Chelsea Widener.

HI Alpha – Hawaii Pacific University

Chapter President – Keila Elderts; 15 Current Members; 1 New Member

Other Spring 2014 Officer: Tara Davis, Corresponding Secretary and Faculty Sponsor

This semester we had a few activities joint with the Science, Technology, Engineering and Math Club: a paper airplane building competition, a pi day celebration, and a celebrate science party. We also had an initiation dinner, although only current members attended, no new members attended. We have a new President but I believe all our other officers have graduated. We are hoping to generate more interest in the honor society during this school year.

New Initiate - Ariana Jamie Castro Doughty.

IA Alpha – University of Northern Iowa

Chapter President –Elizabeth Johnson; 35 Current Members; 5 New Members

Other Spring 2014 Officers: Travis Buhrow, Vice President; Ben Castle, Secretary; Paige Hageman, Treasurer; and Mark D. Ecker, Corresponding Secretary and Faculty Sponsor

Our first spring KME meeting was held on April 3, 2014 at Professor Syed Kirmani's residence where student member Ben Castle talked about his KME paper entitled "A New Perspective on Fermat's Little Theorem." Student member Andrew Powers addressed the spring initiation banquet with "Baseball Pitch Type Analysis on May 1, 2014. Our banquet was held at Godfather's Pizza in Cedar Falls, where five new members were initiated.

New Initiates - Aaron Manternach, Jacob Oswald, Andrew Powers, Sarah Schipper, and Allysha Whitsell.

IA Beta – Drake University

Corresponding Secretary – Lawrence Naylor; 2 New Members

New Initiates - Joel Venzke and Conor Wells.

IA Gamma – Morningside College

Corresponding Secretary – Chris Spicer; 9 New Members

New Initiates - Tiffany Brasby, Crysta Brewer, Neeia Cooperwood, Ashley Fiedler, Andrew Eugene Heffner, Carter Huggins, Katsuhiro Ishii, Zexi Li, and Monta Meirose.

IA Delta – Wartburg College

Chapter President –Bailey Wilson; 43 Current Members; 10 New Members

Other Spring 2014 Officers: Sarah White, Vice President; Kayla Polson, Secretary; Ben Bogard, Treasurer; Brian Birgen, Corresponding Secretary; and Dr. Joy Becker, Faculty Sponsor

In March, ten new initiates were welcomed at our annual banquet and initiation ceremony, which was combined with the initiation ceremony for Sigma Pi Sigma, the Physics Honor Society. Our speaker was Jason Martin-Hiner, Wartburg Alum and KME member. Jason is an education administrator working in improving science curriculum in area schools. In May, together with the Physics and Computer Science clubs, we hosted the departmental end of the year picnic.

IL Delta – University of St. Francis

Corresponding Secretary – Richard J. Kloser; 16 New Members

New Initiates - Daniel J. Bahret, Julia B. Borel-Donohue, Trevor N. Cherwin, Tyler E. Eakle, Ashley M. Howard, Carol S. Jackson, Julian D. Jankowski, Kaley J. Jendraszak, Austin M. Kelly, Emily A. Kowalski, Kyle J. Loughran, Jacob A. Pratscher, Ashley R. Sichak, Jeremy M. Tobolaski, Leanne A. Vota, and Colleen N. Wagner.

IL Theta – Benedictine University

Chapter President – Miranda Henderson; 14 Current Members; 2 New Members

Other Spring 2014 Officers: Trisha Russo, Vice President; Andy Crutchfield, Secretary; Xinwei Chen, Treasurer; Dr. Thomas Wangler, Corresponding Secretary; and Dr. Anthony DeLegge and Dr. Jeremy Nadolski, Faculty Sponsors

New Initiates - John Doherty III and Rachel Nicinski.

IN Alpha – Manchester University

Corresponding Secretary – Jim Brumbaugh-Smith; 3 New Members

New Initiates - Paula Rodriguez, Kaitlyn Taylor, and Loughlin Wylie.

IN Beta – Butler University

Corresponding Secretary – Bill Johnston; 3 New Members

New Initiates - Anna Durham, Kasey Ruppe, and Matt Storey.

KS Gamma– Benedictine College

Chapter President – Jerome Roehm; 17 Current Members; 15 New Members

Other Spring 2014 Officers: Paul Egging, Vice President; Erica Johnson, Secretary and Treasurer; and Eric West, Corresponding Secretary and Faculty Sponsor

New Initiates - Mariah Barnes, Danielle Blongewicz, Dr. Yanran Chen, Dr. Leah Childers, Katelyn Dery, Paula Egging, Luke Friess, Erica Johnson, Katherine Kennedy, Nathaniel McDonough, Eric Parks, Jerome Roehm, Julie Schneier, Elizabeth Schneider, and Deborah Schulte.

KS Epsilon – Fort Hays State University

Corresponding Secretary – Jeffrey Sadler; 11 New Members

New Initiates - Luke Abbott, Amanda Barnum, Dr. Soumya Bhoumik, Krysten Brake, Seonyeong Hu, Sydney Lower, Thuy An Nguyen, Tanner Reece, Cinthia Rodriguez, Charlee Samuelson, and Rachel Schmidt.

LA Delta– The University of Louisiana at Monroe

Chapter President – Sailesh Wagle; 264 Current Members; 8 New Members

Other Spring 2014 Officers: Jenna Lee, Vice President; Cody Grimsley, Secretary and Treasurer; and Dr. Brent Strunk, Corresponding Secretary and Faculty Sponsor

New Initiates - Bryson Belaire, Abigail Gould, Lucas McHan, MacKenzie Miller, Maroutcha Mouawad, Simon Oetter, Zachary Streeter, and Cortney Wells.

MA Beta – Stonehill College

Chapter President – Katherine Osgood; 23 Current Members; 6 New Members

Other Spring 2014 Officers: Andrea Monterotti, Vice President; and Timothy Woodcock, Corresponding Secretary and Faculty Sponsor

On Friday, March 28, 2014 Massachusetts Beta held its annual initiation ceremony, welcoming six new members to the chapter. A buffet dinner, along with good mathematical conversation, was enjoyed by faculty, students, friends and family alike. During the final exam period, the student members of our chapter volunteered to staff a number of drop-by help sessions, open to all students preparing for final exams in calculus.

MD Alpha – Notre Dame of Maryland University

Corresponding Secretary – Margaret Sullivan; 31 New Members

New Initiates - Gabrielle Baldon, Brittany Dunkerly, Kimberly Cadle, Marie Diop, Nissa Dirige, Ajiri Eroraha, Ashley Fancher, Jennifer Georges, Ines Gouri, Suzanne Hamdy, Samantha Hartig, Gabrielle Jacobson, Kiara Jones, Mattie Kobus, Khadija Lake, Alice Lilly, Amal Malik, Rebecca Malone, Victoria Meadows, Marie Angele Messou, Lauren Miller, Danielle Neumeister, Kirsten Newman, Laura O'Brien, Laura Porter, Zainab Raza,

Cheryl Schmidt, Saba Shahzad, Annu Sharma, Caitlin Still, and Huyen Melissa Vu.

MD Delta – Frostburg State University

Chapter President – Chris Colwander; 32 Current Members; 8 New Members

Other Spring 2014 Officers: Jen Scudder, Vice President; Nick Torgerson, Secretary; Meghan Voelkel, Treasurer; Mark Hughes, Corresponding Secretary and Faculty Sponsor; and Frank Barnet and Justin Dunmyre, Faculty Sponsors

Eight new members joined the Maryland Delta Chapter at our March 2, 2014 initiation ceremony. The ceremony featured a presentation by faculty sponsor Dr. Frank Barnet on the mathematics behind Bitcoin. In mid-March our annual Pi Day Bake Sale was quite successful as was our sale of candy Easter Eggs. During our March meeting after spring break, we watched some math videos from the "Dimensions" series. Our last meeting of the semester in late April featured faculty sponsor Justin Dunmyre showing attendees how to play the game Hanabi which was a lot of fun. This meeting also saw the election of new officers for the coming year. Chris Colwander will continue to serve as President with David Foerster elected as Vice President. Olivia Elisio will serve as secretary and Michelle Welch will be the treasurer. In May, we had an enjoyable end-of-semester cookout. Finally, we offer best wishes to graduating seniors Nick Torgerson, Jen Kleponis, DeVonte' McGee, Meghan Voelkel and Jen Scudder.

MD Epsilon – Stevenson University

Chapter President – Harriet Adutwum; 33 Current Members; 13 New Members

Other Spring 2014 Officers: Hassan Zaheer, Vice President; Rebecca Wong, Secretary; David Allison, Treasurer; and Dr. Christopher E. Barat, Corresponding Secretary and Faculty Sponsor

MI Alpha – Albion College

Corresponding Secretary – Mark Bollman; 7 New Members

New Initiates - Tram Hoang, Paxton Mueller, Katie Strunk, Timothy Szocinski, Jonathan Takeshita, Robin Todd, and Brian Wu.

MI Delta – Hillsdale College

Chapter President – Abigail Loxton; 50 Current Members; 14 New Members

Other Spring 2014 Officers: Matthew Raffin, Vice President; Arena Govier, Secretary; Joshua Mirth, Treasurer; and Dr. David Murphy, Corresponding Secretary and Faculty Sponsor

During the Spring 2014 semester, we initiated 14 new members. We held our Spring 2014 initiation on April 2 to celebrate the start of Math

Awareness Month. In addition to our initiation ceremony, Math Awareness Month included our participation in the Honorama bowling tournament for honor societies at Hillsdale College (April 17), an origami party in which we watched the documentary "Between the Folds" and then made our own origami models (April 23), and then took part in the Lyceum society's Honorary Colloquium discussing readings from Pierre Manent (April 25). Earlier in the term, on February 7, we had a dinner party, which was a fun opportunity to socialize. Besides these events, our KME chapter enjoyed several math talks including "Bernoulli Numbers and Polynomials" by Dr. David Gaebler (Hillsdale College), "Just how non-commutative is the Heisenberg group?" by Dr. John Fink (Kalamazoo College), and "Morphisms in Musical Analysis" by Dr. Thomas Fiore (University of Michigan-Dearborn). The semester closed with our annual math department faculty/student lunch recognizing our graduating majors on Thursday, May 8.

MO Alpha – Missouri State University

Chapter President – Rebecca Wood; 33 Current Members, 13 New Members

Other Spring 2014 Officers: Julie Barnum, Vice President; Casey Cornelius, Secretary; Marissa Mullen, Treasurer; and Jorge Rebaza, Corresponding Secretary and Faculty Sponsor

As every semester, we had three seminars along with pizza and soda. Seminar 1 was Wednesday, February 5 where Brittany Street, a double major (Mathematics and Economics, talked about "Data mining techniques and their applications." Seminar 2 was Thursday, March 20 where Professor Harbaugh, from the Mathematics Department at MSU, talked about "Cardano's Solution of the Cubic." Seminar 3 was Tuesday, April 22 where two students from the Senior Seminar class (MTH 497) presented their papers: "Dividing and multiplying fractions with manipulatives," by Brianna Wilson, and "Diophantine equations," by Zachary Easley. Also, KME treasurer, Marissa Mullen, talked about her experience at participating math conferences; in particular about her participation at the Nebraska Conference for Undergraduate Women in Mathematics. We also had an end-of-semester party on Thursday, May 8, the last day of classes. We had lots of games, music, food, drinks, and desserts.

MO Beta – University of Central Missouri

Chapter President – LeighAnn Sherfey; 25 Current Members, 5 New Members

Other Spring 2014 Officers: Alex Card, Vice President; Tifini Gast, Secretary; Thomas Yoder, Treasurer; Rhonda McKee, Corresponding Secretary;

and Dale Bachman and Steve Shattuck, Faculty Sponsors
Nine students and three faculty members from the Missouri Beta chapter attended the KME National Convention. Three chapters from the Midwest traveled together to the convention in a chartered bus. The students enjoyed having the time on the bus to get to know members of other chapters. During the convention, Leigh Ann Sherfey served on the Awards Committee and Rhonda McKee presided over the business meetings. KME members enjoyed a variety of presentations during the year. Topics included Merlin and Magic Squares, by Dr. Baeth; Crazy Curves and Spirograph Math, by Dr. McKee; reports from several students who held internships; and a mathematical comedy video titled, Derivative vs Integral—The Final Smackdown. Other activities included pumpkin carving, book sales and a holiday party. The Claude H. Brown Freshman (2012-2013) Mathematics Achievement Award and Cooper Scholarship went to Amos Bailey and Emily Arnold. The Claude H. Brown Senior Mathematics Achievement Award went to LeighAnn Sherfey.

New Initiates - Emily Arnold, Xiaofei Chang, Richard J. Miskimins, Angel Nabatyanga, and April Shea.

MO Epsilon – Central Methodist University

Chapter President – Novy Foland; 18 Current Members, 8 New Members
Other Spring 2014 Officers: Tabatha Hoback, Vice President; Kelsey Beeler, Secretary; Julia Weber, Treasurer; Pam Gordy, Corresponding Secretary and Faculty Sponsor
We had some activities around Pi Day which were fun.

MO Eta – Truman State University

Corresponding Secretary – David Garth; 6 New Members

New Initiates - Dr. Michael Adams, Dr. Donald Bindner, Zachary Busch, Dr. David Garth, Destry Newton, and Nicole Scheulen.

MO Theta – Evangel University

Chapter President – Kaitlyn Hong; 11 Current Members; 7 New Members
Other Spring 2014 Officers: Bethany VanderMolen, Vice President; and Don Tosh, Corresponding Secretary and Faculty Sponsor
Meetings were held monthly. In January, we initiated seven new members and elected new officers. In April, Dr. Tosh and six students attended the national convention at Jacksonville State University in Alabama. Also, in April we had our end-of-year ice cream social at the home of Don Tosh, where honor cords were presented to graduating seniors.

MO Kappa – Drury University

Corresponding Secretary – Carol Browning; 14 New Members

New Initiates - Kathryn Auner, Jared Bishop, Alexa Busch, Marina Davis, Hoang Nhat Minh Duong, Evan Johnson, Parker LiaBraaten, Jordan Mason, Deborah Peana, Daniel

Richards, Courtney Tay, Guy Tennison, Isaac Weber, and Ashton Wedemeyer.

MO Lambda – Missouri Western State University

Chapter President – Grace Chester; 36 Current Members, 10 New Members

Other Spring 2014 Officers: Sean Callen, Vice President; Joel Henningsen, Secretary; Katelyn Gutteridge, Treasurer; Dr. Steve Klasson, Corresponding Secretary; and Dr. Jennifer Hegeman, Faculty Sponsor

New Initiates - Cody Buehler, Sean Callen, Linnea Edlin, Katelyn Gutteridge, Jordan Lovejoy, Dustin Payne, Steven Warren, Ian Watson, Ian White, and Kerry Wittrock.

MO Mu – Harris-Stowe State University

Corresponding Secretary – Ann Podleski; 5 New Members

New Initiates - Meaghan Effan, Jordan Fowlkes, Kevin Jones, Deodat Kimuene, and Erica Reese.

MS Alpha – Mississippi University for Women

Corresponding Secretary – Joshua Hanes; 2 New Members

New Initiates - Nadeema Appukutti and Sumitra Karki.

NC Zeta – John C. Smith University

Corresponding Secretary – Doug Brown; 9 New Members

New Initiates - Brooke Baumgarten, Jonathon Boles, Nadine Brockman, Juliana Conte, Davis Emerson, Maria Gurski, Nathan Hill, Alex Lee, and Olivia Myers.

NE Beta – University of Nebraska Kearney

Corresponding Secretary – Dr. Katherine Kime; 3 New Members

New Initiates - Tori Beye, Thomas Ostdiek, and Sophia Weinert.

NE Delta – Nebraska Wesleyan University

Chapter President – Jayme Prenosil; 13 Current Members; 8 New Members

Other Spring 2014 Officers: Leanne Hinrichs, Vice President; Frankie Smith, Secretary and Treasurer; and Melissa Erdmann, Corresponding Secretary and Faculty Sponsor

New Initiates - Mathison Anglin, Connor Bohlken, Curtis Dlouhy, Carey Haefele, Leanne Hinrichs, Sheridan Mason, Nicholas Palacio, and Corey Jones.

NH Alpha – Keene State College

Corresponding Secretary – Vincent J. Ferlini; 8 New Members

New Initiates - Jackie Cobleigh, Randall Dunton, Michael Freedman, Sarah Glotzbach, Christina Hadfield, Caroline Hird, Nancy Johnson, and Kelly Welch.

NJ Delta – Centenary College of New Jersey

Chapter President – Johanne Barthelemy; 15 Current Members; 4 New Members

Other Spring 2014 Officers: Cory Vernon, Vice President; Rob Linpensil, Secretary; Linda Ritchie, Treasurer; and Kathy Turrisi, Corresponding

Secretary and Faculty Sponsor

New Initiates - Britney Elise Allison, Tyler Laurance Corby, Amira Guerrero, and Jose J. Torres.

NY Kappa – Pace University

Corresponding Secretary – Shamita Dutta Gupta; 3 New Members

New Initiates - Kalli Bader, Kevin Hankins, and Kaylee Pina.

NY Lambda – LIU Post

Corresponding Secretary and Faculty Sponsor – James Peters; 16 New Members

The NY Lambda chapter initiated 16 more students into the society. The initiations took place at our annual banquet on April 6, held at the Greenvale Town House restaurant in Glen Cove, NY. We present department awards at the banquet and also here a brief talk from a former student. The talk was given by a student who began in Adolescent Education, Mathematics but then completed enough credits for physics certification. After graduation, she obtained a master's degree in Systems Engineering and was just offered a position by our electric utility company.

New Initiates - Michelle Arenella, Nicole Contrada, Kathryn Del Prete, Jenna Dilorenzo, Erica Gershkowitz, Daniel Goldsmith, Rebecca Greenberg, Ryan Haffner Margaret Kumpas, Matthew La Rose, John Logan, Xiangru Lui, Davanjit Parmar, Carly Pasetti, Rebecca Phillips, and Hilary Ratner.

NY Mu – St. Thomas Aquinas College

Corresponding Secretary – Heather A. Rave; 16 New Members

New Initiates - Michael Annitto, Miguel Davila, Nicole D'Ascoli, Tatiana Del-Solar, Sean Feeley, Joanna Felice, Kimberly Fox, Natasha Ireifej, Patrick Merdian, Heather Palmer, Kevin Roth, Lydia Sullivan, Alyssa Voelker, Justine Wamsley, Tiffany White, and Abby Williams.

NY Nu – Hartwick College

Chapter President – Kyle Murray; 20 Current Members; 13 New Members

Other Spring 2014 Officers: Tricia Phillips, Vice President; Jack Miller, Secretary; Nigel Rambhujun, Treasurer; and L. Gerald Hunsberger, Corresponding Secretary and Faculty Sponsor

New Initiates - Urandari Byambadalai, James Canal, Matthew Cucciarre, Ryan Gill, Emma Heritage, James Irvine, Jack Miller, Tricia Phillips, Nigel Rambhujun, Jessica Riesel, Hannah Strom, Christo Tarazi, and Megan Van der Horst.

NY Omicron – St. Joseph's College

Chapter President – Stephen A. Bates; 50 Current Members; 31 New Members

Other Spring 2014 Officers: Janéce Guerra, Vice President; Daniel Ferguson, Secretary; Carl Baurle, Treasurer; Dr. Elana Reiser, Corresponding

Secretary; and Dr. Donna Marie Pirich, Faculty Sponsor
This semester we had an initiation ceremony on our Patchogue campus and our Brooklyn campus. We had members volunteer at our Saturday morning math clinic, which is free for local high school students to get tutored in math. We also held a fundraiser in order to be able to buy Easter baskets for a local charity. We celebrated our tenth anniversary this year.

OH Alpha – Bowling Green State University
Chapter President – Mike Hughes; 3 New Members
Other Spring 2014 Officers: John Maddrey, Vice President; Mark Medwig, Treasurer; Steven Seubert, Corresponding Secretary; and Jim Albert, Faculty Sponsor
The Ohio Alpha Chapter is proud to announce that they initiated three new members this Spring into KME in April.

OK Alpha – Northeastern State University
Chapter President – JeAnna Philpot; 61 Current Members, 13 New Members
Other Spring 2014 Officers: Caleb Stubbs, Vice President; Steven Sly, Secretary; James Townsend, Treasurer; and Dr. Joan E. Bell, Corresponding Secretary and Faculty Sponsor
Our spring initiation brought twelve students and one math faculty member into our chapter. At the first meeting of the semester, Dr. Demitri Plessas, NSU, spoke on "How Recreational Mathematics Becomes Applied Mathematics." The last meeting of the year was our annual ice cream social. Dr. Plessas gave a presentation on "Generating Caves in Video Games."
New Initiates - Hudson Baab, Brooke Bratu, Jacob Cook, Micheal Crockett, Whitney Dushane, Tommy Gonzales, Karmyn Grigson, Luther Langston, Julia Markle, John Moore, Andrea Morgan, Haylee Phillips, and Dr. Demitri Plessas.

OK Epsilon – Oklahoma Christian University
Chapter President – Jonathan McCallum; 22 Current Members; 10 New Members
Other Spring 2014 Officers: Jean Pierre Mutanguha, Vice President; Jennifer Loe, Secretary; Dr. Jennifer Bryan, Corresponding Secretary; and Dr. Craig Johnson, Faculty Sponsor

PA Alpha – Westminster College
Corresponding Secretary – Pamela Richardson; 7 New Members
New Initiates - Richard Beaudoin, Valerie DeSilva, Nicholas Hainsey, Amanda Kowalczyk, Julie Powell, Brandon Saber, and Rachel Simko.

PA Beta – La Salle University
Chapter President – Dominick Macaluso; 35 Current Members; 18 New Members
Other Spring 2014 Officers: David Comberiate, Vice President; Olivia

Shoemaker, Secretary; Daniel Bowers, Treasurer; and Janet Fierson, Corresponding Secretary and Faculty Sponsor

Last semester, La Salle's KME chapter sponsored a late-night Pi Day event, which included a T-shirt making station, a contest to guess the circumference of a pizza pie and a dessert pie, and various games. The organization also held a board game night. Members of the group volunteered at the Philadelphia Science Festival's "Playing with Numbers" event; they helped to design and staff tables at which attendees could discover the mathematics behind some popular games. The Joint Mathematics Meetings and the Nebraska Conference for Undergraduate Women in Mathematics were attended by members of the chapter, with one student receiving external funding to present her research at both conferences. Members of the organization also began visiting a local elementary school (St. Athanasius School in Philadelphia) weekly to provide exciting enrichment activities for some of the school's strongest mathematics students.

PA Gamma – Waynesburg University

Corresponding Secretary – James R. Bush; 9 New Members

New Initiates - Nicolas C. Frazee, Emily S. Hoffman, David A. Leach, Michael A. Lopuchovsky, Benjamin D. McAuley, Raymond M. Melone, Joseph R. Obradovich, William S. Sungala, and Tah Ngijoi-Yogo.

PA Delta – Marywood University

Corresponding Secretary – Thomas Kent; 5 New Members

New Initiates - Joshua Carey, Susan Durand, Johnny Gallis, Kelly Kirchner, and Melissa Williams.

PA Zeta – Indiana University of Pennsylvania

Corresponding Secretary – Gary Stoudt; Faculty Sponsor; 10 New Members

New Initiates - Francisco Alarcón, Janell Connelly, Emily Downs, Timothy Flowers, Derek Hanely, Sarah Letender, Matthew McBurney, Shawn Alexandria Mosley, Jenna Scherrah, and Gary S. Stoudt.

PA Kappa – Holy Family University

Chapter President – Rebecca Gaetani; 8 Current Members; 3 New Members

Other Spring 2014 Officer: Sister Marcella Wallowicz, Corresponding Secretary and Faculty Sponsor

Chapter President Rebecca Gaetani presented her research on Euler and Perfect Numbers at the SEPCHE (Southeastern Pennsylvania Consortium for Higher Education) Honors Conference on March 24 and on April 9 during Scholars Week at Holy Family. Arielle Brady, Jared DeLeo and Brandon Schaeffer were initiated into KME on April 4, 2014. A pizza party honoring the 2014 KME graduating seniors (Sheridan Goodwill, Ben

Savidge and Brandon Schaeffer) was held on April 24, 2014.

PA Lambda – Bloomsburg University of Pennsylvania

Corresponding Secretary – Dr. Eric Kahn; 23 Current Members; 8 New Members

Other Spring 2014 Officer – Faculty Sponsor, Dr. William Calhoun

Two of our students attended the MAA EPaDel section conference at the University of Scranton in April.

PA Mu – Saint Francis University

Chapter President – Sean Veights; 43 Current Members; 20 New Members

Other Spring 2014 Officers: Elise Löfgren, Vice President; Ryan Ickes, Secretary; Maggie Waldron, Treasurer; Dr. Peter Skoner, Corresponding Secretary; and Dr. Brendon LaBuz, Faculty Sponsor

Initiation ceremonies were held on Thursday, January 30, 2014 in DiSepio 213. The evening began with a prayer by chapter chaplain and member Fr. Joseph Chancler, was followed with dinner, continued with a talk "What Can i Do for You?: Imaginary excursions through the complex plane" by Professor of Chemistry and Great Lakes regional director Dr. Pedro Muíño, and concluded with the initiation ceremony for the twenty new members. The annual Pi Day celebration was held on Friday, March 14, 2014; faculty, students, and staff enjoyed taste testing an assortment of "pi" served by members of Kappa Mu Epsilon throughout the day. Two faculty members and three students attended the 40th National Convention held April 3-6, 2014 at Jacksonville State University in Jacksonville, Alabama. KME students and faculty served as judges for the 2014 Pennsylvania Statistics Poster Competition, hosted for the sixth year by Saint Francis University. A large number of posters (524) were received, cash awards were given for first through fourth place in each of four grade level categories, and winning posters were submitted to the National Statistics Poster Competition, coordinated by the American Statistical Association.

PA Nu – Ursinus College

Corresponding Secretary – Jeff Neslen; 12 New Members

New Initiates - Daniel Bekier, Kevin Cox, Andrew Farris, Jamie Hammell, Joshua Hoffman, Jacob Hollingsworth, Tanner Johnson, Jenna Koch, David Martin, Jake Neiman, Jason Rudich, and Michael Vennettilli.

PA Pi – Slippery Rock University

Chapter President – Christina Rajchel; 15 Current Members; 9 New Members

Other Spring 2014 Officers: Miranda Ryan, Vice President; Danielle Faggioli, Secretary; Kalene Ireland, Treasurer; Elise Grabner, Corresponding Secretary; and Richard Marchand, Faculty Sponsor

PA Sigma – Lycoming College

Chapter President – Carrie Tubbs; 15 Current Members; 9 New Members

Other Spring 2014 Officers: Meghan Hughes, Vice President; Matt DaVolio, Secretary; Erin Fite, Treasurer; Santu de Silva, Corresponding Secretary; and Dr. Eileen Peluso, Faculty Sponsor

There was not a lot of activity this semester, except for initiating an unusually (for us) large number of new members. Our faculty sponsor, Dr. Eileen Peluso, has been absorbed into the administration, which means that it will become necessary to formally appoint a new Faculty Sponsor. The new chair is Dr. Gene Sprechini, who may be expected to be the Faculty Sponsor.

New Initiates - Sarabeth Ciccarelli, Matthew DaVolio, Richard Frederick, Meghan Hughes, Caroline Lapano, Tai Nguyen, Tung Nguyen, Viet Ba Nguyen, and Carrie Tubbs.

PA Tau – DeSales University

Chapter President – Angela M. Ulrich; 4 Current Members; 11 New Members

Other Spring 2014 Officers: Zachary Sikanowicz, Vice President; Jaquelin M. Pastor, Secretary; Keith T. Crozier, Treasurer; and Br. Daniel P. Wisniewski, O.S.F.S., Corresponding Secretary and Faculty Sponsor

On Sunday, April 27, 2014, the PA Tau Chapter of Kappa Mu Epsilon at DeSales University (DSU) initiated eleven new KME members. The event included a presentation entitled "Life After Receiving My Mathematics Degree" by Mr. Zane N. Kershner, Energy Trader at PPL Corporation (Allentown, PA), who received his B.S. in mathematics & computer science in 2003 and his M.B.A. in 2010, both from DSU. In attendance were family and friends of the new and current KME members, as well as two KME alumni.

New Initiates - Drew H. Borger, Joshua R. Brobst, Nathan Furman, Kristen M. Juhrden, Matthew P. Kastner, Jacob M. Kean, Zane N. Kershner, Alison Malatesta, Sean H. Raiser, Kathleen M. Ryan, and Peter T. Stadtmueller.

RI Beta – Bryant University

Chapter President – James Wood; 42 Current Members; 21 New Members

Other Spring 2014 Officers: Andrew DiFronzo, Vice President; Delaney Carr, Secretary; Summer Lyons, Treasurer; John Quinn, Corresponding Secretary; and Alan Olinsky, Faculty Sponsor

We met with our student executive board several times to plan the nomination process for new members and to arrange the initiation ceremony and dinner, which was held on April 28, 2014. Two of our students (James Wood and Summer Lyons) presented at the KME National Convention at Jacksonville State University - "Benefit Reserves in Actuarial Mathematics." In addition, Professor Rick Smith also attended the national conven-

tion and participated in the Resolutions Committee. One of our new KME students (Stephen Lamontagne) wrote a paper with Professor Smith and submitted it to KME's journal, The Pentagon. The paper is entitled "The Number of Constant Terms Remaining in a Telescoping Series." The paper was accepted, with some revisions, and has been revised and resubmitted. Billie Anderson is the new Corresponding Secretary for the Rhode Island Beta chapter (Bryant University). We have just elected our student executive board for the fall 2014 semester and the student board is made up of the following students: Andrew DiFronzo, President; Stephen Lamontagne, Vice President; Allison Orr, Treasurer; and Jerlyn Crowley, Secretary.

New Initiates - Samantha Alicandro, Dominic Cauteruccio, Yu-Shan Chang, Jerlyn Crowley, Gabriella Diamandis, Michael DiTocco, Enxhi Elezi, Sarah French, Jennifer Gagnon, Huy Huynh, Nicholas Johnson, Stephen Lamontagne, Christian Lemieux, Amanda LoBello, Kasey Mazza, Pascal Mihigo, Janhavi Nerurkar, Allison Orr, Krystin Sinclair, Jessica Soojian, and Stephanie Torgan.

SC Gamma – Winthrop University

Corresponding Secretary – Matthew R. Clark; 4 New Members

New Initiates - Martin Hanner, Lynnique Johnson, Stephen McFall, and Abigail Roush.

SD Beta – Black Hills State University

Chapter President – Rachel Solano; 9 Current Members

Other Spring 2014 Officers: Zachary Zenk, Vice President; Kelsey Dalzell, Secretary; Keenan Justice, Treasurer; Ms. Kristel Ehrhardt, Corresponding Secretary; and Dr. Hui Ma, Faculty Sponsor

This spring we met monthly to discuss ideas of what we wanted our chapter of KME to accomplish in the future. We also discussed fundraising efforts. Our first fundraiser was tending the concession stand at one of our sports events. This helped us raise $100 as well as $33 in tips. It's not much, but it's a start. We hope to attend a KME Conference in the future and hope to continue our fundraising efforts this fall.

TN Beta – East Tennessee State University

Chapter President – Dustin Chandler; 823 Current Members; 14 New Members

Other Spring 2014 Officers: Derek Kiser, Vice President; Cory Ball, Secretary; Thomas Robacker, Treasurer; and Robert "Dr. Bob" Gardner, Corresponding Secretary and Faculty Sponsor

Our chapter webpage is http://faculty.etsu.edu/gardnerr/KME/KME.html. On April 11, 2014, the Tennessee Beta chapter of Kappa Mu Epsilon held the initiation ceremony for new members. The ceremony was held at the ETSU Department of Mathematics and Statistics' annual Honors Banquet. A total of 14 new members were initiated, bringing the chapter's cumula-

tive membership to 823. Officers were elected at a meeting of the chapter.
New Initiates - Cory Ball, Carrie Elliot, Russell Harper, Katie Heidt, Miranda Hilton, Amanda Justus, Derek Kiser, David Lewis, Leanna Murdock, Thomas Robacker, Brett Shields, Alex Smyth, Rebekah White, and Alyssa Williams.

TN Gamma – Union University

Chapter President – Alexandra Archer; 24 Current Members; 10 New Members

Other Spring 2014 Officers: Grace Morriss, Vice President; Megan Mouser, Secretary/Treasurer; Lydia DeWolf, Webmaster and Historian; George Moss, Corresponding Secretary; and Matt Lunsford, Faculty Sponsor

New Initiates - Nicole Bantz, Zachary Benson, Stephen Clement, Jonathan Eldridge, Michael Kelly, Danielle Lamb, Chance Mattox, Vicki Searl, Levi Stanfield, and Benjamin Walker.

TX Alpha – Texas Tech University

Chapter President and Treasurer – Saba Nafees; 18 Current Members; 5 New Members

Other Spring 2014 Officers: Annabel Offer, Vice President; and Dr. Magdalena Toda, Treasurer, Corresponding Secretary, and Faculty Sponsor

On February 27, we held an initiation and orientation Meeting, coordinated by the advisor and secretary, Dr. Toda. She invited the new initiates and gave a small speech regarding the importance of being in a group that promotes Mathematics such as KME, a brief history of KME, and encouraged students to become more involved and attend the KME National Conference. The students worked on a simple math problem proposed by Dr. Toda and prizes were given out. The student members decided to give graduation cords for members graduating this semester. Pi Day was held on March 14 with KME members along with the Math Ambassadors working together to engage students in mathematics and sciences. This was a university-wide event involving the Student Government Association, Tech Activities Board, Center for Campus Life, KME, and the Tech Math Ambassadors. It was designed to include upper administration such as the president, provost of the university along with college deans and departmental chairs. The Office of the Provost provided funding for the pies that were distributed to the student body by mathematics professors along with the department's chairperson, Honors College Dean, and various other faculty members throughout the university. In April, the Department of Mathematics and Statistics Banquet was held. Last spring, Ms. Nafees proposed a new award to be given by KME every year. This award is called Outstanding Mathophile Award. It was named by Dr. Edward Allen and is given to someone who shows extreme spirit and passion for the art and study of Mathematics. This unique individual dedicates their lifetime to the advancement and service of Mathematics. This individual

can be any undergraduate or graduate student, any faculty or staff. The first recipient of this award was Dr. Udaya Jayatilake, who received his Ph.D. in Mathematics in May, 2013 from Texas Tech University, and received it from his graduate advisor Dr. Roger Barnard. Retired Texas Tech professor Dr. Dean Victory received the Outstanding Mathophile Award at the 41st Annual Math Banquet. The winner of the KME Professor of the Year was Dr. Brock Williams. During the spring of 2014 five new students were initiated, bringing the chapter to eighteen members. Chapter President Saba Nafees graduated with a bachelor of science in mathematics and is continuing her education at Texas Tech as a graduate student in the math department. Annabel Offer, an undergraduate student pursuing a math degree, will serve as the interim president until an official election is held in March 2015.

TX Gamma – Texas Woman's University

Corresponding Secretary – Mark Hamner; 7 New Members

New Initiates - Keri Brookshire, Alexandra Chavarria, Jaynie Jones, Taylor Kirkpatrick, Karen Lindsey Smith, Jennifer McCluskey, and Cassandra Rowland.

TX Iota – McMurry University

Corresponding Secretary – Dr. Kelly McCoun; 3 New Members

New Initiates - Richard L. Barton, Jacob Howdeshell, and Trevor May.

TX Kappa – University of Mary Hardin-Baylor

Chapter President – Kendall Pye; 15 Current Members; 11 New Members

Other Spring 2014 Officers: Shelby Prather, Vice President; Alex Oard, Secretary; Peter H. Chen Corresponding Secretary; and Maxwell Hart, Faculty Sponsor

New Initiates - Kristen Cain, Alicia Colclasure, Macy Moore, Sharon Moore, Alex Oard, Shangrila Pathak, Jared Pitts, Shelby Prather, Kendall Pye, Brian Sides, and Chi Zhang.

VA Gamma – Liberty University

Corresponding Secretary – Dr. Tim Van Voorhis; 7 New Members

New Initiates - Bradley Carney, Christopher Chung, Heath Detweiler, Chad Docksteader, James Latshaw, Scott Long, and Walter Schultz.

VA Delta – Marymount University

Chapter President and Secretary – Marina Romadan; 32 Current Members; 2 New Members

Other Spring 2014 Officers: Lucy Ogbole, Vice President and Treasurer; Secretary; William Heuett, Corresponding Secretary; and Elsa Schaefer, Faculty Sponsor

New Initiates - Lucy Ogbole and Marina Romadan.

WV Alpha – Bethany College

Chapter President – Melinda S. Bierhals; 14 Current Members; 3 New

Members

Other Spring 2014 Officers: Jacob E. Fischer, Vice President; Tyler N. Pannebaker, Secretary and Treasurer; and Adam C. Fletcher, Corresponding Secretary and Faculty Sponsor

West Virginia Alpha chapter assisted the Bethany College Mathematics and Computer Science Club in hosting the eighth annual Bethany College Math/Science Day competition in March. Initiations were also held in March, with three new members being initiated. For the first time in recent memory, West Virginia Alpha attended the National KME Conference at Jacksonville State, where the contingent consisted of the four members of the executive board: President Mindy Bierhals, Vice President Jacob Fischer, Secretary/Treasurer Tyler Pannebaker, and Corresponding Secretary Professor Adam Fletcher.

New Initiates - Keith R. Bartlett, Jr., James A. Long, Jr., and Joseph Michael Morgan.

Active Chapters of Kappa Mu Epsilon

Listed by date of installation

Chapter	Location	Installation Date
OK Alpha	Northeastern State University, Tahlequah	18 Apr 1931
IA Alpha	University of Northern Iowa, Cedar Falls	27 May 1931
KS Alpha	Pittsburg State University, Pittsburg	30 Jan 1932
MO Alpha	Missouri State University, Springfield	20 May 1932
MS Alpha	Mississippi University for Women, Columbus	30 May 1932
MS Beta	Mississippi State University, Mississippi State	14 Dec 1932
NE Alpha	Wayne State College, Wayne	17 Jan 1933
KS Beta	Emporia State University, Emporia	12 May 1934
AL Alpha	Athens State University, Athens	5 Mar 1935
NM Alpha	University of New Mexico, Albuquerque	28 Mar 1935
IL Beta	Eastern Illinois University, Charleston	11 Apr 1935
AL Beta	University of North Alabama, Florence	20 May 1935
AL Gamma	University of Montevallo, Montevallo	24 Apr 1937
OH Alpha	Bowling Green State University, Bowling Green	24 Apr 1937
MI Alpha	Albion College, Albion	29 May 1937
MO Beta	University of Central Missouri, Warrensburg	10 Jun 1938
TX Alpha	Texas Tech University, Lubbock	10 May 1940
KS Gamma	Benedictine College, Atchison	26 May 1940
IA Beta	Drake University, Des Moines	27 May 1940
TN Alpha	Tennessee Technological University, Cookeville	5 Jun 1941
MI Beta	Central Michigan University, Mount Pleasant	25 Apr 1942
NJ Beta	Montclair State University, Upper Montclair	21 Apr 1944
IL Delta	University of St. Francis, Joliet	21 May 1945
KS Delta	Washburn University, Topeka	29 Mar 1947
MO Gamma	William Jewell College, Liberty	7 May 1947
TX Gamma	Texas Woman's University, Denton	7 May 1947
WI Alpha	Mount Mary College, Milwaukee	11 May 1947
OH Gamma	Baldwin-Wallace College, Berea	6 Jun 1947
CO Alpha	Colorado State University, Fort Collins	16 May 1948
MO Epsilon	Central Methodist College, Fayette	18 May 1949
MS Gamma	University of Southern Mississippi, Hattiesburg	21 May 1949
IN Alpha	Manchester College, North Manchester	16 May 1950
PA Alpha	Westminster College, New Wilmington	17 May 1950
IN Beta	Butler University, Indianapolis	16 May 1952
KS Epsilon	Fort Hays State University, Hays	6 Dec 1952
PA Beta	LaSalle University, Philadelphia	19 May 1953
VA Alpha	Virginia State University, Petersburg	29 Jan 1955
IN Gamma	Anderson University, Anderson	5 Apr 1957
CA Gamma	California Polytechnic State University, San Luis Obispo	23 May 1958
TN Beta	East Tennessee State University, Johnson City	22 May 1959
PA Gamma	Waynesburg College, Waynesburg	23 May 1959
VA Beta	Radford University, Radford	12 Nov 1959
NE Beta	University of Nebraska—Kearney, Kearney	11 Dec 1959
IN Delta	University of Evansville, Evansville	27 May 1960

OH Epsilon	Marietta College, Marietta	29 Oct 1960
MO Zeta	University of Missouri—Rolla, Rolla	19 May 1961
NE Gamma	Chadron State College, Chadron	19 May 1962
MD Alpha	College of Notre Dame of Maryland, Baltimore	22 May 1963
CA Delta	California State Polytechnic University, Pomona	5 Nov 1964
PA Delta	Marywood University, Scranton	8 Nov 1964
PA Epsilon	Kutztown University of Pennsylvania, Kutztown	3 Apr 1965
AL Epsilon	Huntingdon College, Montgomery	15 Apr 1965
PA Zeta	Indiana University of Pennsylvania, Indiana	6 May 1965
AR Alpha	Arkansas State University, Jonesboro	21 May 1965
TN Gamma	Union University, Jackson	24 May 1965
WI Beta	University of Wisconsin—River Falls, River Falls	25 May 1965
IA Gamma	Morningside College, Sioux City	25 May 1965
MD Beta	McDaniel College, Westminster	30 May 1965
IL Zeta	Dominican University, River Forest	26 Feb 1967
SC Beta	South Carolina State College, Orangeburg	6 May 1967
PA Eta	Grove City College, Grove City	13 May 1967
NY Eta	Niagara University, Niagara University	18 May 1968
MA Alpha	Assumption College, Worcester	19 Nov 1968
MO Eta	Truman State University, Kirksville	7 Dec 1968
IL Eta	Western Illinois University, Macomb	9 May 1969
OH Zeta	Muskingum College, New Concord	17 May 1969
PA Theta	Susquehanna University, Selinsgrove	26 May 1969
PA Iota	Shippensburg University of Pennsylvania, Shippensburg	1 Nov 1969
MS Delta	William Carey College, Hattiesburg	17 Dec 1970
MO Theta	Evangel University, Springfield	12 Jan 1971
PA Kappa	Holy Family College, Philadelphia	23 Jan 1971
CO Beta	Colorado School of Mines, Golden	4 Mar 1971
KY Alpha	Eastern Kentucky University, Richmond	27 Mar 1971
TN Delta	Carson-Newman College, Jefferson City	15 May 1971
NY Iota	Wagner College, Staten Island	19 May 1971
SC Gamma	Winthrop University, Rock Hill	3 Nov 1972
IA Delta	Wartburg College, Waverly	6 Apr 1973
PA Lambda	Bloomsburg University of Pennsylvania, Bloomsburg	17 Oct 1973
OK Gamma	Southwestern Oklahoma State University, Weatherford	1 May 1973
NY Kappa	Pace University, New York	24 Apr 1974
TX Eta	Hardin-Simmons University, Abilene	3 May 1975
MO Iota	Missouri Southern State University, Joplin	8 May 1975
GA Alpha	State University of West Georgia, Carrollton	21 May 1975
WV Alpha	Bethany College, Bethany	21 May 1975
FL Beta	Florida Southern College, Lakeland	31 Oct 1976
WI Gamma	University of Wisconsin—Eau Claire, Eau Claire	4 Feb 1978
MD Delta	Frostburg State University, Frostburg	17 Sep 1978
IL Theta	Benedictine University, Lisle	18 May 1979
PA Mu	St. Francis University, Loretto	14 Sep 1979
AL Zeta	Birmingham-Southern College, Birmingham	18 Feb 1981
CT Beta	Eastern Connecticut State University, Willimantic	2 May 1981
NY Lambda	C.W. Post Campus of Long Island University, Brookville	2 May 1983
MO Kappa	Drury University, Springfield	30 Nov 1984
CO Gamma	Fort Lewis College, Durango	29 Mar 1985

NE Delta	Nebraska Wesleyan University, Lincoln	18 Apr 1986
TX Iota	McMurry University, Abilene	25 Apr 1987
PA Nu	Ursinus College, Collegeville	28 Apr 1987
VA Gamma	Liberty University, Lynchburg	30 Apr 1987
NY Mu	St. Thomas Aquinas College, Sparkill	14 May 1987
OH Eta	Ohio Northern University, Ada	15 Dec 1987
OK Delta	Oral Roberts University, Tulsa	10 Apr 1990
CO Delta	Mesa State College, Grand Junction	27 Apr 1990
PA Xi	Cedar Crest College, Allentown	30 Oct 1990
MO Lambda	Missouri Western State College, St. Joseph	10 Feb 1991
TX Kappa	University of Mary Hardin-Baylor, Belton	21 Feb 1991
SC Delta	Erskine College, Due West	28 Apr 1991
SD Alpha	Northern State University, Aberdeen	3 May 1992
NY Nu	Hartwick College, Oneonta	14 May 1992
NH Alpha	Keene State College, Keene	16 Feb 1993
LA Gamma	Northwestern State University, Natchitoches	24 Mar 1993
KY Beta	Cumberland College, Williamsburg	3 May 1993
MS Epsilon	Delta State University, Cleveland	19 Nov 1994
PA Omicron	University of Pittsburgh at Johnstown, Johnstown	10 Apr 1997
MI Delta	Hillsdale College, Hillsdale	30 Apr 1997
MI Epsilon	Kettering University, Flint	28 Mar 1998
KS Zeta	Southwestern College, Winfield	14 Apr 1998
TN Epsilon	Bethel College, McKenzie	16 Apr 1998
MO Mu	Harris-Stowe College, St. Louis	25 Apr 1998
GA Beta	Georgia College and State University, Milledgeville	25 Apr 1998
AL Eta	University of West Alabama, Livingston	4 May 1998
NY Xi	Buffalo State College, Buffalo	12 May 1998
NC Delta	High Point University, High Point	24 Mar 1999
PA Pi	Slippery Rock University, Slippery Rock	19 Apr 1999
TX Lambda	Trinity University, San Antonio	22 Nov 1999
GA Gamma	Piedmont College, Demorest	7 Apr 2000
LA Delta	University of Louisiana, Monroe	11 Feb 2001
GA Delta	Berry College, Mount Berry	21 Apr 2001
TX Mu	Schreiner University, Kerrville	28 Apr 2001
NJ Gamma	Monmouth University, West Long Branch	21 Apr 2002
CA Epsilon	California Baptist University, Riverside	21 Apr 2003
PA Rho	Thiel College, Greenville	13 Feb 2004
VA Delta	Marymount University, Arlington	26 Mar 2004
NY Omicron	St. Joseph's College, Patchogue	1 May 2004
IL Iota	Lewis University, Romeoville	26 Feb 2005
WV Beta	Wheeling Jesuit University, Wheeling	11 Mar 2005
SC Epsilon	Francis Marion University, Florence	18 Mar 2005
PA Sigma	Lycoming College, Williamsport	1 Apr 2005
MO Nu	Columbia College, Columbia	29 Apr 2005
MD Epsilon	Stevenson University, Stevenson	3 Dec 2005
NJ Delta	Centenary College, Hackettstown	1 Dec 2006
NY Pi	Mount Saint Mary College, Newburgh	20 Mar 2007
OK Epsilon	Oklahoma Christian University, Oklahoma City	20 Apr 2007
HA Alpha	Hawaii Pacific University, Waipahu	22 Oct 2007
NC Epsilon	North Carolina Wesleyan College, Rocky Mount	24 Mar 2008

CA Zeta	Simpson University, Redding	4 Apr 2009
NY Rho	Molloy College, Rockville Center	21 Apr 2009
NC Zeta	Catawba College, Salisbury	17 Sep 2009
RI Alpha	Roger Williams University, Bristol	13 Nov 2009
NJ Epsilon	New Jersey City University, Jersey City	22 Feb 2010
NC Eta	Johnson C. Smith University, Charlotte	18 Mar 2010
AL Theta	Jacksonville State University, Jacksonville	29 Mar 2010
GA Epsilon	Wesleyan College, Macon	30 Mar 2010
FL Gamma	Southeastern University, Lakeland	31 Mar 2010
MA Beta	Stonehill College, Easton	8 Apr 2011
AR Beta	Henderson State University, Arkadelphia	10 Oct 2011
PA Tau	DeSales University, Center Valley	29 Apr 2012
TN Zeta	Lee University, Cleveland	5 Nov 2012
RI Beta	Bryant University, Smithfield	3 Apr 2013
SD Beta	Black Hills State University, Spearfish	20 Sept 2013
FL Delta	Embry-Riddle Aeronautical University, Daytona Beach	22 Apr 2014
IA Epsilon	Central College, Pella	30 Apr 2014