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Knot Theory and the Structure of DNA

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Abstract

A mathematical knot is a tangled piece of string with its ends attached. A given knot can be quite complex. Understanding and classification of knots is an important area of study in mathematics. Knot theory can be generalized to ribbon theory. A ribbon is a twisted thickened knot. One can apply ribbon theory to DNA analysis by considering its double-helix structure as a ribbon. In this expository paper, we show how studying knots and ribbons can give us insight into DNA structure and the phenomena of supercoiling.

1. Introduction

The goal of this paper is to clearly explain the phenomena of supercoiling of DNA. The mathematical subject that is related to this phenomenon is knot theory. Knot theory has been used to explain supercoiling of DNA ([1], Section 7.1). However, we found the existing literature to be inadequate and unclear.

Many of the definitions in this paper already exist and none of the results are original. We hope that this expository paper will make the connection between Knot Theory and DNA more accessible for undergraduate students.

2. Knots

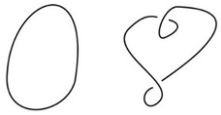
Imagine you have a string and it is tangled about itself as many times as

you please. If you attach the two ends together, this is what is considered a mathematical *knot*. If you were to attach the two ends without tangling the string at all, you get the *unknot*. The unknot is the most basic knot. Figure 1 shows how to build a nontrivial knot, which is called the *trefoil*. The first examples of nontrivial knots include the trefoil and the figure-eight knot.

Figure 1: This image shows the creation of a trefoil knot. [1]

Another nontrivial knot is what is called the *figure-eight knot*.

A particular knot may be drawn in multiple ways, called *projections*. For example, below are two projections of the unknot.



We get different projections of a knot by manipulating it without cutting it. In a drawing of a knot, a *crossing* is where the knot crosses itself. Crossings can be added or removed by twisting the knot or moving one part of the knot over another. The knot can be pulled and moved around to look different from the original, creating a different projection. Various projections of a particular knot are called *equivalent* knots. In essence, they represent the same knot.

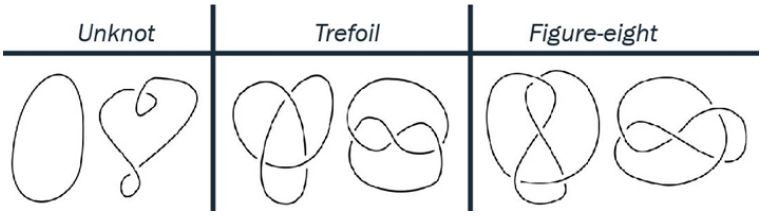


Figure 2: Some basic knots, represented by multiple projections.

There are two types of crossings, *over* and *under*. They are clearly

represented in the drawing. The over crossing is the visible, full line and the under crossing is where there is a gap in the line. The knot does not break at this point, it is drawn this way to represent an under crossing.

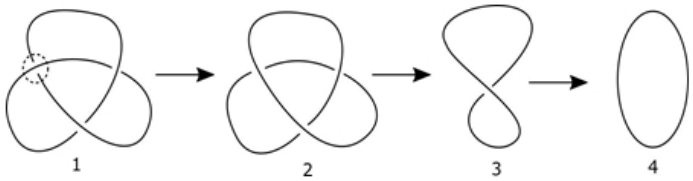
Practice Problem 1: On your own, try creating a different projection of the trefoil different from the two shown in Figure 2.

In general, it is not easy to prove that two knots are not equivalent. For example, how can we be sure that no one can manipulate the trefoil to get the unknot? How do we know we cannot play around with the figure-eight knot to get the trefoil? If the trefoil is made out of a piece of string, it turns out no matter how much time is spent, one will not be able to transform it into the unknot without cutting. One way to distinguish knots from each other is using knot invariants. A *knot invariant* is a number associated to each knot which does not change under various projections of that knot. Equivalent knots will have the same invariants. One such invariant is the unknotting number.

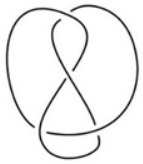
Definition: The *unknotting number* of a knot is the minimum number of crossings that need to be changed to transform the knot into the unknot in any projection of that knot. We denote the unknotting number of a knot *k* by $u(k)$.

To find the unknotting number, one picks a crossing in the knot and alters it. If it is an over crossing, change it to an under crossing. If it is an under crossing, change it to an over crossing. If the resulting knot can be drawn as the unknot, the unknotting number of the knot is 1. Otherwise, one must pick another crossing. If changing any of crossings does not give rise to the unknot, one must pick another crossing and repeat the same steps. Then, if that produces the unknot, the unknotting number would be 2, and so on.

Example: Let's find the unknotting number of the trefoil. In step (1), we pick a crossing to alter. In step (2), we change the crossing from over to under. In steps (3) and (4), we move the knot around to untangle it without breaking the knot at all. Eventually, we are left with the unknot. Since we only changed one crossing, we know that $u(k) = 1$.



Practice Problem 2: Find the unknotting number of the figure-eight knot below.



If two knots do not have the same unknotting number, they cannot be identical. For example, we know that the trefoil is not the same as the unknot because the unknotting number of the trefoil is 1, but the unknotting number of the unknot is 0.

Definition: An *orientation* of a knot is a choice of direction in which to travel along the knot, so there are two possible orientations for a given knot. Once the orientation is established, we have an *oriented knot*.

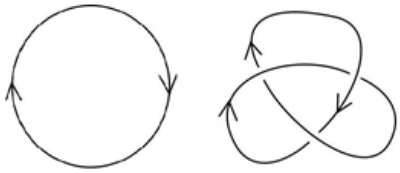
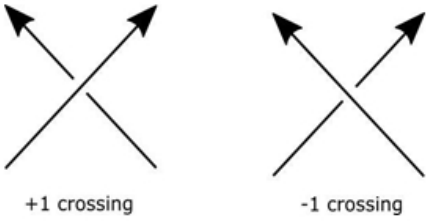


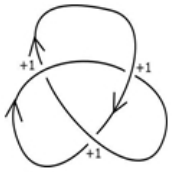
Figure 3: Oriented unknot and oriented trefoil knot.

Definition: The *writhe* of an oriented knot is an integer associated to that knot which roughly describes how twisted the knot is. To define writhe, we use the Right Hand Rule. To each crossing in an oriented knot, we assign either a $+1$ or -1 using this rule. The right hand rule is executed by placing the right hand, palm down, with all four fingers in the direction of the over crossing. Then, if the thumb is pointing in the same direction as the under crossing's orientation, the crossing is assigned $+1$. Otherwise, the crossing is assigned -1.



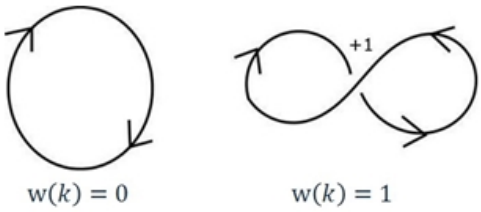
The writhe of an oriented knot is the sum of the $+1$ and -1s in the given projection. We denote the writhe of a knot k to be $w(k)$.

Example: The writhe of the oriented knot below is $w(k) = 1+1+1 = 3$.



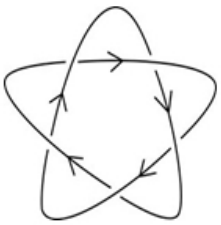
Note that different projections of an oriented knot may not have the same writhe and hence writhe is not a knot invariant.

Example: Below are two projections of the unknot with different writhe.



Practice Problem 3: Find the writhe of the oriented knot below.

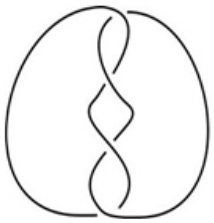
Practice Problem 4: Find the writhe of the oriented knot below.



Remark: The writhe of an oriented knot depends on the orientation. If the orientation is reversed, the sign of the writhe will change from positive to

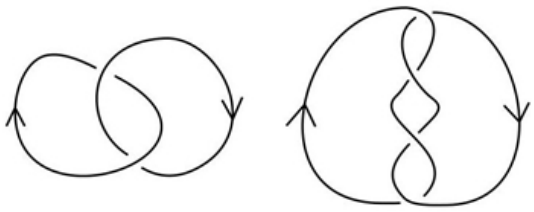
negative or from negative to positive. (You can prove this on your own by observing what happens when you change the orientations in the figures above)

Definition: A *link* is a collection of knots linked together. A link may have multiple loops, called *components.* For example, here is a link with two components.



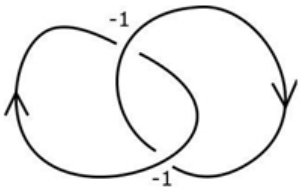
In this paper, we are only concerned with links having two components. Note that links are generalizations of knots since a knot is a link with only one component.

An orientation of a link is a choice of orientation for each component of the link. Once one chooses an orientation for each component, one will have an *oriented link*. Below are some examples of oriented links. If the orientation of any of the components is changed, a new oriented link will be created.



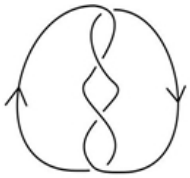
Definition: The *linking number* measures how tangled up the two components are. To find the linking number of an oriented link, we use the Right Hand Rule to assign $+1$s and -1s to each crossing that occurs between the two components (ignoring the crossings with the component and itself). Once we assign $+1$s and -1s, the linking number is the sum of all of the $+1$s and -1s divided by 2. The linking number of a knot k is denoted $LK(k)$.

Example: Below, we find the linking number of the example above. First, we labeled the $+1$s and -1s, then we added them to get -2 and divided by 2 to get $LK(k) = -1$.



$LK(k) = \frac{-1-1}{2} = -\frac{2}{2} = -1$

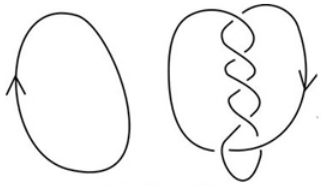
Practice Problem 5: Find the linking number of the link below.



Just as with knots, a link may have different projections. By moving a link around, we can get different projections of that link as long as the link is not cut.

Definition: A link is *splittable* if the components can be pulled apart without cutting the link.

Example: The link below is splittable. The two components can be taken apart without cutting the link.



$LK(k) = 0$

$LK(k) = 0$

Remark 1: The linking number depends on the orientation. If the orientation of both components are reversed at once, the linking number remains unchanged. (Try this with the Practice Problem above to see for yourself.) If the orientation of one component is reversed while preserving the orientation of the other component, the linking number will change sign. (Prove this!) In knot theory, it is proved that the linking number of an oriented link does not depend on the projection used ([1], Section 1.4). This means that the computed linking number will always be the same, no matter what projection of the link is used to compute it. In particular, this

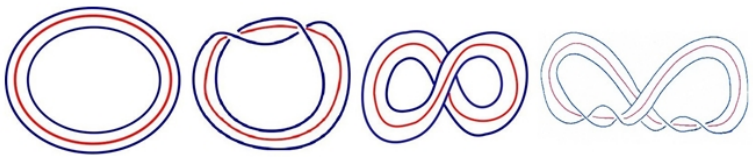
implies that linking number is an invariant of links. We will use this fact when we discuss supercoiling.

Remark 2: The linking number of a splittable link is 0. To see this, note that since the linking number is invariant under various projections, we may use a projection in which the two components are completely separated. For such a projection the linking number is 0 since there are no crossings between the two components.

Remark 3: The linking number of any link of two components is an integer. To see this, it is enough to prove that the sum in the numerator in the linking number is even. Given any link of two components, there are a series of crossings between the components that can be changed that would turn the link into a splittable link (similar to knots when crossings are changed to find the unknotting number). Changing a crossing will change the sign of that crossing. Thus, the effect of altering a crossing on the sum in the numerator is either 2 (if the crossing changes from -1 to $+1$) or -2 (if the crossing changes from $+1$ to -1). Since the linking number of a splittable link is 0, the sum in the numerator is always a multiple of 2. This proves that the linking number is an integer.

3. Ribbons

Definition: A *ribbon* is a thickened knot which may contain twists. If one takes a belt and twists and tangles it any way and then attaches the two ends, one will have a ribbon. Below, there are some examples of ribbons. Note that the two edges of a ribbon form a link. The *axis* of a ribbon is the knot that traverses the ribbon halfway between the edges. It is shown in red for each ribbon. The simplest ribbon is the *unribbon* which is shown at the most left below.



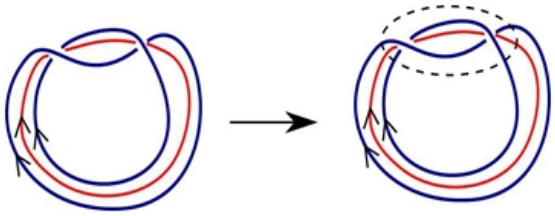
To study ribbons, we associate invariants to each ribbon that give us information about that ribbon. In particular, we want to know when two ribbons are not "the same." More precisely, how do we know we cannot continuously transform one ribbon into another ribbon without cutting. The ribbon invariants that we study in this paper are twist number, writhe, and linking number.

Definition: The *twist number* of a ribbon R, denoted by $Tw(R)$, measures just how much the ribbon twists around its axis. To define it, we assign a $+1$ to each full positive twist as shown in Figure 4. A full negative twist would look similar, but with opposite crossings as shown in Figure 5, and to which we assign a -1.

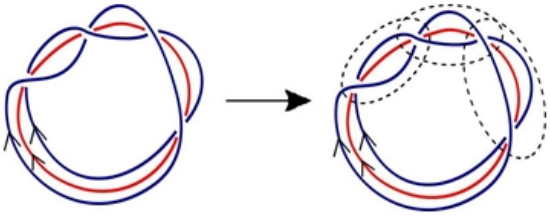
Figure 4: One complete twist, $+1$.

Figure 5: One complete negative twist, -1.

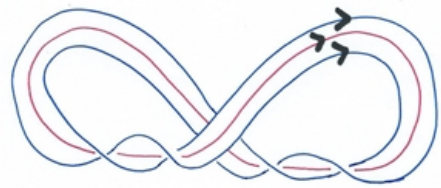
Example 1: The twist number of the ribbon below is 1 because we can see one full positive twist in the ribbon, as highlighted.



Example 2: The twist number of the ribbon below is 3. We have circled the full three positive twists in the image below.



Practice Problem 6: Find the twist number of the ribbon below.



In theory, the twist number may not be an integer. For example, there could be half of a twist. Note that one can increase the twist number by twisting the ribbon without cutting the ribbon. This means that the twist number is not an invariant of ribbons.

It is clear that the two edges of a ribbon form two identical knots. By giving identical orientations to the edges, we can orient the ribbon. The *writhe* of an oriented ribbon R, denoted by $Wr(R)$, is defined to be the writhe of one of the edges of the ribbon, as defined above. The writhe of a ribbon measures how much the axis of the ribbon is contorted in space.

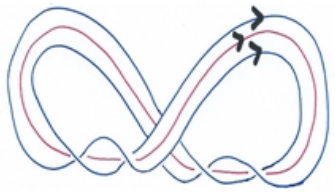
Example: To find the writhe of the oriented ribbon below, we consider the outside edge of the ribbon to be a knot. Then, we use the right hand rule to find the writhe of the knot and that is the writhe of the ribbon.

Figure 6: $Wr(R) = 1$.

Example: Using the same steps as the example above, we find the writhe of the ribbon below.

Figure 7: $Wr(R) = -2$.

Practice Problem 7: Find the writhe of the ribbon below.



Remark: Without cutting, we can change the writhe of an oriented ribbon by flipping the ribbon over itself a number of times. In essence, the resulting ribbon would be equivalent, but with different writhe. This implies that writhe is not an invariant of ribbons.

As we mentioned before, the two edges of a ribbon form a link. The *linking number* of an oriented ribbon R, denoted $Lk(R)$, is the linking number of this link.

Example: Let's find the linking number of the oriented ribbon below. We colored each of the edges to distinguish between them as the two different components of our link. We label the $+1$s and -1s where the blue and green links overlap. Notice that we do not count the crossings in which the components cross themselves.

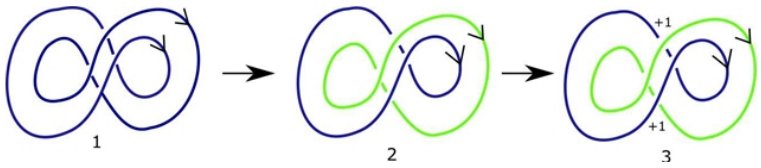
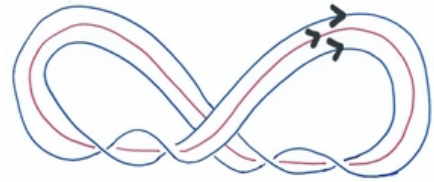


Figure 8: $Lk(R) = \frac{1+1}{2} = 1$.

Practice Problem 8: Find the linking number of the ribbon below.



Remark: We mentioned above that the linking number of a link is invariant under continuous transformations of the link. This implies that if one were to twist and flip a ribbon over itself, the linking number will not change. Thus, the linking number is invariant under continuous transformations.

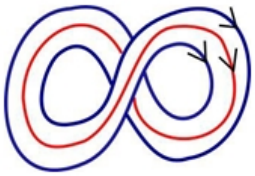
4. Conservation Law

Definition: The twist number, writhe, and linking number of a ribbon are related via what is called the *Conservation Law*. It states that for any ribbon R, the twist number and the writhe add up to the linking number.

$$Lk(R) = Tw(R) + Wr(R).$$

This law is actually hard to prove but was accomplished by J. White, B. Fuller, and G. Calugareanu ([1], Section 7.1).

Example: Consider the ribbon below.

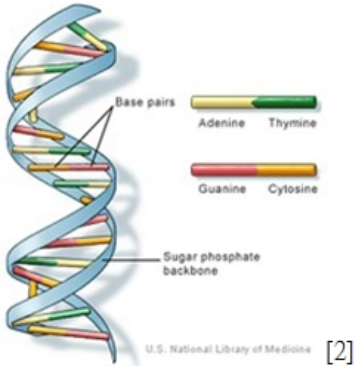


We already found that $Wr(R) = 1$ and $Lk(R) = 1$. This ribbon has no twists so $Tw(R) = 0$. We see that in this example, $Lk(R) = Tw(R) + Wr(R)$.

Practice Problem 9: Show that the Conservation Law holds for the ribbon below. You should have already solved for the twist, writhe, and linking number, so this should be simple!

5. DNA Structure and Supercoiling

DNA is made up of long molecular strands of alternating sugars and phosphates. The bases are Adenine, Thymine, Cytosine, and Guanine and they are bonded together by ladder-like rungs that are spiraled around each other. The rungs are hydrogen bonds between the pairs Adenine and Thymine and Cytosine and Guanine.



[2]

If one were to cut up a piece of DNA and attach the ends, the DNA will form a ribbon. This actually occurs naturally sometimes and is called

double cyclic DNA. For example, plasmids are small, circular, double-stranded DNA molecules that naturally exist in bacterial cells, and they also occur in some eukaryotes. This is where a connection between DNA and ribbon theory emerges.

The two strands of the DNA form the two edges of the ribbon. Thus, we can define twist number, writhe, and linking number for DNA. The Conservation Law will then hold consequently.


[3]

In the natural world, the double cyclic DNA comes in a coiled form, as shown in the picture below. The question is why does this happen?


[3]

To explain this, we can use ribbon theory developed above.

The Conservation Law becomes extremely interesting in this connection between ribbons and DNA. The law says that if we move a ribbon to a different position in space, any change in the twist of the ribbon must be balanced out by a change in writhe because the linking number will remain unchanged by the movement. When DNA is relaxed, it twists around it's own axis at a certain, controlled rate. The twisting happens by the way the sugars and base pairs bond together. If the DNA is more tightly twisted, it is manifested in the twist number. By folding over itself, the DNA is balancing it's writhe to compensate for the change in the twist, since the sum of writhe and twist number must remain constant. The process of DNA folding over itself and coiling up is called *supercoiling*.

Example: You have probably seen the phenomena of supercoiling that happens to telephone cords as well. Telephone cords coil up, as you can see in the picture below.

[4]

We can explain this phenomena using a similar argument as above. Since the two ends of the cord are fixed, we can think of it as a ribbon, even though the two ends are not attached. As the cord twists around itself, the twist number can increase or decrease and this has to be compensated by the writhe. The cord coils up in the appropriate direction to make up for the twist number. Again, this is because the sum of the twist number and writhe must remain unchanged.

References

[1] C. C. Adams, The Knot Book: An Elementary Introduction to the Mathematical Theory of Knots. New York: W.H. Freeman, 1994.

[2] http://www.sciencebuddies.org/blog/2013/04/celebrating-dna-and-the-history-of-the-double- helix.php

[3] http://cmgm.stanford.edu/

[4] http://www.siumed.edu/ bbartholomew/-lectures/Supercoiling.pdf

Tailored Quartic Roots

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Abstract

This paper makes use of a Möbius transformation to map the roots of a quartic equation to the roots of another quartic equation. The transformed roots can be tailored as desired, by expressing them in terms of an unknown parameter in the transformation. We discuss two cases; one in which the roots are reciprocals of each other, and the second in which the two roots are squares of the other two roots.

1. Introduction

Can one transform a given quartic equation so that the roots of the transformed quartic can be tailored as needed? For instance, can a root of the transformed quartic be the reciprocal of another root, or the square of another root? In this paper, we explore this issue by making use of the Möbius transformation [1, 2]. This transformation, proposed by the German mathematician August Ferdinand Möbius (1790 - 1868) in the nineteenth century, is expressed in its general form as

$$y = (Ax + B)/(Cx + D),$$

where x and y are variables in the transformation, and A, B, C, and D are numbers which can, in general, be complex.

The transformation is used to map the roots of a polynomial equation in one domain to the roots of a transformed equation in another domain; however, as we show in this paper, the transformed roots can be tailored as desired by suitably choosing an unknown parameter in the transformation. In this context, we propose the following:

Theorem: Consider the reduced quartic equation,

$$x^4 + bx^2 + cx + d = 0, \quad b,\ c,\ d\ \in\ \Re,$$

and its resolvent cubic equation,

$$8z^3 + 4bz^2 - 8dz + c^2 - 4bd = 0,$$

with a real root p. The Möbius transformation, $y = (x + f)/(gx + h)$, with f and h defined as

$$f^2 - [c/(2p + b)]f + p = 0,$$

and

$$h = gp/f,$$

transforms the above quartic equation in x to an even-powered quartic equation in y. If g is chosen as

$$g = \pm N^{1/4},\ \text{or},\ g = \pm N^{1/4}i$$

where

$$N = \frac{f^4 + bf^2 - cf + d}{(p/f)^4 + b(p/f)^2 - c(p/f) + d},$$

a pair of roots will be reciprocals of the other pair of roots, and if g is chosen as

$$g = \pm\frac{\left(-L + \sqrt{L^2 - 4N}\right)^{3/2}}{2\sqrt{2N}},\ \text{or},\ g = \pm\frac{\left(-L - \sqrt{L^2 - 4N}\right)^{3/2}}{2\sqrt{2N}},$$

where

$$L = \frac{6(p^2 + d) + b[f^2 + 4p + (p/f)^2] - 3c[f + (p/f)]}{(p/f)^4 + b(p/f)^2 - c(p/f) + d},$$

a pair of roots will be squares of the other pair of roots.

2. Transforming a quartic equation

Since a general quartic equation of the form $x^4+ax^3+bx^2+cx+d = 0$ can be converted to a *reduced* quartic equation (which has no x^3 term) using a simple linear transformation, we consider here,without loss of any generality, the following reduced quartic equation:

$$x^4 + bx^2 + cx + d = 0 \tag{1}$$

where b, c, and d are real coefficients. We plan to transform the quartic equation (1) in x to one in y using the Möbius transformation

$$y = (x + f)/(gx + h), \tag{2}$$

where f, g, and h are unknown numbers. In order to use this transformation in (1), the variable x in (2) is expressed as, $x = -(hy - f)/(gy - 1)$.

Substituting for x in (1) yields

$$[(hy-f)/(gy-1)]^4+b[(hy-f)/(gy-1)]^2-c[(hy-f)/(gy-1)]+d=0.$$

Expanding the above expression and rearranging it in descending powers of y, and further normalizing the y^4 term leads to the following transformed quartic equation in y:

$$y^4+ky^3+ly^2+my+n=0, \tag{3}$$

where k, l, m, and n are given by

$$\begin{aligned}
k &= \frac{-4fh^3-2bgh(fg+h)+cg^2(3h+fg)-4dg^3}{h^4+bg^2h^2-cg^3h+dg^4},\\
l &= \frac{6f^2h^2+b(f^2g^2+h^2+4fgh)-3cg(h+fg)+6dg^2}{h^4+bg^2h^2-cg^3h+dg^4},\\
m &= \frac{-4f^3h-2bf(fg+h)+c(h+3fg)-4dg}{h^4+bg^2h^2-cg^3h+dg^4},\\
n &= \frac{f^4+bf^2-cf+d}{h^4+bg^2h^2-cg^3h+dg^4}.
\end{aligned} \tag{4}$$

Setting the coefficients of y^3 and y in (3) to zero (i.e., $k=m=0$) converts (3) to a quadratic equation in y^2:

$$y^4+ly^2+n=0, \tag{5}$$

as well as resulting in the following two expressions:

$$-4fh^3-2bgh(fg+h)+cg^2(3h+fg)-4dg^3=0, \tag{6}$$

and

$$-4f^3h-2bf(fg+h)+c(h+3fg)-4dg=0. \tag{7}$$

Multiplying (6) by f and (7) by gh, and subtracting results in

$$4f^2h^2+cg(h+fg)-4dg^2=0. \tag{8}$$

Similarly, multiplying (7) by g^2 and subtracting the result from (6) leads to

$$(2fh+bg)(h+fg)-cg^2=0. \tag{9}$$

Eliminating the term $(h+fg)$ from (8) using (9) yields

$$8(fh/g)^3+4b(fh/g)^2-8d(fh/g)+c^2-4bd=0. \tag{10}$$

Notice that (10) is a cubic equation in (fh/g). In order to solve a quartic equation, we need to solve an associated cubic equation, which is known as the *resolvent* cubic equation. The cubic (10) is a resolvent cubic equation, with real coefficients and therefore it has at least one real root, say p, which can be determined from the well known Cardano formulae [2]. So, now $(fh/g)=p$ is a known number, and its use in (9) yields a quadratic

equation in f,

$$f^2 - [c/(2p+b)]f + p = 0;$$

and solving this we can determine f. Subsequently (h/g) is determined from the relation $(h/g) = (p/f)$. The quartic equation (5) is quadratic in y^2 and therefore its two solutions are

$$y^2 = \left(-l \pm \sqrt{l^2 - 4n}\right) \Big/ 2.$$

Further taking the square root of above expression yields four solutions of quartic equation (5):

$$y = \pm\sqrt{\left(-l \pm \sqrt{l^2 - 4n}\right) \Big/ 2}. \tag{11}$$

The expressions for l and n in terms of f and p (which are known quantities) are obtained by using the substitution $(h/g) = (p/f)$ in (4), giving us $l = L/g^2$, where

$$L = \frac{6(p^2+d) + b[f^2 + 4p + (p/f)^2] - 3c[f + (p/f)]}{(p/f)^4 + b(p/f)^2 - c(p/f) + d},$$

and $n = N/g^4$, where

$$N = \frac{f^4 + bf^2 - cf + d}{(p/f)^4 + b(p/f)^2 - c(p/f) + d}.$$

Making use of these expressions, the four roots of the quartic equation (5) obtained earlier in (11) are now expressed in terms of L, N and g as,

$$\begin{aligned} y_1 &= \sqrt{-L + \sqrt{L^2 - 4N}} \Big/ \left(\sqrt{2}\, g\right), \\ y_2 &= \sqrt{-L - \sqrt{L^2 - 4N}} \Big/ \left(\sqrt{2}\, g\right), \\ y_3 &= -y_1, \qquad y_4 = -y_2. \end{aligned} \tag{12}$$

From the expressions in (12), it is clear that the unknown g is yet to be determined. By tailoring the roots of (5) as desired, one can determine g. We consider here two cases; in the first case, the two roots of the transformed quartic are reciprocals of the other two roots; and in the second case, the two roots are squares of the other two roots.

2.1. Two roots of a transformed quartic are reciprocals of other the two roots

Let the root y_2 be the reciprocal of the root y_1 i.e., $y_2 = 1/y_1$, which

results in,

$$\frac{\sqrt{-L-\sqrt{L^2-4N}}}{\sqrt{2}\,g} = \frac{\sqrt{2}\,g}{\sqrt{-L+\sqrt{L^2-4N}}}.$$

Simplifying the above expression, we obtain four expressions for g as:

$$g = \pm N^{1/4},\ \ \pm N^{1/4}\,i.$$

Note that any one of the above four expressions can be used to determine g. Let $g = N^{1/4}$ so from (12) we have

$$y_1 = \sqrt{Q+\sqrt{Q^2-1}},\ \text{ where }\ Q = -L/(2\sqrt{N}). \tag{13}$$

Consequently, we determine the remaining three roots of the quartic equation (5) as: $y_2 = 1/y_1$, $y_3 = -y_1$, and $y_4 = -1/y_1$. Now, the roots of quartic equation (1) can be obtained from the roots of quartic equation (5) using the transformation $x = -(hy-f)/(gy-1)$ [see (2)].

To illustrate the proposed method with a numerical example, consider the following quartic equation in x,

$$x^4 + 3x^2 - 6x + 10 = 0,$$

and mapping its roots to the roots of a quartic in y using the transformation, $y = (x+f)/(gx+h)$, with the condition that the roots of transformed quartic equation are reciprocals of each other. The resolvent cubic equation (10) is

$$(fh/g)^3 + 1.5(fh/g)^2 - 10(fh/g) - 10.5 = 0.$$

Solving this cubic, we obtain three roots: $3, 1$, and 3.5. Choosing $p = (fh/g) = 1$, the quadratic equation in f is $f^2+6f-1 = 0$, and hence $f = -3\pm\sqrt{10}$. Choosing $f = -3+\sqrt{10}$, we determine that $(h/g) = (p/f) = -3-\sqrt{10}$. Using these values, L and N are determined respectively as 0.0392427038 and 0.0072293966.

Next, from the relation, $g = N^{1/4}$, we determine that $g = 0.2915919377$ and $h = g(p/f) = 1.7968704838$. From the relation $Q = -L/\sqrt{N}$, we determine $Q = -0.2307692308$. From (14) we find the root y_1 to be

$$y_1 = 0.6201736729 + 0.7844645405i,$$

and consequently y_2, y_3, and y_4 are found to be:

$$y_2 = 0.6201736729 - 0.7844645405i,$$
$$y_3 = 0.6201736729 - 0.7844645405i,$$
$$y_4 = -0.6201736729 + 0.7844645405i.$$

Using these, the roots of the given quartic equation $x^4+3x^2-6x+10 = 0$ are determined through the transformation $x = -(hy-f)/(gy-1)$, as:

$$x_1 = -1-2i,\ \ x_2 = -1+2i,\ \ x_3 = 1+i,\ \ x_4 = 1-i.$$

The use of the other three expressions for g to obtain the roots of y and x is left as an exercise for the reader.

2.2. Two roots of transformed quartic are squares of other two roots

In this case, we can have either y_2 be the square of y_1, or y_1 be the square of y_2. When $y_2 = y_1^2$, we obtain:

$$\sqrt{-L - \sqrt{L^2 - 4N}} = \left(-L + \sqrt{L^2 - 4N}\,\right) \Big/ \left(\sqrt{2}\, g\right),$$

which after simplification yields two expressions for g as,

$$g = \pm \left(-L + \sqrt{L^2 - 4N}\right)^{3/2} \Big/ \left(2\sqrt{2N}\right). \tag{14}$$

Any one of the above expressions can be used to determine the roots of (5) so, using the one with positive sign, we determine y_1 [see (12)],

$$y_1 = Q - \sqrt{Q^2 - 1}, \;\; \text{where} \;\; Q = -L/(2\sqrt{N}), \tag{15}$$

and the remaining three roots of (5) are obtained as: $y_2 = y_1^2$, $y_3 = -y_1$, and $y_4 = -y_1^2$. The roots of quartic equation (1) are now obtained from the roots of (5) using the transformation, $x = -(hy - f)/(gy - 1)$.

Let us solve one numerical example using the same quartic equation considered in Section 2.1. Note that we can use the same values of f, p, (h/g), L, N, and Q, which have been determined in the previous case. Using (14), g is found to be:

$$g = \pm(0.2643011706 - 0.1231695960i)$$

and choosing $g = 0.2643011706 - 0.1231695960i$, we determine that

$$h = -1.6286971990 + 0.7590052499i.$$

Next y_1 is evaluated from (15), and consequently y_2, y_3, and y_4 are obtained as:

$$\begin{aligned} y_1 &= -0.2307692308 - 0.9730085108i \\ y_2 &= -0.8934911243 + 0.4490808511i \\ y_3 &= 0.2307692308 + 0.9730085108i \\ y_4 &= 0.8934911243 - 0.4490808511i. \end{aligned}$$

The roots of the quartic equation $x^4 + 3x^2 - 6x + 10 = 0$ are obtained from the roots of y using the transformation $x = -(hy - f)/(gy - 1)$, as:

$$x_1 = 1 + i, \quad x_2 = 1 - i, \quad x_3 = -1 - 2i, \quad x_4 = -1 + 2i.$$

One may use the other value of g, and obtain the roots of the above quartic equation.

When $y_1 = y_2^2$, we obtain two expressions for g as

$$g = \pm \left(-L - \sqrt{L^2 - 4N}\right)^{3/2} \Big/ \left(2\sqrt{2N}\right),$$

and choosing the one with positive sign, we determine y_2 to be

$$y_2 = Q + \sqrt{Q^2 - 1}, \text{ where } Q = -L/(2\sqrt{N}).$$

The remaining roots of (5) are:

$$y_1 = y_2^2, y_3 = -y_2^2, \text{and } y_4 = -y_2.$$

The roots of (1) are determined from the roots of (5) using the transformation, $x = -(hy - f)/(gy - 1)$. We invite the interested reader to obtain the roots of y and x for the numerical example $x^4 + 3x^2 - 6x + 10 = 0$ using the condition, $y_1 = y_2^2$.

3. Summary

In this paper we have made use of a Möbius transformation to map the roots of a quartic equation to roots of another quartic equation in such a way that the new roots can be tailored as needed. This is accomplished by first determining two of the three unknowns in the transformation, which will lead to determination of the roots of the new quartic in terms of the third unknown, which is later used to tailor the roots. We have illustrated the proposed method by considering two cases with numerical examples; in the first case, the two roots of the transformed quartic equation are reciprocals of the other two roots, and in the second case the two roots are squares of the other two roots.

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[1] Douglas N. Arnold and Jonathan Rogness, "Möbius transformations revealed", Notices of the American Mathematical Society, Vol. 55, No. 10, November 2008, pp. 1226 - 1231.

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The Problem Corner

Edited by Pat Costello

The Problem Corner invites questions of interest to undergraduate students. As a rule, the solution should not demand any tools beyond calculus and linear algebra. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following new problems should be submitted on separate sheets before October 1, 2017. Solutions received after this will be considered up to the time when copy is prepared for publication. The solutions received will be published in the Fall 2017 issue of *The Pentagon*. Preference will be given to correct student solutions. Affirmation of student status and school should be included with solutions. New problems and solutions to problems in this issue should be sent to Pat Costello, Department of Mathematics and Statistics, Eastern Kentucky University, 521 Lancaster Avenue, Richmond, KY 40475-3102 (e-mail: pat.costello@eku.edu, fax: (859) 622-3051).

NEW PROBLEMS 789-797

Problem 789. *Proposed by Daniel Sitaru, Colegiul National Economic Theodor Costescu, Drobeta Turnu – Severin, Mehedinti, Romania.*

In triangle ABC, let $I =$ the incenter, $O =$ the circumcenter, $G =$ the centroid, and a, b, c the lengths of the sides. Prove that

$$\left(\sum IA\right)\left(\sum OA\right)\left(\sum GA\right) < (a+b)(b+c)(c+a).$$

Problem 790. *Proposed by Daniel Sitaru, Colegiul National Economic Theodor Costescu, Drobeta Turnu – Severin, Mehedinti, Romania.*

Prove that if $a, b \in \mathbb{R}$ with $a < b$, then

$$\ln\left|\left(\frac{2+\sin 2b}{2+\sin 2a}\right)\right| \leqslant \frac{2\sqrt{3}}{3}(b-a).$$

Problem 791. *Proposed by Jose Luis Diaz-Barrero, School of Civil Engineering, Barcelona Tech - UPC, Barcelona, Spain.*

Determine whether the real number

$$\frac{\ln(11+5\sqrt{2})}{\ln(5+11\sqrt{2})}$$

is rational or not.

Problem 792. *Proposed by Jose Luis Diaz-Barrero, School of Civil Engineering, Barcelona Tech - UPC, Barcelona, Spain.*

Let $x_1, x_2, \ldots, x_n$ be real numbers lying in the interval $(0, \pi/2)$. Prove that

$$\left(\frac{1}{n}\sum_{k=1}^{n}\sin x_k\right)\left(\frac{1}{n}\sum_{k=1}^{n}\cos x_k\right) \leqslant \frac{1}{2}.$$

Problem 793. *Proposed by D.M. Batinetu-Giurgiu, "Matei Basarab" National College, Bucharest, Romania, Neculai Stanciu, "George Emil Palade", Buzau, Romania.*

If $a \in [0, \pi/4]$, compute $\int_0^a (x^2 - ax + a^2)(\ln(1 + \tan x \tan a))dx$.

Problem 794. *Proposed by D.M. Batinetu-Giurgiu, "Matei Basarab" National College, Bucharest, Romania, Neculai Stanciu, "George Emil Palade", Buzau, Romania.*

Let a, b, c be positive real numbers. Prove that

$$(1+a)(1+b)(1+c) \geqslant \left(1+\frac{2ab}{a+b}\right)\left(1+\frac{2bc}{b+c}\right)\left(1+\frac{2ca}{c+a}\right).$$

.

Problem 795. *Proposed by Michal Kremzer, Glicice, Silesia, Poland.*

Let $\mathbb{Q}$ be the set of rational numbers. Does there exist a function $f : (\mathbb{Q} - \{0\}) \to (\mathbb{Q} - \{0\})$ so that $f(x) < f(3x) < f(2x)$ for all x in the set $(\mathbb{Q} - \{0\})$?

Problem 796. *Proposed by Kadir Altintas, Turkey and Leonard Giugiuc, Romania.*

If A, B and C are the angles of a triangle, prove that

$$\sqrt{6\left(1+\cos A\cos B\cos C\right)} \geqslant 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}.$$

Problem 797. *Proposed by the editor.*

Let integer n be called a *consecutives concatenated* number when n is formed by concatenating two consecutive integers. For example, 67 and 1314 are consecutives concatenated numbers. 67 is prime, but 1314 is composite. It turns out there are lots of consecutives concatenated primes. Find a consecutives concatenated prime of 20 digits where the first integer of the two concatenated is divisible by $2^3 * 3^3$.

SOLUTIONS TO PROBLEMS 769-779

Problem 769. *Proposed by the Northwest Missouri State University Problem Solving Group, Maryville, MO.*

Let $T_k = \frac{k(k+1)}{2}$ be the k^{th} triangular number.

1. Under what condition(s) on $n \in \mathbb{N}$ does 13 divide $2(T_{3^n} - 1)$?
2. Under what condition(s) on $n \in \mathbb{N}$ does 13 divide $2T_{3^n} + 1$?

Solution *by Henry Ricardo, New York Math Circle, NY.*

1. We have $2(T_{3^n} - 1) = 2\left(\frac{3^n(3^n+1)}{2} - 1\right) = 3^{2n} + 3^n - 2$. If $n = 3k$ for some positive integer k, then $3^n = 3^{3k} = (3^3)^k \equiv 1 \pmod{13}$. Consequently, $3^{2n} + 3^n - 2 = (3^{3k})^2 + 3^{3k} - 2 \equiv 1^2 + 1 - 2 \equiv 0 \pmod{13}$. If $n = 3k+1$, then $3^{2n}+3^n-2 = (3^{3k})^2\cdot 3^2+3^{3k}\cdot 3-2 \equiv 9+32 \equiv 10$. If $n = 3k + 2$, then $3^{2n} + 3^n - 2 = (3^{3k})^2 \cdot 3^4 + 3^{3k} \cdot 3^2 - 2 \equiv 81 + 9 - 2 \equiv 10$. So 13 divides $2(T_{3^n} - 1)$ if and only if n is a positive multiple of 3.

2. We have $2T_{3^n} + 1 = 2\left(\frac{3^n(3^n+1)}{2}\right) + 1 = 3^{2n} + 3^n + 1$. Since $3^{3k} \equiv 1 \pmod{13}$, $3^{2n} + 3^n + 1 = (3^{3k})^2 + 3^{3k} + 11^2 + 1 + 1 \equiv 3 \pmod{13}$. If $n = 3k+1$, then $3^{2n}+3^n+1 = (3^{3k})^2\cdot 3^2+3^{3k}\cdot 3+1 \equiv 9+3+1 \equiv 0$. If $n = 3k+2$, then $3^{2n}+3^n+1 = (3^{3k})^2\cdot 3^4+3^{3k}\cdot 3^2+1 \equiv$

$81+9+1 \equiv 0$. So 13 divides $2T_{3^n}+1$ if and only if n is *not* a multiple of 3.

Also solved by the Missouri State University Problem Solving Group, Missouri State University, Springfield, MO; Jeremiah Bartz, University of North Dakota, Grand Forks, ND; Ioan Viorel Codreanu, Satulung, Maramures, Romania; and the proposers.

Problem 770. *Proposed by Jose Luis Diaz-Barrero, School of Civil Engineering, Barcelona Tech - UPC, Barcelona, Spain.*

Let $f : [0,1] \to \mathbb{R}$ be a continuous concave function. Prove that

$$\frac{3}{4}\int_0^{1/7} f(t)\,dt + \frac{1}{12}\int_0^{2/7} f(t)\,dt \leq \frac{2}{3}\int_0^{3/14} f(t)\,dt.$$

Solution *by the proposer.*

If f is concave in $[0,1]$, then for all $x, y \in [0,1]$ and for all $\lambda \in [0,1]$, we have

$$f(\lambda x + (1-\lambda)y) \geqslant \lambda f(x) + (1-\lambda)f(y). \tag{1}$$

Setting $\lambda = 3/4$ and $x = t/7, y = (3t)/7$, $0 \leq t \leq \frac{7}{3}$ into (1) we get

$$(3/4)\, f\,(t/7) + (1/4)\, f\,(3t/7) \leqslant f\,(3t/14)\,.$$

Integrating on $[0,1]$ yields

$$\frac{3}{4}\int_0^1 f(t/7)dt + \frac{1}{4}\int_0^1 f(3t/7)dt \leqslant \int_0^1 f(3t/14)dt. \tag{2}$$

Setting $u = t/7$, we have

$$\frac{3}{4}\int_0^1 f(t/7)dt = \frac{21}{4}\int_0^{1/7} f(u)du.$$

Likewise, setting $u = 3t/7$, we get

$$\frac{1}{4}\int_0^1 f(3t/7)dt = \frac{7}{12}\int_0^{3/7} f(u)du.$$

Setting $u = 3t/14$, we obtain

$$\int_0^1 f(3t/14)dt = \frac{14}{3}\int_0^{3/14} f(u)du.$$

Substituting these into (2) and dividing by 7, we get

$$\frac{3}{4}\int_0^{1/7} f(u)du + \frac{1}{12}\int_0^{2/7} f(u)du \leqslant \frac{2}{3}\int_0^{3/14} f(u)du.$$

Problem 771. *Proposed by Jose Luis Diaz-Barrero, School of Civil Engineering, Barcelona Tech - UPC, Barcelona, Spain.*

Let $a < b$ be positive real numbers and let $f : [a, b] \to \mathbb{R}$ be a continuous function. Prove that there exists $c \in (a, b)$ such that

$$2f(c) = \frac{1}{\sqrt{c}}\left[\frac{\sqrt{a}+\sqrt{c}}{a-c} + \frac{\sqrt{b}+\sqrt{c}}{b-c}\right]\int_a^c f(t)\ dt.$$

Solution *by the proposer.*

Consider the function $F : [a, b] \to \mathbb{R}$ defined by

$$F(x) = (\sqrt{x}-\sqrt{a})(\sqrt{x}-\sqrt{b})\int_a^x f(t)dt.$$

The function $F(x)$ is continuous on $[a, b]$, differentiable on (a, b), and $F(a) = F(b) = 0$. Therefore, according to Rolle's Theorem, there exists $c \in (a, b)$ such that $F'(c) = 0$. That is

$$\frac{1}{2\sqrt{c}}(\sqrt{c}-\sqrt{b})\int_a^c f(t)dt + \frac{1}{2\sqrt{c}}(\sqrt{c}-\sqrt{a})\int_a^c f(t)dt + (\sqrt{c}-\sqrt{a})(\sqrt{c}-\sqrt{b})f(c) = 0.$$

Dividing both sides by $(\sqrt{c}-\sqrt{a})(\sqrt{c}-\sqrt{b})$ we get

$$\frac{1}{2\sqrt{c}}\left[\frac{1}{\sqrt{c}-\sqrt{a}} - \frac{1}{\sqrt{c}-\sqrt{b}}\right]\int_a^c f(t)dt + f(c) = 0.$$

Equivalently, $\frac{1}{2\sqrt{c}}\left[\frac{\sqrt{c}+\sqrt{a}}{c-a} - \frac{\sqrt{c}+\sqrt{b}}{c-b}\right]\int_a^c f(t)dt + f(c) = 0$. From this, we have $2f(c) = \frac{1}{\sqrt{c}}\left[\frac{\sqrt{a}+\sqrt{c}}{a-c} + \frac{\sqrt{b}+\sqrt{c}}{b-c}\right]\int_a^c f(t)dt$.

Problem 772. *Proposed by Marcel Chirita, Bucharest, Romania.*

Solve in positive integers the equation $x^2 - 97y! = 2015$.

Solution *by Jonathan Mangum (student), Missouri State University, Springfield, MO.*

We consider solutions on a case by case basis. For $y = 0$ and $y = 1$ we do not have integer solutions for x. For $y = 2$, we have $x^2 - 194 = 2015$ and $x = 47$ is a positive integer solution. We now show that for all other values of $x, y!$ is not divisible by 3 and therefore y is not an integer. First, we rearrange the equation to $y! = (x^2-2015)/97$. For the right had side of this equation to be an integer, we need $x^2 \equiv 2015 \pmod{97}$, which reduces to $x^2 \equiv 75 \pmod{97}$. Computation shows that 75 is a quadratic residue of 97 with roots 47 and 50. So when $y!$ is an integer, x must be of the form $47 + 97t$ or $50 + 97t$. This leaves us with equations $y! = 97t^2 + 94t + 2$ or $y! = 97t^2 + 100t + 5$. But when $y \geq 3$, $y! \equiv 0 \pmod{3}$. Checking cases mod 3:

t	$97t^2 + 94t + 2$	$97t^2 + 100t + 5$
0	2	2
1	1	1
2	2	2

Since none of the entries in the last 2 columns are 0, the only solution is $x = 47, y = 2$.

Also solved by Jeremiah Bartz, University of North Dakota, Grand Forks, ND; Ioan Viorel Codreanu, Satulung, Maramures, Romania; Missouri State Problem Solving Group, Missouri State University, Springfield, MO; and the proposer.

Problem 773. *Proposed by Marcel Chirita, Bucharest, Romania.*

Let a, b, c be real numbers greater than or equal to 3. Prove that

$$\min\left(\frac{a^2b^2+3b^2}{b^2+27}, \frac{b^2c^2+3c^2}{c^2+27}, \frac{c^2a^2+3a^2}{a^2+27}\right) \leq \frac{abc}{9}.$$

Solution *by Ioan Viorel Codreanu, Satalung, Maramures, Romania.*

We have $\frac{a^2b^2+3b^2}{b^2+27} = (a^2+3)\frac{b^2}{b^2+27}$. Observe that

$$\frac{b^2}{b^2+27} \leqslant \frac{b^3}{9(b^2+3)} \Leftrightarrow 9b^2 + 27 \leqslant b^3 + 27b \Leftrightarrow (b-3)^3 \geqslant 0 \Leftrightarrow b \geqslant 3.$$

Using similar relations, we get

$$\begin{aligned}
&\min\left(\frac{a^2b^2+3b^2}{b^2+27}, \frac{b^2c^2+3c^2}{c^2+27}, \frac{c^2a^2+3a^2}{a^2+27}\right)\\
&\leqslant \sqrt[3]{\frac{a^2b^2+3b^2}{b^2+27}\cdot\frac{b^2c^2+3c^2}{c^2+27}\cdot\frac{c^2a^2+3a^2}{a^2+27}}\\
&\leqslant \sqrt[3]{\frac{(a^2+3)b^3}{9(b^2+3)}\cdot\frac{(b^2+3)c^3}{9(c^2+3)}\cdot\frac{(c^2+3)a^3}{9(a^2+3)}}\\
&= \frac{abc}{9}.
\end{aligned}$$

Note that equality holds iff $a = b = c = 3$.

Also solved by the proposer.

Problem 774. *Proposed by Mohammad K. Azarian, University of Evansville, Evansville, Indiana.*

If both x and y are positive real numbers, then find y as a function of x, provided

$$y' + (y+1)\ln(y+1)[1 - (\ln(y+1))^{-2}((1/4)x^{-1} + x^{1/2})]x^{1/2} = 0.$$

Solution *by the proposer.*

If we let $z = y + 1$, then we have

$$z' + z\ln z\left[1 - (\ln z)^{-2}\left((1/4)\,x^{-1} + x^{1/2}\right)\right]x^{1/2} = 0.$$

If we let $w = lnz$, then since $w > 0$, the above can be rewritten as

$$w' + w\left[1 - w^{-2}\left((1/4)\,x^{-1} + x^{1/2}\right)\right]x^{1/2} = 0,$$

which is equivalent to the Bernoulli equation

$$w' + x^{1/2}w = \left((1/4)\,x^{-1/2} + x\right)w^{-1}.$$

To solve this equation, we let $s = w^2$ and obtain the linear differential equation

$$s' + 2x^{1/2}s = (1/2)\,x^{-1/2} + 2x.$$

Hence

$$\begin{aligned}
s &= e^{-2\int x^{1/2}dx}[\int ((1/2)x^{-1/2} + 2x)e^{2\int x^{1/2}dx}dx + C]\\
&= e^{-4/3x^{3/2}}[\int ((1/2)x^{-1/2} + 2x)e^{4/3x^{3/2}}dx + C]\\
&= e^{-4/3x^{3/2}}[(1/2)\int x^{-1/2}e^{4/3x^{3/2}}dx + 2\int xe^{4/3x^{3/2}}dx + C].
\end{aligned}$$

Now, if we integrate the second integral using integration by parts letting $u = x^{1/2}$ and $dv = x^{1/2}e^{4/3x^{3/2}}dx$, then $\int vdu$ will cancel with the first integral and we will have

$$s = e^{-4/3x^{3/2}}[x^{1/2}e^{4/3x^{3/2}} + C] = x^{1/2} + Ce^{-4/3x^{3/2}}.$$

Therefore,

$$w = \pm(x^{1/2} + Ce^{-4/3x^{3/2}})^{1/2}.$$

Consequently,

$$y(x) = -1 + e^{\pm(x^{1/2}+Ce^{-4/3x^{3/2}})^{1/2}}.$$

Also partially solved by Madison Estabrook, Missouri State University, Springfield, MO.

Problem 775. *Proposed by Mohammad K. Azarian, University of Evansville, Evansville, Indiana.*

Determine y explicitly as a function of x provided

$$x(1+\sin x)y' + [(x^2+y^2+4) - (-3+\sin x)y - 2(1+\sin x)] = 0,$$

with $y \neq -2$ and $x \neq k\pi$.

Solution *by the proposer.*

We note that the given differential equation can be rewritten as

$$\left[\left(y^2+x^2+4y+4\right) - (y+2)(1+\sin x)\right]dx + [x+\sin x]\,dy = 0,$$

which is equivalent to

$$\left[\left((y+2)^2+x^2\right) - (y+2)(1+\sin x)\right]dx + [x+\sin x]\,dy = 0.$$

Since $((y+2)^2+x^2)(1+\sin x) \neq 0$, we divide the equation by this to get

$$\left[\frac{1}{1+\sin x} - \frac{y+2}{(y+2)^2+x^2}\right]dx + \left[\frac{x}{(y+2)^2+x^2}\right]dy = 0$$

Substitute $u = y+2$ in the above equation and obtain

$$\left[\frac{1}{1+\sin x} - \frac{u}{u^2+x^2}\right]dx + \left[\frac{x}{u^2+x^2}\right]dy = 0. \tag{1}$$

Since $\frac{\partial}{\partial u}\left[\frac{1}{1+\sin x} - \frac{u}{u^2+x^2}\right] = \frac{\partial}{\partial x}\left[\frac{x}{u^2+x^2}\right]$, (1) is an exact differential

equation. Thus, there must exist a continuous function $g(x,u)$ such that

$$\begin{aligned} g(x,u) &= \int \frac{1}{1+\sin x} - \frac{u}{u^2+x^2}dx \\ &= (\tan x - \sec x) - \tan^{-1}(x/u) + h(u). \end{aligned}$$

Hence

$$g_u(x,u) = \frac{x}{u^2+x^2} + h'(u) = \frac{x}{u^2+x^2}.$$

Consequently, $h(u) = C$ and therefore the general solution of the differential equation is

$$g(x,y) = (\tan x - \sec x) - \tan^{-1}(x/(y+2)) = C.$$

Finally, if we solve this equation for y, we obtain

$$y = -2 + x[\tan(\tan x - \sec x - C)]^{-1}.$$

Also partially solved by Madison Estabrook, Missouri State University, Springfield, MO.

Problem 776. *Proposed by Natanael Karjanto, University College, Suwon, Republic of Korea.*

Show that for $\alpha > 0$ and $n \in \mathbb{N}$, the harmonic number H_n can be represented by the following integral:

$$\begin{aligned} H_n &= \sum_{k=1}^{n} \frac{1}{k} \\ &= \frac{1}{2}\sum_{k=1}^{n} \int_{-\infty}^{\infty} e^{-\alpha|x|}\,\text{sech}^{k+1}x\,dx \\ &\qquad + \frac{1}{2}\sum_{k=1}^{n} \frac{\alpha-(k-1)}{n} \int_{-\infty}^{\infty} e^{-(\alpha+1)|x|}\,\text{sech}^{k}x\,dx. \end{aligned}$$

Solution *by the proposer.*

Since both integrands of the integrals on the right-hand sides are even functions, they are equivalent to twice the integral from 0 to ∞ (or $-\infty$ to 0). Without loss of generality, we consider the proof by integrating through

the former one. Thus, we need to show that

$$H_n = \sum_{k=1}^{n} \frac{1}{k} = \sum_{k=1}^{n} \left(I_{k+1} + \frac{\alpha - (k-1)}{k} J_k \right)$$

where

$$I_{k+1} = \int_0^\infty e^{-\alpha x} \operatorname{sech}^{k+1} x dx \qquad \alpha > 0, k \in \mathbb{N}$$
$$J_k = \int_0^\infty e^{-(\alpha+1)x} \operatorname{sech}^{k} x dx \qquad \alpha > 0, k \in \mathbb{N}.$$

We will show this using induction. For $k = 1$, we need to show that for $\alpha > 0$ we have $I_2 + J_1 = 1$. Consider I_2. By expressing $\operatorname{sech} x = 2e^x/(1 + e^{2x})$ and letting $y = e^x$, we get

$$I_2 = \int_0^\infty e^{-\alpha x} \operatorname{sech}^2 x dx = \int_1^\infty \frac{2^2 y^{-\alpha+2}}{(1+y^2)^2} \frac{dy}{y} = \int_1^\infty 2y^{-\alpha} \frac{d(1+y^2)}{(1+y^2)^2}.$$

Implementing integration by parts on the last expression and returning to the original variable x, yields

$$\begin{aligned} I_2 &= \lim_{b\to\infty} \left(-y^{-(\alpha+1)} \frac{2y}{1+y^2} \right) |_1^b - \alpha \int_1^\infty y^{-(\alpha+1)} \frac{2y}{1+y^2} \frac{dy}{y} \\ &= \lim_{b\to\infty} (-e^{-(\alpha+1)x} \operatorname{sech} x)|_0^b - \alpha \int_0^\infty e^{-(\alpha+1)x} \operatorname{sech} x dx \end{aligned}$$

and $I_2 + \alpha J_1 = 1, \alpha > 0$. Now assume the case of $k = n$ is true. We want to show the case $k = n + 1$ is also true. It is sufficient to show that for $\alpha > 0$

$$I_{n+2} + \frac{\alpha - n}{n+1} J_{n+1} = \frac{1}{n+1}.$$

Employing a similar technique to that of the base case, we observe that

$$\begin{aligned} I_{n+2} = \int_0^\infty e^{-\alpha x} \operatorname{sech}^{n+2} x dx &= \int_1^\infty \frac{2^{n+2} y^{-(\alpha-(n+2))}}{(1+y^2)^{n+2}} \frac{dy}{y} \\ &= \int_1^\infty 2^{n+1} y^{-(\alpha-n)} \frac{d(1+y^2)}{(1+y^2)^{n+2}}. \end{aligned}$$

After integrating by parts and returning to the variable x, we obtain

$$\begin{aligned} I_2 &= \lim_{b\to\infty}\left[-\frac{1}{n+1}y^{-(\alpha+1)}\left(\frac{2y}{1+y^2}\right)^{n+1}\right]_1^b \\ &\quad -\frac{\alpha-n}{n+1}\int_1^\infty y^{-(\alpha+1)}(\frac{2y}{1+y^2})^{n+1}\frac{dy}{y} \\ &= \lim_{b\to\infty}(-e^{-(\alpha+1)x}/(n+1)\text{sech}^{n+1}x)|_0^b \\ &\quad -\frac{\alpha-n}{n+1}\int_0^\infty e^{-(\alpha+1)x}\text{sech}^{n+1}x dx \end{aligned}$$

and $I_{n+2}+\frac{\alpha-n}{n+1}J_{n+1}=\frac{1}{n+1}$, $\alpha>0$. This completes the induction step. Then

$$\begin{aligned} RHS &= \sum_{k=1}^{n+1}\left(I_{k+1}+\frac{\alpha-(k-1)}{k}J_k\right) \\ &= \sum_{k=1}^{n}\left(I_{k+1}+\frac{\alpha-(k-1)}{k}J_k\right)+I_{n+2}+\frac{\alpha-n}{n+1}J_{n+1} \\ &= \sum_{k=1}^{n}\frac{1}{k}+\frac{1}{n+1}=\sum_{k=1}^{n+1}\frac{1}{k}=LHS. \end{aligned}$$

Combining with the other part where we integrate from $-\infty$ to 0, we obtain the desired result.

Problem 777. *Proposed by Robert Gardner and William Ty Frazier (graduate student), East Tennessee State University, Johnson City, TN.*

Let $[x]$ represent the floor (or greatest integer) function. Let $n, m \in \mathbb{N}$ with $2 \le m \le n-1$ and $k \in \{0, 1, 2, \ldots, m-1\}$. Use the floor function to express the smallest integer N greater than or equal to n which is congruent to k modulo m.

Solution *by Jeremiah Bartz, University of North Dakota, Grand Forks, ND.*

Let $n, m \in \mathbb{N}$ with $2 \le m \le n-1$ and $k \in \{0, 1, 2, \ldots, m - 1\}$. We claim

$$N = m\left[1+\frac{n-k-1}{m}\right]+k.$$

Observe that the right hand side is an integer congruent to k modulo m. Since $n > m$, we can write $n = mq+r$ with $q \in \mathbb{N}$ and $r \in \{0, 1, 2, \ldots, m - 1\}$

so that

$$m\left[1+\frac{n-k-1}{m}\right]+k = m\left[1+\frac{mq+r-k-1}{m}\right]+k$$
$$= m\left[1+q=\frac{r-k-1}{m}\right]+k.$$

We consider two cases, namely $0 \leq r \leq k$ and $r \geq k+1$. If $0 \leq r \leq k$, then

$$m\left[1+q+\frac{r-k-1}{m}\right]+k = mq+k \geqslant mq+r = n.$$

If $r \geq k+1$, we see

$$m\left[1+q+\frac{r-k-1}{m}\right]+k = m(q+1)+k > mq+r = n.$$

In both cases, the expression is greater than or equal to n. Lastly, we show the expression is the smallest such integer with the desired properties. Suppose $N' \equiv k \pmod m$ with $N' < m\left[1+\frac{n-k-1}{m}\right]+k$, so in particular $N' \leqslant \left(m\left[1+\frac{n-k-1}{m}\right]+k\right)-m$. If $0 \leq r \leq k$, then

$$N' \leqslant (m\left[1+q+\frac{r-k-1}{m}\right]+k)-m = m(q-1)+k < mq+r = n.$$

If $r \geq k+1$, then

$$N' \leqslant (m\left[1+q+\frac{r-k-1}{m}\right]+k)-m = mq+k < mq+k+1 \leqslant mq+r = n.$$

In both cases, $N' < n$. It follows that $m\left[1+\frac{n-k-1}{m}\right]+k$ is the desired expression.

Also solved by the proposers.

Problem 778. *Proposed by Thomas Chu (graduate student), Western Illinois University, Macomb, IL.*

Let p_1 and p_2 be distinct odd primes both congruent to 1 or $3 \bmod 4$. Prove that

$$\gcd\left(\frac{p_1+p_2}{2}, \frac{|p_1-p_2|}{4}\right) = 1.$$

Solution *by the Missouri State University Problem Solving Group, Missouri State University, Springfield, MO.*

We will show more generally that if p_1 and p_2 are distinct relatively prime odd integers such that $p_1 \equiv p_2 \pmod 4$, then the result follows.

Without loss of generality, we may assume that $p_1 > p_2$ and dispense with the absolute value. Letting $a = (p_1 + p_2)/2$ and $b = (p_1 - p_2)/4$, we have $p_1 = a + 2b$ and $p_2 = a - 2b$. Since $gcd(p_1, p_2) = 1$, there are integers s and t such that $p_1 s + p_2 t = 1$. Therefore, $a(s + t) + b(2s - 2t) = 1$ and hence $gcd(a, b) = 1$.

Also solved by Ioan Viorel Codreanu, Satulung, Maramures, Romania; Jeremiah Bartz, University of North Dakota, Grand Forks, ND; and the proposer.

Problem 779. *Proposed by the editor.*

Use all the digits 1, 2, 3, ..., 9 without repeats to create two primes such that their product is a maximum. Each digit should be used in only one of the two numbers.

Solution *by Jeremiah Bartz, University of North Dakota, Grand Forks, ND.*

We claim the two primes are 8641 and 97523 with product M = 842696243. A maximal product will occur for two primes which begin with the digits 8 and 9. Also note the two primes will contain k and $9 - k$ digits for $1 \leq k \leq 8$. Thus one of the primes will have 4 or less digits. Thus it is sufficient to identify primes with distinct digits under 10000 which begin with 8 or 9 and then investigate the maximal products. Both 8 and 9 are composite, so there are no single digit primes to consider. For two digit primes, the only ones are 83 and 97. Observe that 83*9765421 = 810529943 and 97*8654321 = 839469137. Both products are less than the above M value. There are 11 primes of the desired form consisting of 3 digits from 821 to 971. The largest product of 863*975421 = 841788323 is less than the above M value. There are 53 primes of the desired form consisting of four digits. Of these there are three pairs which consist of the same four digits, namely (8123, 8231), (8713, 8731), and (9613,9631). The largest product is formed with the larger prime in each pair. This gives 50 cases to investigate. The second factor is the largest possible number not using the leading digit of the first factor. It follows that we only need to consider 4-digit primes in the intervals [8630,8799] and [9614,9799]. There are 11 such primes. However, 8713 and 8731 use the same digits so we only need to consider the larger. For the primes 8731 and 8741, all rearrangements beginning with 9 and using the remaining digits are composite. Below are listed the maximal product in each case where the second factor is the largest prime using the remaining digits:

8641*97523 = 842696243 = M

8647*95231 = 823462457

8753*94621 = 828217613

9623*87541 = 842407043

9631*85427 = 822747437

9643*87251 = 841361393

9721*86453 = 840409613

9743*85621 = 834205403.

We see the largest product comes from 8641 and 97523.

Also solved by the proposer.

Kappa Mu Epsilon News

Edited by Peter Skoner, Historian
Updated information as of January 2017

Another Historian was elected in April 2017, so news of chapter activities and other noteworthy KME events should now be sent to
Cynthia Huffman, KME Historian
Pittsburg State University
Mathematics Department
1701 S. Broadway
Pittsburg, KS 66762
or to
cjhuffman@pittstate.edu

Chapter News

AL Alpha – Athens State University
Corresponding Secretary – Patricia Glaze; 6 New Members
New Initiates – Dustin James Lorance, Laura Martin, Savannah McCullough, Mallory Ann Patterson, Taylor Lynn Peters, and Lydia Jean Urick.

AL Beta – University of North Alabama
Corresponding Secretary – Ashley Johnson; 13 New Members
New Initiates – Aaron Avery, Erin Cosby, Eileen Drass, Katelynn Gordon, Hannah Hopkins, Victoria Krohn, Jesse Laster, Jonathon Marlar, Joseph Schafer, Nealey Sims, Kaitlin Snyder, Jenna Thompson, and Tyler Yasaka.

AL Gamma – University of Montevallo
Corresponding Secretary – Scott Varagona; 12 New Members
New Initiates – Chase Baker, Ryan Baker, Kate Davis, Pamela Faddis, Elizabeth Hawk, Michael LaRiviere, Cailin Monroe, Drew Mullinax, Stephen Navarro, Illyssa Overton, Katherine Terino, and Brooke Warren.

AL Zeta – Birmingham-Southern College
Chapter President – Sam Crowder; 51 Current Members; 26 New Members
Other Spring 2016 Officers: Julia Creager, Vice President; Andy Crowder, Secretary; Nirja Patel, Treasurer; and Maria Stadnik, Corresponding Secretary and Faculty Sponsor
This spring we held a Pi Day Pizza Pie Party on Monday, March 14, 2016. We had a student recite the first 55 digits of pi, and we played board games and ate pizza. At our spring initiation, we were excited to initiate 26 new members to KME.

AL Theta – Jacksonville State University

Chapter President – Daniel Miradakis; 60 Current Members; 24 New Members

Other Spring 2016 Officers: Timothy Garrett, Vice President; Jasmine Beaudette, Secretary; James Thompson, Treasurer; and Dr. David Dempsey, Corresponding Secretary and Faculty Sponsor

On February 29, 2016, the Alabama Theta chapter initiated 24 new student members. New members received their certificates, pins, and honor cords in a ceremony held on the 11th floor of Houston Cole Library. Spring activities included outings for bowling, theater, and LockedIn (escaping from a room by solving puzzles), as well as several pizza and game nights. New officer elections were held April 12 at our last meeting and end-of-semester game night.

AR Beta – Henderson State University

Corresponding Secretary – Dr. Fred Worth; 6 New Members

New Initiates – China Banks, William Blair, Megan Buxbaum, Shana Campbell, Hannah Oden, and Miranda Welch.

CA Gamma – California Polytechnic State University

Corresponding Secretaries and Faculty Sponsors – Robert Easton and Erin Pearse

CA Epsilon – California Baptist University

Corresponding Secretary – James Buchholz; 1 New Member

New Initiates – Megan Rusokoff.

CA Eta – Fresno Pacific University

Chapter President – David Maes; 20 Current Members; 7 New Members

Other Spring 2016 Officers: Elaine Draper, Vice President and Secretary; Kim Raulino, Treasurer; Terence Yi, Corresponding Secretary; and Ron Pratt, Faculty Sponsor

New Initiates – Reyna Benitez, Elaine Draper, Joshua Ewert, James Lopez, Andrew Martinez, Kimberlie Raulino, and Nicole Zalewski.

CT Beta – Eastern Connecticut State University

Corresponding Secretary – Mehdi Khorami; 463 Current Members; 20 New Members

New Initiates – Melanie Barney, Gabriela Brown, Meghan Condren, Kayla Croft, Jennifer DuBois, Lisa Ferrari, Morgan Guimond, Michaela Hanjack, Quiana Johnson, Zachary Johnson, Kasandra Kelley, Caroline Mooney, Peter Morrow, Kelsey Palazzo, Elise Petersen, Danielle Robillard, Lindsey Schaffrick, Anna Shannon, Wesley Velazquez, and Nicole Vigorita.

FL Beta – Florida Southern College

Corresponding Secretary – Lisa De Castro; 11 New Members

New Initiates – William Albert Duffie, Kristen Felgar, Jessica Flickinger, Kay Levin Hoff-

mann, Isabel Loyd, Vrund Patel, David Pollack, Vincent Ragusa, Jordan Rassmann, Carolyn Sellick, and Amelia Summersbee.

FL Gamma – Southeastern University

Corresponding Secretary – Dr. Berhane Ghaim; 7 New Members

New Initiates – Emiene Amali Adekwu, Jenna Harwick, Alissa Hernandez, Quintan Rossow, Hayly Spires, Mercy Toma, and John White.

GA Zeta – Georgia Gwinnett College

Chapter President – Shahriyar Roshan Zamir; 42 Current Members; 8 New Members

Other Spring 2016 Officers: Bess Burnett, Vice President; Heather McAfee, Secretary; Christopher Lohrmann, Treasurer; Dr. Jamye Curry, Corresponding Secretary; and Drs. Jenny Sinclair and Livy Uko, Faculty Sponsors

James Frye (class of 2015) was accepted to math graduate school at Louisiana State University.

New Initiates – Alison Blavesciunas, Alicia Crewey, Peter Fischer, Jordyn Fones, Maegan Lawrence, Antoinette Miezan, Cristian Retana, and Joshua Sims.

HI Alpha – Hawaii Pacific University

Chapter President – Dyon Buitenkamp; 15 Current Members; 5 New Member

Other Spring 2016 Officers: Tara Davis, Corresponding Secretary and Faculty Sponsor

We had our initiation dinner in April 2016.

New Initiates – Dyon Buitenkamp, Bennett Hazelgrove, Phuong Hue Ly, Samantha Esther Rivera, and Thomas R West Jr.

IA Alpha – University of Northern Iowa

Chapter President – Jacob Oswald; 26 Current Members; 6 New Members
Other Spring 2016 Officers: Julie Kirkpatrick, Vice President; Lindsey Pederson, Secretary; Toby Maggert, Treasurer; and Mark D. Ecker, Corresponding Secretary and Faculty Sponsor

Our first spring KME meeting was held on February 24, 2016, at Professor Mark Ecker's residence where student member Lindsey Pedersen talked about her KME paper entitled "The Determinants of Wine Quality." Our second meeting was held on March 30, 2016, at Professor Syed Kirmani's residence where student member Jordan Ratz talked about his paper "Return on Investment for Graduate School." Student member Jake Oswald addressed the spring initiation banquet with "Determinants of Fantasy Football Points per Game" on April 27, 2016. Our banquet was held at Godfather's Pizza in Cedar Falls, where six new members were initiated.

New Initiates – Destiny Leitz, Dalton Lillie, Tim Norton, Ange Rehnstrom, Allie Waters,

and Jake Weber.

IA Gamma – Morningside College

Corresponding Secretary and Faculty Sponsor – Chris Spicer; 6 New Members

New Initiates – Alex Boettger, Merle Bublitz, Derek Delzell, Carlie Maasz, Suzanne Ras, and Joe Schueller.

IA Delta – Wartburg College

Chapter President – Ashlyn Bagge; 58 Current Members; 20 New Members

Other Spring 2016 Officers: Holli Gorman, Vice President; Johanna Ender, Secretary; Kelsey Miner, Treasurer; Brian Birgen, Corresponding Secretary; and Dr. Joy Becker, Faculty Sponsor

In March, twenty new initiates were welcomed at our annual banquet and initiation ceremony. Our speaker was Kayla Schwickerath Hazelton, a 2005 Wartburg Alum and KME member. Kayla is a senior group leader for Target Distribution based in Cedar Falls. In May, we hosted the departmental end of the year picnic.

New Initiates – Samuel Brooks, Bobbie Burrows, Johanna Ender, Alaina Feltes, Jarren Ford, Kerri Golinghorst, Holli Gorman, Jakob Hamilton, Amy Isvik, Madalynn McKelvey, Sarah Mullinax, Morgan Neuendorf, Austin Pauling, Joseph Rottinghaus, Sarah Schirmer, Jennifer Seubert, Tristen Sima, Carly Sis, Madison Thomas, and Cassandra Woodcock.

IA Epsilon – Central College

Chapter President – Paige Wilkin, 22 Current Members; 8 New Members

Other Spring 2016 Officers: Katherine Todd, Vice President; and Dr. Russell E. Goodman, Corresponding Secretary and Faculty Sponsor

The Iowa Epsilon Chapter initiated eight new members this spring and said goodbye to nine graduating members!

IL Delta – University of St. Francis

Corresponding Secretary – Richard J. Kloser; 14 New Members

New Initiates – Miguel Aldana, Lauren Burkhart, Nicholas Collofello, Darren Desmarais, Lauren Douglas, Krystal Garritson, Jordan Giddings, Daniel Healy, Matthew Lough, Johnathan Marquardt, Kayla Paeth, John Rivera, Shawn Roberts, and Logan Timmons.

IL Theta – Benedictine University

Corresponding Secretary – Dr. Thomas Wangler; 7 New Members

New Initiates – Andrew Cate, Matthew Gilsdorf, Jakub Jancek, Kyle Keen, Erik Kerber, Juveriya Khatoon, and Marko Saric.

IL Iota – Lewis University

Corresponding Secretary – Margaret M. Juraco; 13 New Members

New Initiates – Gail Theresa Bragg, Francisco Cano, Robert Dudasik, Grecia R. Equihua, Joe Garcia, Alexander Heldt, Catherine Jasionowski, Brandon Joutras, Bryon Nush, Kyle

Ruiter, Rachel Seiberlich, Steven Suggett, and Matthew Welch.

IL Kappa – Aurora University

Corresponding Secretary – Sebastian Wyman; 11 New Members

New Initiates – Heather Gehlhaar, Teasia Kimmons, Omar Mendez, Michelle Murray, Nicole Noonan, Lauren Sander, Sean Smith, Stephanie Stellman, Jacob Stockman, Matt Swanson, and Brayden Teele.

IN Gamma – Anderson University

Corresponding Secretary – Courtney Taylor; 9 New Members

New Initiates – Madeline Diniz, Michael R. Horner, YeRim Kang, Katy Kidman, Love Sa Rang Lee, Maxwell P. Luetkemeier, Michaela Richardson, Isaac C. Voegtle, and Benjamin Yoder.

KS Alpha – Pittsburg State University

Corresponding Secretary – Tim Flood; 28 New Members

New Initiates – Hamad Albishi, Dalal Almutairi, Amber Bartlett, Joseph Bullock, Benjamin Coltharp, Avery Coronado, Kennedy Dujakovich, Ian Dungan, Emily Feldman, Nathan Flood, Michael Fuhrmeister, Dalton Gregory, Djavan Hairabedian, Hana Hays, Andrew Huffman, Cole Hurley, Jennifer Magee, Hannah Norris, Vincent Piccini, Andrea Price, Ilona Robinson, Olivia Roudebush, Jacob Rowley, Rance Schoenhals, Hannah Skidmore, Payton Smith, Bailey Titus, and Tam Tran.

KS Beta – Emporia State University

Corresponding Secretary – Thomas Mahoney; 13 New Members

New Initiates – Alexus Atchinson, Rob Catlett, Morgan Flowers, Ryan Frier, Amy Fugit, Md. Ibrahim Kholil, Caelob King, Shuangohuang Liang, Peng Shi, Rose Stuhlsatz, Saugat U. Subedi, Regan Wright, and Tingting Wu.

KS Delta – Washburn University

Chapter President – Branden Childers; 24 Current Members; 8 New Members

Other Spring 2016 Officers: Katelynn Robinson, Vice President and Secretary; Jonathan Tyler, Treasurer; and Kevin Charlwood, Corresponding Secretary and Faculty Sponsor

Kansas Delta's chapter of KME met three times this spring with our Math Club for pizza lunches. The meetings often featured a speaker; we had two local actuaries give presentations to our group, and one of our faculty gave a presentation for Pi Day on March 11 prior to spring break. We traveled to the KME Regional Convention at the University of Nebraska-Kearney April 1-2, taking 5 students and 3 faculty to the meeting. Taylor Balsmeier gave a presentation at the convention on his research work, "Statistical Analysis of Trends in Ranking and Salaries for Washburn University" and

won a prize for third place in the judged competition.

MA Alpha – Assumption College

Corresponding Secretary – Dr. Robert Fry; 8 New Members

New Initiates – Kelsey B. Adkins, Rachel I. Cowen, Emma L. Machado, Mallory A. Monaco, Sarah F. Small, Minh T. Nguyen, Tanyalak Vattansasil, and Caryna A. Wright.

MD Alpha – Notre Dame of Maryland University

Corresponding Secretary – Margaret Sullivan; 8 New Members

New Initiates – Summara Abaid, Darcy Conant, Sarah Faress, Marguerite Linz, Chinwendu Nwokeabia, Meghan O'Connor, Kaitlyn Sommer, and Fareeha Syed.

MD Delta – Frostburg State University

Chapter President – Dustin Ullery; 34 Current Members; 9 New Members Other Spring 2016 Officers: Morgan Robertson, Vice President; Amanda Monahan, Secretary; Olivia Elisio, Treasurer; Mark Hughes, Corresponding Secretary and Faculty Sponsor; and Frank Barnet and Justin Dunmyre, Faculty Sponsors

Nine new members were welcomed to the Maryland Delta Chapter during our Initiation Ceremony on February 28. The ceremony featured a lecture by faculty sponsor Dr. Mark Hughes entitled "Huygens, Curvature and the Pendulum Clock." During March, our chapter conducted our annual fundraisers, namely, the Pi-Day Bake Sale and the sale of candy Easter Eggs. Our March and April meetings featured math videos and pizza with the April meeting also seeing the election of our new officers: Rebecca Lee as President, James West as Vice President and Emma Seibert as Treasurer. We finished the semester with a picnic. Finally, we offer best wishes to graduating seniors Dustin Ullery, Morgan Robertson, Olivia Elisio, Michael Shannon and Jocelyn Williams.

MD Epsilon – Stevenson University

Chapter President – Jeremy Kline; 15 Current Members; 9 New Members Other Spring 2016 Officers: Robert Chen, Vice President; Sarah Modzelewski, Secretary; Clayton Foxwell, Treasurer; Benjamin Wilson, Corresponding Secretary and Faculty Sponsor

On October 23, 2015, nine new members (7 students and 2 faculty) were initiated into the Chapter. President and senior Applied Mathematics major Jeremy Kline and the other officers led the ceremony which was followed by lunch with members of the Math department and family and friends of the initiates. Other events in the fall included a service activity where we made math flash cards and tangrams to donate to local elementary and middle schools, a math movie night, and an afternoon of card and board games. Our biggest event in the spring was hosting the 1st annual Spring Spectacular: a day of exciting and interactive math and science demonstrations, games, and exhibits. Some of the exhibits included a vacuum can-

non, liquid nitrogen ice cream, a giant Tower of Hanoi puzzle, the Chaos Game fractal generator, minimal surfaces with soap bubbles, a Prisoner's Dilemma simulation, and the Monty Hall Problem probability game.

MI Delta – Hillsdale College

Chapter President – Michael Tripepi; 35 Current Members; 17 New Members

Other Spring 2016 Officers: Sarah Onken, Vice President; David Peters, Secretary; Linnet Mbogo, Treasurer; and Dr. David Gaebler, Corresponding Secretary and Faculty Sponsor

The Michigan Delta chapter held its 2015–2016 initiation on Pi Day, welcoming 17 new members to the ranks. We also held a panel discussion on mathematics graduate school, which was well-attended by students of all grade levels.

MI Epsilon – Kettering University

Corresponding Secretary – Boyan Dimirov

Our Michigan Epsilon Chapter passes through some difficult time. The former two sponsors Dr. Ruben Hyrapetyan and Dr. Ada Cheng resigned a year ago. Our founder Professor Dr. Brian McCartin passed away on January 29 this year. It is my duty to say some words in his honor and memory. Dr. McCartin joined GMI (now Kettering University since 1999) in 1993. He retired in September 2015 due to severe illness. In 22 years, he contributed over 163 publications, 135 academic presentations, eight book chapters and four books. He earned Outstanding Teacher 8 times, Outstanding Researcher in 2000 and 2010, his name at the Kettering Hall Wall of Fame, and international awards, including the esteemed Chauvenet Prize in 2010 from the Mathematical Association of America for his remarkable article "e: The Master of All," Mathematical Intelligencer, Vol. 28, No. 2, 2005, pp. 10-21. It shows lots of interesting facts about the Euler constant "e," and will stand as a symbol of great achievement for him. The U-tube link `https://www.youtube.com/watch?v=W7ZH8efXm4g`, offers more insight into his professional contributions to the geometry of music, and this is a link to his obituary, `http://obits.mlive.com/obituaries/flint/obituary.aspx?pid=177522061`. More about Brian is on his site `https://paws.kettering.edu/~bmccarti/`. His former student and now new faculty member Dr. Matthew Causley is working to continue the heritage of Professor Brian McCartin. This spring we lost also our retired faculty colleague John Dulin, who taught math and statistics for 40 years. John was a voracious reader and book collector. The Flint water crisis excited the entire world in 2015. The Mathematics Department collected every available statistical data, and engaged the students in an appropriate statistical study

of the problem. Here are a few excerpts from student essays. "I found the MDHHS report extremely interesting, especially during analysis. The data showed that the proportion of elevated lead blood level cases was greater in 2010 than in 2015. This was probably one of the most interesting projects I've completed in college." "Seeing the actual numbers of the Flint Water Problem makes the crisis more real for me." Finally, we held our Kettering Homecoming in May 2016, 3–4 days to see the progress at the Institution. It includes a Poster Session and the Math Department had six posters, including one about the Flint Water Crisis study.

MO Alpha – Missouri State University

Chapter President – Mena Whalen; 35 Current Members; 12 New Members

Other Spring 2016 Officers: Benjamin Borgstede, Vice President; Ashley Kingston, Secretary; Paige Buchmueller, Treasurer; and Jorge Rebaza, Corresponding Secretary and Faculty Sponsor

We had the following three seminars: Wednesday, January 20, Bob Garino (National Agricultural Statistics Service), talked about "USDA Statistics: Making a Significant Contribution." Pizza and soda were served; Thursday, February 25, Dr. Sean Maher (Biology Department) talked about "Models to Assess Biotic Responses to Climate Change." Pizza and soda were served. The event was very special because we celebrated "Pi-Day" with participation of faculty and students, and lots of pies on faces! Tuesday, April 19, three students from the Senior Seminar class (MTH 497) presented their papers: "Chi-square Distributions and Caesar Shifts," by Kelsie Stewart, "A Mathematical Look at Craps," by John Talarico, and "Blackjack Betting Systems," by Spencer Adams. Pizza and soda were served. We celebrated Pi-Day on Monday, March 14. Dr. Les Reid talked about "The History and Importance of Pi". We ate lots of pie, and also had pizza and soda. We also had an end-of-semester party on Thursday May 5th, the last day of classes. We had lots of games music, food, drinks, and desserts.

MO Beta – University of Central Missouri

Chapter President – Amos Bailey; 30 Current Members; 7 New Members

Other Spring 2016 Officers: Madison Ultican, Vice President; Christina Duerr, Secretary; Nicholas Purcell, Treasurer; Rhonda McKee, Corresponding Secretary; Steve Shattuck and Nicholas Baeth, Faculty Sponsors

In the spring semester, the Missouri Beta chapter enjoyed programs such as the mathematics behind the game set and a talk from an alumna who works in operations research. We also took a field trip to the DaVinci exhibit at Union Station in Kansas City. As always, we held a book sale as a fundraiser. Nine members attended the regional KME convention in

Kearney, NE.

New Initiates – Paige Crain, Dylan Ellis, Ashley M. Garrard, Joshua Haunty, Casey Thomas Kelley, Timothy Morris, and MacKenzie Snyder.

MO Epsilon – Central Methodist University

Chapter President – Kelsey Beeler; 16 Current Members; 6 New Members
Other Spring 2016 Officers: Julia Weber, Vice President; Alexandra Surgeon, Secretary; Sam Pollock, Treasurer; Pam Gordy, Corresponding Secretary and Faculty Sponsor

New Initiates – Zachary Adams, Bayley Bellers, Darcy Rae Latham, Brittany Nicole Lawson, Savanna Nault, and Kristy E. Thomas.

MO Zeta – Missouri University of Science and Technology

Corresponding Secretary – Sandy Gu; 52 New Members

New Initiates – Dalton Akley, Lyndon Allen, Kyle Avola, Jake Beinart, Jordan Bodenbach, Auric Brockfeld, Jacob Brockmeyer, Nikita Gahr, John Gallion, Kent Gorday, Darci Graefser, Austin Granger, Liyang Gu, Sandy Gu, James Hamm, Wesley Harris, Jack Hoerchler, Shelby Kapperman, Christina King, Matt Klosterman, Alyssa Knight, Nicole Korklan, Seth Lanius, Krishna Lella, Kelechi Madubuko, Frances Manahan, Hunter Matthews, Kelly Mauntel, Travis McGuire, Stephen Miller, Pabalelo Nkhwalume, Rachel O'Neal, Meyyammai Palani, Krystal Peterson, Kole Rakers, Daniel Roesch, Chris Rosemann, Trevor Rucker, Emily Schmitter, Owen Smith, Ransom Stamps, Matthew Stanfield, Joseph Sullivan, Skye Tackkett, Caleb Trecazzi, Alyssa Wagner, Brandon Ware, Andrew Watson, Lucas Weiler, Greg Westphal, Shannah Withrow, and Kelcy Yunghans.

MO Theta – Evangel University

Chapter President – Kevin Grimes; 22 Current Members; 10 New Members
Other Spring 2016 Officers: Samantha Orr, Vice President; and Don Tosh, Corresponding Secretary and Faculty Spons

Meetings were held monthly. In January, we initiated 10 new members and elected new officers. In April, Dr. Tosh, Dianne Twigger, and seven students attended the regional convention at the University of Nebraska at Kearney. Two of the students, John Vallelonga and Nathan Dahlin, jointly presented a paper at the conference. Also, in April, we had our end-of-year barbeque at the home of Dianne Twigger, where honor cords were presented to graduating members.

MO Iota – Missouri Southern State University

Corresponding Secretary – Dr. Charles Curtis; 9 New Members

New Initiates – Amila Appuhany, Jacob Boswell, Caitlin Brock, Keith Geller, Laura Lora, Alexander Salgado, Merry Shackelford, Steven Stokes, and Cory Williams.

MO Kappa – Drury University

Corresponding Secretary – Dr. Carol Browning; 6 New Members

New Initiates – David Barberis, Clayton Brinkley, Xiao Chang, Swapnaneel Nath, Kylie

Pfaff, and Bijan Pourmand.

MO Lambda – Missouri Western State University

Corresponding Secretary – Dr. Steven Klassen; 6 New Members

New Initiates – Autumn Cross, Jaimee Jordan, Kameron Kelly, Andrea Koch, Jeremee Nute, and Sarah Thomas.

MO Nu – Columbia College

Corresponding Secretary – Kenny Felts; 3 New Members

New Initiates – Rotshak Dakup, Dasril Dasril, and Oran White.

MO Xi – William Woods University

Chapter President – Kiersten Lockman; 4 Current Members; 2 New Members

Other Spring 2016 Officers: Anne Wehner, Vice President; and Chris Schneider, Corresponding Secretary and Faculty Sponsor

The Missouri XI Chapter of Kappa Mu Epsilon at William Woods University was initiated on February 17, 2016. Dr. Rhonda McKee, President of Kappa Mu Epsilon, was on campus to initiate the chapter and initiate our charter members. Several Mathematics and Science faculty and students were also in attendance, along with faculty sponsor Chris Schneider. Refreshments were enjoyed afterwards.

MS Alpha – Mississippi University for Women

Chapter President and Treasurer – Audra Polk; 13 Current Members; 3 New Members

Other Spring 2016 Officers: Ciara Peoples, Vice President; Mandy Elam, Secretary; Joshua Hanes, Corresponding Secretary and Faculty Sponsor

In the spring, we held officer elections for the fall. We initiated three new members. Some of us participated in the MS Walk in Tupelo, MS.

New Initiates – Sugam Bhattarai, Aastha Ghimire, and Ramesh Pandey.

NC Epsilon – North Carolina Wesleyan College

Chapter President – Jasmine Edgren; 5 Current Members; 9 New Members

Other Spring 2016 Officer: Bill Yankosky, Corresponding Secretary and Faculty Sponsor

NE Beta – University of Nebraska Kearney

Corresponding Secretary – Dr. Katherine Kime; 5 New Members

New Initiates – Kato Craig, Chenli Huang, Youngin Kim, Alexis Stockton, and Jianbai Xu.

NE Delta – Nebraska Wesleyan University

Chapter President – Connor Bohlken; 14 Current Members; 7 New Members

Other Spring 2016 Officers: Spencer Randazzo, Vice President; William Reimer, Secretary and Treasurer; and Kristin Pfabe, Corresponding Sec-

retary and Faculty Sponsor

We hosted five events this spring. In February, Dr. Erin Carmody, visiting professor at Nebraska Wesleyan University, gave a talk on infinity. In March, three math graduate students from the University of Nebraska-Lincoln gave talks on several novel problems. We also had a Pi Day Run which drew about 45 runners. We took a field trip to Kansas City where we visited the Money Museum at the Federal Reserve and also saw a collection of rare math books at Linda Hall Library. Among those books was a first edition copy of Euclid's Elements, printed in 1482. We initiated seven new KME members and had a picnic in April.

NJ Beta – Montclair State University

Corresponding Secretary – Jonathan Cutler; 15 New Members

New Initiates – William Anderson, Alexa Aucoin, Jalia Carter, Tyler N. Clark, Travis Henion, Albert Jarvis, Jessica Kerslake, Nicholas Mariani, Shazeb Zahid Munir, Andrew Pallotto, Amanda Provost, Hector I. Reyes Jr., Marc Riemann, Seth Stadtlander, and Kevin Weatherwalks.

NY Kappa – Pace University

Corresponding Secretary – Shamita Dutta Gupta; 3 New Members

New Initiates – Caitlyn Brehm, Daniel Buffone, and Abigail Lamonica.

NY Lambda – LIU Post

Chapter President – Erica Gershkowitz; 14 New Members

Other Spring 2016 Officers: Elizabeth Hartmann, Vice President; Christina Wolf, Secretary; Clifford Clark, Treasurer; and Corbett Redden, Corresponding Secretary and Faculty Sponsor

The NY Lambda chapter of KME held its annual banquet and initiation on April 19, 2015.

New Initiates – Rebecca Blitzer, Christina Cirami, Anthony Cirisano, Christopher Guevara, Kyle Henry, David Mannlein, Christian Nothdurft, Nicholas Rehder, Trevor Rodowicz, Tina Schwartz, Anna Sellis, Courtney Simone, Kathleen Vertolomo, and Qi Wei.

NY Nu – Hartwick College

Corresponding Secretary and Faculty Sponsor – L. Gerald Hunsberger; 10 New Members

New Initiates – Mary Buntrock, Sierra Bentley, Joanna Carber, Michael Dolan, Thomas Dwyer, John Garcia, Justine Kozubal, Krishna Pokharel, Alyssa Ralph, and Nicholas Ryan.

NY Omicron – St. Joseph's College

Chapter President – Matthew R. Stitt; 29 Current Members; 14 New Members

Other Spring 2016 Officers: Sara E. Schmidt, Vice President; Nicole A. Danisi, Secretary; Giovanni Mayo, Treasurer; Elana Reiser, Corresponding Secretary; and Donna Pirich, Faculty Sponsor

This spring we held an initiation ceremony and welcomed new members.

We also did an Easter basket drive and donated over 100 baskets to a local charity. Our members volunteered their time to tutor in our math clinic for local high school students.

OH Gamma – Baldwin Wallace University

Chapter President – Lizzie Cherry; 42 Current Members; 23 New Members

Other Spring 2016 Officers: Stevan Zlojutro, Vice President; Heather Knotts, Treasurer; David Calvis, Corresponding Secretary and Faculty Sponsor

OH Epsilon – Marietta College

Corresponding Secretary – John Tynan; 35 New Members

New Initiates – Michael Bowen, Devon Butler, Pauline Clarchick, Sydney Clottey, Ryan Eberle, Alfred Edwards, Jesse Eicher, Rachele Farr-Haught, Adam J. Garlow, Andrej Goreta, Steven Gott, Stephanie Grube, Paige Haught, Mary Catherine Hornbrook, Zachary Janus, Emily Lang, Adam Majer, Sheldon Mullet, Braden Natoli, Michael Nauls, Karl Ni, Tanner W. Palmer, Matthew Pintell, Katherine Plas, Yizhi Ren, Andrew Phillip Ribbe, Ted Rutkowsky, Erin Shade, Tyler Stockdale, Adam Stupak, Hardik Tripathi, Matthew Ian Valverde, Nathan Walker, Eric White, and Eric Wilken.

OH Zeta – Muskingum University

Corresponding Secretary – Richard Daquila; 7 New Members

New Initiates – Qin Deng, Yuezhi Li, Joelle Miles, Amanda Nemeth, Courtney Nestoff, Abigail Orr, and Robert Warne.

OH Theta – Capital University

Chapter President – Abbie Neininger; 8 Current Members; 12 New Members

Other Spring 2016 Officers: Julia Kunkel, Vice President; Jaime Ashworth, Secretary; Oscar O'Flaherty, Treasurer; Paula Federico, Corresponding Secretary; and Jonathan Stadler, Faculty Sponsor

During the Spring Semester our Chapter had only 4 student members because most of the members initiated in Spring 2015 had graduated that semester. The chapter hosted a presentation by Dr. Jon Stadler entitled "The Mathematics of the Mysterious Towers of Hanoi." We also co-organized with the MAA Student Chapter a Pi Day bake sale. Twelve new members were initiated during our Initiation Ceremony on April 1, 2016 (picture below). Mr. Matthew McMullen, Senior Instructor of Mathematics at Otterbein University was our guest speaker. He treated us with a presentation was entitled "A Drunken Walk in Las Vegas – Catalan Numbers and Gambling." We are looking forward to the Fall 2016 semester in which will have ten active student members.

OK Alpha – Northeastern State University

Chapter President – Brooke Bratu; 58 Current Members; 13 New Members

Other Spring 2016 Officers: Erinn Lawson, Vice President; Frances Millspaugh, Secretary; Whitney Dushane, Treasurer; and Dr. Demitri Plessas, Corresponding Secretary and Faculty Sponsor

Our spring initiation brought nine students and four faculty into our chapter. During the second meeting of the semester, Dr. Bloomfield, NSU, spoke on "Algebra as Language: a Case Study." During the last meeting of the semester, NSU student Chris Stratton spoke on his undergraduate research project, "Love Letter: Endgame Strategies."

New Initiates – Garrett L. Butler, Lauren A. Goekler, Joshua R. Killer, Steven M. O'Dell, Melissa I. Sam, Chetan D. Munsell, Dr. Ummugul Bulut, Dr. Nathan Bloomfield, Dr. Richard Hasenauer, Kayla M. Tanner, Jacklyn L. Wilkes, Katherine G. Williams, and Dr. Rui Zhang.

OK Epsilon – Oklahoma Christian University

Chapter President – Kaylee Eubank; 15 Current Members; 10 New Members

Other Spring 2016 Officers: Josh Bilello, Vice President; Aubrey Gonzalez, Secretary; Dr. Jennifer Bryan, Corresponding Secretary; and Dr. Craig Johnson, Faculty Sponsor

New Initiates – Kourtney Bradbeary, Bradly Browning, Kara Conway, Chaunicie Ehrlich, Parker LasMascus, Tristan Minor, Martha Owino, Marina Pendleton, Seanhenry Vandyke,

and Austin Wondra.

PA Alpha – Westminster College

Corresponding Secretary – Pamela Richardson; 9 New Members

New Initiates – Trevor Arrigoni, Nicholas Caiazza, Christopher Caroff, Bryan Gallo, Tyler Heintz, Ava Hoag, Shawna Howard, Rachael Huff, and Brittany Slupe.

PA Beta – La Salle University

Corresponding Secretary and Faculty Sponsor – Janet Fierson; 14 New Members

New Initiates – Jordan Agzigian, Nicole Costa, Deja Davis, Jessica Dopsovic, Eric Frazier, Julie Gutekunst, Toni Massetti, Robert McDonough, Zachary McNulty, Katherine Ortega, Catherine Oseguera, Nichelle Short, Howard Stickley, and Jackson Swindells.

PA Gamma – Waynesburg University

Corresponding Secretary – James R. Bush; 4 New Members

New Initiates – Brady Cameron, Connor James Dayton, Michelle Karnavas, and Daniel A. Marvin Jr.

PA Epsilon – Kutztown University

Corresponding Secretary – Dr. Lyn McQuaid; 8 New Members

New Initiates – Casey Charterina, Jacob Christ, Logan Hartman, Odysseus Fox, Jacob Kramer, Lisa Lawson, Jiao Xu, and Guozhi Zhang.

PA Kappa – Holy Family University

Chapter President – Dominic McAllister; 6 Current Members; 5 New Members

Other Spring 2016 Officer: Sister Marcella Wallowicz, Corresponding Secretary and Faculty Sponsor

The honor society assisted the math club in planning the Pi week festivities which included a pie eating contest. A mathematics game show night was held on February 26, 2016. The honor society and math club worked jointly on this successful inaugural event.

PA Lambda – Bloomsburg University of Pennsylvania

Corresponding Secretary – Eric Kahn; 10 New Members

New Initiates – Barbara Dressler, Blake Durante, Nicole Hausleben, Alyssa Jones, Tim Mackiw, Katherine Mullen, Todd Poe, Abbey Remley, Anastasia Timofeeva, and Michaela Wagner.

PA Mu – Saint Francis University

Chapter President – Casey Gallaher; 49 Current Members; 19 New Members

Other Spring 2016 Officers: Lydia Mignogna, Vice President; William Shee, Secretary; Hannah Patton and Jay Pillot, Treasurers; Josh Vinglish, Historian; Dr. Peter Skoner, Corresponding Secretary; and Dr. Brendon LaBuz, Faculty Sponsor

The annual Pi Day celebration was held on Monday, March 14, 2016. Fac-

ulty, students, and staff enjoyed taste testing an assortment of "pi" served by members of Kappa Mu Epsilon throughout the day. Initiation ceremonies were held on Tuesday, February 2, 2016, in DiSepio 213. The evening began with a prayer by chapter chaplain and member Fr. Joseph Chancler, T.O.R., followed with dinner, and continued with a talk "Mathematical Modeling on a Limestone Channel," by Dave Wolfe, senior chemistry/mathematics major, continued with the initiation ceremony for the 19 new members, and concluded with remarks by corresponding secretary Dr. Peter Skoner. Two faculty members and four students attended the Great Lakes Regional Convention, held in collaboration with the Ohio MAA sectional meeting, held on April 8 and 9, 2016, at Ohio Eta at Ohio Northern University in Ada, Ohio. KME students and faculty served as judges for the 2016 Pennsylvania Statistics Poster Competition, hosted for the eighth year by Saint Francis University. A large number of posters (584) were received, cash awards were given for first through fourth place in each of four grade level categories, and winning posters were submitted to the National Statistics Poster Competition, coordinated by the American Statistical Association.

PA Xi – Cedar Crest College

Corresponding Secretary – Joshua Harrington; 4 New Members

New Initiates – Hope Hurd, Elizabeth Reichard, Ashley Santangelo, and Donna Sarara.

PA Pi – Slippery Rock University

Chapter President – Kallie Simpson; 15 Current Members; 5 New Members

Other Spring 2016 Officers: Sean Ingimarson, Vice President; Rebekah Bright, Secretary and Treasurer; Elise Grabner, Corresponding Secretary; and Richard Marchand, Faculty Sponsor

New Initiates – Kristina Bell, Rebekah Bright, Jacob Edmonds, Sean Ingimarson, and Kallie Simpson.

PA Rho – Thiel College

Corresponding Secretary – Dr. Russell Richins

New Initiates – Rebecca Adams, Amanda Dobi, Michael Long, Dugan Paxton, Jesse Sealand, and John Thiel.

RI Beta – Bryant University

Chapter President – Stephen Lamontagne; 46 Current Members; 21 New Members

Other Spring 2016 Officers: William Kelley, Vice President; Emma Wieduwilt, Secretary; Krystin Sinclair, Treasurer; John Quinn, Corresponding Secretary; Alan Olinsky, Faculty Sponsor

We met with our student executive board to plan modifying our chapter by-laws and to arrange the initiation ceremony for new member which

was held on May 4, 2016. The changes to the by-laws were subsequently approved on May 13, 2016. One of our new initiates, Max Vogt, just graduated as an actuarial major and was a varsity athlete on the tennis team, playing at the number 1 singles spot and also played doubles. He was just named the 2015-16 Northeast Conference Student-Athlete of the Year. He already passed two actuarial exams and will be working for Willis Towers Watson in Boston, MA.

New Initiates – Riley Barrows, Danielle Bergner, Krystin Bernacki, Hannah Bradley, Christopher Buccheri, Brianna Cote, Elena Freedman, Ryan Goldberg, Emily Gustafson, Caitlin Hannagen, James Heyden, Nathaniel Morgan, Matthew Orsi, Joseph Paparelli, Anna Rodier, Brittany Sarza, Bryanna Seefeldt, Annmarie Tuxbury, Gianna Vallante, Max Vogt, and Owen Wrinn.

SC Epsilon – Francis Marion University

Corresponding Secretary and Faculty Sponsor – Jeremiah Bartz; 13 New Members

New Initiates – Teryese Grant, Kendrick D. Hardison, Javier Bustos Jaimes, Alexander L. Joyce, Dylan Leon, Jackson K. McDonald, Mary Mulholland, Camille T. Cardona Rivera, Phillip M. Rouse, Justin R. Sims, Jared Brett Singleton, Aaron Stafford, and Cody H. Williams.

TN Beta – East Tennessee State University

Chapter President – William Ty Frazier; 40 Current Members; 12 New Members

Other Spring 2016 Officers: Samuel Green, Vice President; Kyle Murphy, Secretary; Haley Russell, Treasurer; and Robert Gardner, Corresponding Secretary and Faculty Sponsor

TN Gamma – Union University

Chapter President – Nicole Bantz; 15 New Members

Other Spring 2016 Officers: Dillon Lisk, Vice President; Rachel Brewer, Secretary and Treasurer; Joshua Stuckey, Webmaster and Historian; Bryan Dawson, Corresponding Secretary; and Matt Lunsford, Faculty Sponsor

TN Gamma held its annual initiation banquet at the Old Country Store on April 18. Earlier in April, two of our students attended the North Central regional convention in Nebraska and one of them, Joshua Stucky, won the prize for best presentation.

New Initiates – Meghan Aranda, Kaylee Barker, Lydia Black, Andrew Edmiston, Graham Gardner, Ainsley Hunt, Samuel Jeong, Michayla Kramer, Rachel McCann, Benjamin Melton, Amy Murdaugh, Matthew Owen, Mason Ruby, David Taylor, and Seth Thibado.

TN Delta – Carson-Newman University

Chapter President – Ryan Eberle; 13 Current Members; 6 New Members

Other Spring 2016 Officers: Mitchell Benjamin, Vice President; and Ken-

neth Massey, Corresponding Secretary and Faculty Sponsor
The Carson-Newman KME club has taken a fascination with board games. There is an ongoing community chess battle, and the students hold regular 'game nights.' Some favorites are "Settlers of Catan," and "Stratomatic Baseball." One student has chosen strategy board games as the motivating topic for his honors thesis on game theory.
New Initiates – Trish Gordon, Katherine Knight, Natalie Levengood, Caryn Mays, Justin Patterson, and Taryn Springer.

TN Zeta – Lee University
Corresponding Secretary – Caroline Maher-Boulis; 15 New Members
New Initiates – Amanda Michelle Akin, Nicholas Baker, Benjamin Benavides, Allison Bernhard, Kaitlyn Burk, Josey Carroll, Robert Kyle Chaney, Chanda Hughes, Anne Kelton, Jeremy Newton, Blessing Okenye, Elizabeth Rawson, Merrily Suits, Philip Winn, and Lauren Wood.

TX Alpha – Texas Tech University
Corresponding Secretary – Giorgio Bornia; 39 New Members
New Initiates – Ikechukwu Achonye, Maria Bajayo, Oluwaseyi Balogun, Allison Baniukiewicz, Chandler Barrow, Levi Box, Tania Brandao, Brittany Burdine, Gage Davis, Scott English, Sean Flaherty, Holden Fried, Kenneth Garcia, Warner Gibson, Alexis Gomez, Jose Gomez, Lino Virgen Gracia, Evan Gring, Glenn Gross, Michael Hand, Karen Houliston, William Krause, Megan Lightborn, Shambhavi Makeswaran, Mario Martinez, Ryan Matuszak, Cody Maupin, Michael McNulty, Pierson Milligan, Peter Minca, Emmanuela Niamkey, Angelique Rangel-Catano, Aspen Shade, Trenton Smith, Douyer Soro, Kim Ngoc Ta, Kyrvin Villarta, Brooke Wasson, and Marcus Yanello.

TX Eta – Hardin-Simmons University
Corresponding Secretary – Jessica Rieger; 15 New Members
New Initiates – Conlan D. Aguirre, Tracy E. Desrochers, Alicia M. Finn, Amanda L. Glover, Sunny S. Helms, Will L. Howard, Shannon Johnson, Genia M. Jones, Kaitlin Elizabeth Key, Misti M. Kingston, Laci Moreland, David A. Offner, Tyler L. Renfro, Caleb Spoon, and Melissa L. Taylor.

TX Kappa – University of Mary Hardin-Baylor
Corresponding Secretary – Peter H. Chen; 15 New Members
New Initiates – Garry Abercrombie, Lauren Addison, Katherine Allen, Steven Alvarez, Jacob Baran, April Brown, Kyle Carey, Tess Dula, Ashley Hastings, Rebecca Kellum, Leah Landry, Jessica Livingston, Haven Neal, Luis Fernando Torres, and Michael Yarberry.

TX Lambda – Trinity University
Chapter President – Zach Tuten; 249 Current Members; 10 New Members Other Spring 2016 Officers: Shelby Luikart, Vice President; David Stroud, Secretary; and Dr. Hoa Nguyen, Corresponding Secretary and Faculty Sponsor
New Initiates – Alexis Daggett, Eshan Prashana Jayamanne Mohottige Don, Giovanni Scott

Giammanco, Danielle King, Dayton James King, Ethan Jesse Krohn, Adrien Lhemann, Niti Nararidh, Gyunghwan Paik, and Owen Thomas Rettenmaier.

TX Mu – Schreiner University

Corresponding Secretary – Clint Coles; 1 New Member

New Initiate – Samantha L. Scudder.

VA Gamma – Liberty University

Corresponding Secretary – Dr. Tim Van Voorhis; 3 New Members

New Initiate – Chelsea Casady, Christopher Pellegrino, and Jennifer Williams.

VA Delta – Marymount University

Chapter President – Bernadette Wunderly; 37 Current Members; 3 New Members

Other Spring 2016 Officers: Kayla Baughman, Vice President; Nicole Ferree, Secretary; Katherine Martin, Treasurer; William Heuett, Corresponding Secretary and Faculty Sponsor

New Initiates – Kayla Baughman, Nicole Ferree, and Katherine Martin.

WI Gamma – University of Wisconsin-Eau Claire

Corresponding Secretary – Carolyn Otto; 11 New Members

New Initiate – Jacob Bartels, Vanessa Van Engelenhoven, McKenzie Hennen, Tennie Jacobson, David Kornack, Mitchell Lemons, Nicolas Lydeen, Auna Nelson, Rita Post, Sarah Reukema, and John Skubal.

WV Alpha – Bethany College

Chapter President – Tess L. Parry; 18 Current Members; 5 New Members
Other Spring 2016 Officers: Jacob T. Riddell, Vice President; Julia A. Mouch, Secretary; Brandon A. Trinh, Treasurer; and Adam C. Fletcher, Corresponding Secretary and Faculty Sponsor

The chapter worked in conjunction with the Mathematics and Computer Science Club to host the tenth annual Math/Science Day on campus. The chapter also brought one student member, one faculty member, and four associate student members to the Great Lakes Regional Convention.

WV Beta – Wheeling Jesuit University

Corresponding Secretary – Marc Brodie; 5 New Members

New Initiate – Sarah K.S. Algee, Patrick J. Chadowski, Jenna Pew, Sean Shields and Wolfgang Zober.

Active Chapters of Kappa Mu Epsilon

Listed by date of installation

Chapter	Location	Installation Date
OK Alpha	Northeastern State University, Tahlequah	18 Apr 1931
IA Alpha	University of Northern Iowa, Cedar Falls	27 May 1931
KS Alpha	Pittsburg State University, Pittsburg	30 Jan 1932
MO Alpha	Missouri State University, Springfield	20 May 1932
MS Alpha	Mississippi University for Women, Columbus	30 May 1932
MS Beta	Mississippi State University, Mississippi State	14 Dec 1932
NE Alpha	Wayne State College, Wayne	17 Jan 1933
KS Beta	Emporia State University, Emporia	12 May 1934
AL Alpha	Athens State University, Athens	5 Mar 1935
NM Alpha	University of New Mexico, Albuquerque	28 Mar 1935
IL Beta	Eastern Illinois University, Charleston	11 Apr 1935
AL Beta	University of North Alabama, Florence	20 May 1935
AL Gamma	University of Montevallo, Montevallo	24 Apr 1937
OH Alpha	Bowling Green State University, Bowling Green	24 Apr 1937
MI Alpha	Albion College, Albion	29 May 1937
MO Beta	University of Central Missouri, Warrensburg	10 Jun 1938
TX Alpha	Texas Tech University, Lubbock	10 May 1940
KS Gamma	Benedictine College, Atchison	26 May 1940
IA Beta	Drake University, Des Moines	27 May 1940
TN Alpha	Tennessee Technological University, Cookeville	5 Jun 1941
MI Beta	Central Michigan University, Mount Pleasant	25 Apr 1942
NJ Beta	Montclair State University, Upper Montclair	21 Apr 1944
IL Delta	University of St. Francis, Joliet	21 May 1945
KS Delta	Washburn University, Topeka	29 Mar 1947
MO Gamma	William Jewell College, Liberty	7 May 1947
TX Gamma	Texas Woman's University, Denton	7 May 1947
WI Alpha	Mount Mary College, Milwaukee	11 May 1947
OH Gamma	Baldwin-Wallace College, Berea	6 Jun 1947
CO Alpha	Colorado State University, Fort Collins	16 May 1948
MO Epsilon	Central Methodist College, Fayette	18 May 1949
MS Gamma	University of Southern Mississippi, Hattiesburg	21 May 1949
IN Alpha	Manchester College, North Manchester	16 May 1950
PA Alpha	Westminster College, New Wilmington	17 May 1950
IN Beta	Butler University, Indianapolis	16 May 1952
KS Epsilon	Fort Hays State University, Hays	6 Dec 1952
PA Beta	LaSalle University, Philadelphia	19 May 1953
VA Alpha	Virginia State University, Petersburg	29 Jan 1955
IN Gamma	Anderson University, Anderson	5 Apr 1957
CA Gamma	California Polytechnic State University, San Luis Obispo	23 May 1958
TN Beta	East Tennessee State University, Johnson City	22 May 1959
PA Gamma	Waynesburg College, Waynesburg	23 May 1959
VA Beta	Radford University, Radford	12 Nov 1959
NE Beta	University of Nebraska—Kearney, Kearney	11 Dec 1959
IN Delta	University of Evansville, Evansville	27 May 1960

OH Epsilon	Marietta College, Marietta	29 Oct 1960
MO Zeta	University of Missouri—Rolla, Rolla	19 May 1961
NE Gamma	Chadron State College, Chadron	19 May 1962
MD Alpha	College of Notre Dame of Maryland, Baltimore	22 May 1963
CA Delta	California State Polytechnic University, Pomona	5 Nov 1964
PA Delta	Marywood University, Scranton	8 Nov 1964
PA Epsilon	Kutztown University of Pennsylvania, Kutztown	3 Apr 1965
AL Epsilon	Huntingdon College, Montgomery	15 Apr 1965
PA Zeta	Indiana University of Pennsylvania, Indiana	6 May 1965
AR Alpha	Arkansas State University, Jonesboro	21 May 1965
TN Gamma	Union University, Jackson	24 May 1965
WI Beta	University of Wisconsin—River Falls, River Falls	25 May 1965
IA Gamma	Morningside College, Sioux City	25 May 1965
MD Beta	McDaniel College, Westminster	30 May 1965
IL Zeta	Dominican University, River Forest	26 Feb 1967
SC Beta	South Carolina State College, Orangeburg	6 May 1967
PA Eta	Grove City College, Grove City	13 May 1967
NY Eta	Niagara University, Niagara University	18 May 1968
MA Alpha	Assumption College, Worcester	19 Nov 1968
MO Eta	Truman State University, Kirksville	7 Dec 1968
IL Eta	Western Illinois University, Macomb	9 May 1969
OH Zeta	Muskingum College, New Concord	17 May 1969
PA Theta	Susquehanna University, Selinsgrove	26 May 1969
PA Iota	Shippensburg University of Pennsylvania, Shippensburg	1 Nov 1969
MS Delta	William Carey College, Hattiesburg	17 Dec 1970
MO Theta	Evangel University, Springfield	12 Jan 1971
PA Kappa	Holy Family College, Philadelphia	23 Jan 1971
CO Beta	Colorado School of Mines, Golden	4 Mar 1971
KY Alpha	Eastern Kentucky University, Richmond	27 Mar 1971
TN Delta	Carson-Newman College, Jefferson City	15 May 1971
NY Iota	Wagner College, Staten Island	19 May 1971
SC Gamma	Winthrop University, Rock Hill	3 Nov 1972
IA Delta	Wartburg College, Waverly	6 Apr 1973
PA Lambda	Bloomsburg University of Pennsylvania, Bloomsburg	17 Oct 1973
OK Gamma	Southwestern Oklahoma State University, Weatherford	1 May 1973
NY Kappa	Pace University, New York	24 Apr 1974
TX Eta	Hardin-Simmons University, Abilene	3 May 1975
MO Iota	Missouri Southern State University, Joplin	8 May 1975
GA Alpha	State University of West Georgia, Carrollton	21 May 1975
WV Alpha	Bethany College, Bethany	21 May 1975
FL Beta	Florida Southern College, Lakeland	31 Oct 1976
WI Gamma	University of Wisconsin—Eau Claire, Eau Claire	4 Feb 1978
MD Delta	Frostburg State University, Frostburg	17 Sep 1978
IL Theta	Benedictine University, Lisle	18 May 1979
PA Mu	St. Francis University, Loretto	14 Sep 1979
AL Zeta	Birmingham-Southern College, Birmingham	18 Feb 1981
CT Beta	Eastern Connecticut State University, Willimantic	2 May 1981
NY Lambda	C.W. Post Campus of Long Island University, Brookville	2 May 1983
MO Kappa	Drury University, Springfield	30 Nov 1984
CO Gamma	Fort Lewis College, Durango	29 Mar 1985

NE Delta	Nebraska Wesleyan University, Lincoln	18 Apr 1986
TX Iota	McMurry University, Abilene	25 Apr 1987
PA Nu	Ursinus College, Collegeville	28 Apr 1987
VA Gamma	Liberty University, Lynchburg	30 Apr 1987
NY Mu	St. Thomas Aquinas College, Sparkill	14 May 1987
OH Eta	Ohio Northern University, Ada	15 Dec 1987
OK Delta	Oral Roberts University, Tulsa	10 Apr 1990
CO Delta	Mesa State College, Grand Junction	27 Apr 1990
PA Xi	Cedar Crest College, Allentown	30 Oct 1990
MO Lambda	Missouri Western State College, St. Joseph	10 Feb 1991
TX Kappa	University of Mary Hardin-Baylor, Belton	21 Feb 1991
SC Delta	Erskine College, Due West	28 Apr 1991
SD Alpha	Northern State University, Aberdeen	3 May 1992
NY Nu	Hartwick College, Oneonta	14 May 1992
NH Alpha	Keene State College, Keene	16 Feb 1993
LA Gamma	Northwestern State University, Natchitoches	24 Mar 1993
KY Beta	Cumberland College, Williamsburg	3 May 1993
MS Epsilon	Delta State University, Cleveland	19 Nov 1994
PA Omicron	University of Pittsburgh at Johnstown, Johnstown	10 Apr 1997
MI Delta	Hillsdale College, Hillsdale	30 Apr 1997
MI Epsilon	Kettering University, Flint	28 Mar 1998
KS Zeta	Southwestern College, Winfield	14 Apr 1998
TN Epsilon	Bethel College, McKenzie	16 Apr 1998
MO Mu	Harris-Stowe College, St. Louis	25 Apr 1998
GA Beta	Georgia College and State University, Milledgeville	25 Apr 1998
AL Eta	University of West Alabama, Livingston	4 May 1998
NY Xi	Buffalo State College, Buffalo	12 May 1998
NC Delta	High Point University, High Point	24 Mar 1999
PA Pi	Slippery Rock University, Slippery Rock	19 Apr 1999
TX Lambda	Trinity University, San Antonio	22 Nov 1999
GA Gamma	Piedmont College, Demorest	7 Apr 2000
LA Delta	University of Louisiana, Monroe	11 Feb 2001
GA Delta	Berry College, Mount Berry	21 Apr 2001
TX Mu	Schreiner University, Kerrville	28 Apr 2001
NJ Gamma	Monmouth University, West Long Branch	21 Apr 2002
CA Epsilon	California Baptist University, Riverside	21 Apr 2003
PA Rho	Thiel College, Greenville	13 Feb 2004
VA Delta	Marymount University, Arlington	26 Mar 2004
NY Omicron	St. Joseph's College, Patchogue	1 May 2004
IL Iota	Lewis University, Romeoville	26 Feb 2005
WV Beta	Wheeling Jesuit University, Wheeling	11 Mar 2005
SC Epsilon	Francis Marion University, Florence	18 Mar 2005
PA Sigma	Lycoming College, Williamsport	1 Apr 2005
MO Nu	Columbia College, Columbia	29 Apr 2005
MD Epsilon	Stevenson University, Stevenson	3 Dec 2005
NJ Delta	Centenary College, Hackettstown	1 Dec 2006
NY Pi	Mount Saint Mary College, Newburgh	20 Mar 2007
OK Epsilon	Oklahoma Christian University, Oklahoma City	20 Apr 2007
HA Alpha	Hawaii Pacific University, Waipahu	22 Oct 2007
NC Epsilon	North Carolina Wesleyan College, Rocky Mount	24 Mar 2008

CA Zeta	Simpson University, Redding	4 Apr 2009
NY Rho	Molloy College, Rockville Center	21 Apr 2009
NC Zeta	Catawba College, Salisbury	17 Sep 2009
RI Alpha	Roger Williams University, Bristol	13 Nov 2009
NJ Epsilon	New Jersey City University, Jersey City	22 Feb 2010
NC Eta	Johnson C. Smith University, Charlotte	18 Mar 2010
AL Theta	Jacksonville State University, Jacksonville	29 Mar 2010
GA Epsilon	Wesleyan College, Macon	30 Mar 2010
FL Gamma	Southeastern University, Lakeland	31 Mar 2010
MA Beta	Stonehill College, Easton	8 Apr 2011
AR Beta	Henderson State University, Arkadelphia	10 Oct 2011
PA Tau	DeSales University, Center Valley	29 Apr 2012
TN Zeta	Lee University, Cleveland	5 Nov 2012
RI Beta	Bryant University, Smithfield	3 Apr 2013
SD Beta	Black Hills State University, Spearfish	20 Sept 2013
FL Delta	Embry-Riddle Aeronautical University, Daytona Beach	22 Apr 2014
IA Epsilon	Central College, Pella	30 Apr 2014
CA Eta	Fresno Pacific University, Fresno	24 Mar 2015
OH Theta	Capital University, Bexley	24 Apr 2015
GA Zeta	Georgia Gwinnett College, Lawrenceville	28 Apr 2015
MO Xi	William Woods University, Fulton	17 Feb 2016
IL Kappa	Aurora University, Aurora	3 May 2016