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Elliptic Curve Cryptography

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Abstract

Elliptic curves are defined by equations of the form $y^2 = p(x)$, where $p(x)$ is a cubic polynomial. Their mathematical properties have made them very useful in some modern encryption algorithms making up the field called Elliptic Curve Cryptography (ECC). Of particular importance is the addition operation that can be performed on an elliptic curve whereby two points on the curve can be "added" to yield a third point on the curve. The geometric version of this addition rule leads to its algebraic equivalent. When the curve's coefficients and solutions are taken from a finite field, the proper setting is formed for applications to cryptography. The fact that the "Elliptic Curve Discrete Logarithm Problem" is difficult to solve is the key to the effectiveness of ECC. The Elliptic Diffie-Hellman Key Exchange algorithm uses this property to perform a secure private key exchange between two parties in the presence of eavesdroppers. This paper investigates the mathematics behind the addition of points on an elliptic curve and how this leads to an effective encryption technique allowing secure key exchanges.

Introduction

Elliptic Curve Cryptography provides a process of sending a private key from one party, Alice, to another party, Bob, securely in the presence of an eavesdropper. In order to do this without hand delivering it, Elliptic Curve Cryptography is used. The private key is used to encrypt and decrypt a message using Data Encryption Standard (DES) or Advanced Encryption Standard (AES).

The Elliptic Curve

The elliptic curve that is used is in the Weierstrass Normal Form [6]:

$$y^2 = x^3 + Ax + B$$

where $(x, y) \in \mathbb{R}^2$ and A and B must satisfy the following to avoid having singular points where a tangent line would not be defined:

$$4A^3 + 27B^2 \neq 0.$$

This equation along with the point at infinity is what defines a curve over the field of real numbers.

$$Y^2 = X^3 - 2X + 8$$
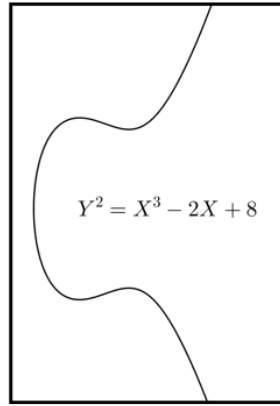
Figure 1

The points of an elliptic curve form an abelian group [4]. Therefore, they have the following properties: closure, associativity, commutativity, an identity and an inverse. The definition of these properties, in the context of elliptic curve addition, are as follows [6]:

1. The closure property: If points P and Q are on the elliptic curve E, then the point $P \oplus Q$ will be on the curve as well.
2. The associative property: If P, Q and R are points on the curve E, then $(P \oplus Q) \oplus R = P \oplus (Q \oplus R)$.
3. The commutative property: If P and Q are points on the curve E then $P \oplus Q = Q \oplus P$.
4. The identity: there exists a point O such that, for all points P on the curve E, $P \oplus O = O \oplus P = P$.
5. The inverse: For all points P on the curve E, there exists a point $-P$ such that $P \oplus (-P) = O$.

The point O is the point at infinity. Projective geometry allows one to define this point rigorously. Here we view O informally as the "horizon

point" common to all vertical lines.

Geometric Definition of the Addition of Two Different Points

In figure 2, we have our elliptic curve $E : y^2 = x^3 - 2x + 8$ with points $P = (-2, 2)$, and $Q = (-1, 3)$ on the curve. We will find $P \oplus Q$. If a line is drawn through the two points, P and Q, the line will always intersect the curve at a third point, R (where R will be taken to be O if P and Q lie on a vertical line). Then, the third point, R, is reflected over the axis of symmetry, the x-axis in this case. The point that is the result of this reflection is the desired result, $P \oplus Q$.
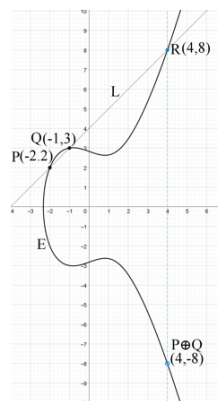
Figure 2

Geometric Definition of the Addition of a Point with Itself

In figure 3, given an elliptic curve $E : y^2 = x^3 - 8x + 4$, with the point $P = (0, 2)$ on the curve, we will find $P \oplus P$. This will be done by counting P as two points. If a line is drawn through P tangent to the curve E, the line will intersect the curve at a third point, R. Then, the third point, R, is reflected over the line of symmetry, the x-axis in this case. The point that is found is $P \oplus P$; as desired.

Figure 3

Algebraic Definition of Addition
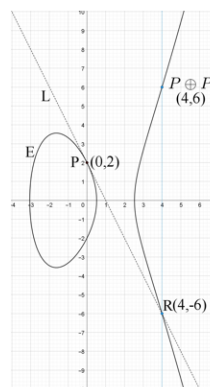
The following theorem is an algebraic approach to what was shown above geometrically.

Theorem 1 *Let P and Q be points on an elliptic curve, E, over the field of real numbers with the equation, $y^2 = x^3 + Ax + B$. Then, the following hold [6]:*

    1. *If $P = O$, then $P \oplus Q = Q = Q \oplus P$.*

    2. *Let $P = (x_1, y_1)$ and $Q = (x_2, y_2)$.*

        a) *If $x_1 = x_2$ and $y_1 = -y_2$, then$P \oplus Q = O = Q \oplus P$.*

        b) *Generally, the point $P \oplus Q = (x_3, y_3)$,*
            where $x_3 = \lambda^2 - x_1 - x_2$ and $y_3 = \lambda(x_1 - x_3) - y_1$ and λ is defined by:

$$\lambda = \begin{cases} \dfrac{y_2 - y_1}{x_2 - x_1} & for\ P \neq Q \\[2ex] \dfrac{3x_1^2 + A}{2y_1} & for\ P = Q \end{cases}.$$

    3. *$P \oplus Q = Q \oplus P$.*

Proof. First, we derive the formulas in (2b). This proof can be found in [6] and [9]. When $P = Q$, we need to compute the slope of the tangent line to the curve. In particular, we will show that the slope of the line through point P tangent to the curve, E, is:

$$\frac{dy}{dx} = \frac{3x_1^2 + A}{2y_1}.$$

Consider:

$$y^2 = x^3 + Ax + B.$$

Taking the derivative, with respect to x, of both sides:

$$\frac{d}{dx}(y^2) = \frac{d}{dx}(x^3 + Ax + B)$$

$$\Rightarrow 2y\frac{dy}{dx} = 3x^2 + A$$

$$\Rightarrow \frac{dy}{dx} = \frac{3x^2 + A}{2y}.$$

Evaluated at point $P = Q = (x_1, y_1)$:

$$\frac{dy}{dx} = \frac{3x_1{}^2 + A}{2y_1},$$

as desired.

    We now prove the formula for x_3 and y_3 in (2b). Let λ be the slope of the line through the two points $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ on the elliptic curve $E : y^2 = x^3 + Ax + B$. Then, the line L through these points,

with slope λ has the following equation:

$$y - y_1 = \lambda \left(x - x_1 \right)$$
$$\Rightarrow y = \lambda \left(x - x_1 \right) + y_1$$
$$\Rightarrow y = \lambda x - \lambda x_1 + y_1$$
$$\Rightarrow y = \lambda x + y_1 - \lambda x_1.$$

For simplicity we let $v = y_1 - \lambda x_1$. Therefore, $L : y = \lambda x + v$. Then, to find where the line intersects the curve again, we plug L into E:

$$\left(\lambda x + v \right)^2 = x^3 + Ax + B$$
$$\Rightarrow \lambda^2 x^2 + 2v\lambda x + v^2 = x^3 + Ax + B$$
$$\Rightarrow 0 = x^3 - \lambda^2 x^2 + \left(A - 2v\lambda \right) x + \left(B - v^2 \right).$$

We know that $x = x_1$ and $x = x_2$ are roots. So, x_3 will be our third point of intersection. Therefore:

$$x^3 - \lambda^2 x^2 + \left(A - 2v\lambda \right) x + \left(B - v^2 \right) = \left(x - x_1 \right) \left(x - x_2 \right) \left(x - x_3 \right)$$
$$\Rightarrow x^3 - \lambda^2 x^2 + \left(A - 2v\lambda \right) x + \left(B - v^2 \right)$$
$$= x^3 - \left(x_1 + x_2 + x_3 \right) x^2 + \left(x_1 x_2 + x_2 x_3 + x_1 x_3 \right) x - x_1 x_2 x_3.$$

It must be the case that the coefficient of x_2 on the right must be equal to the coefficient of x_2 on the left. Therefore, $x_1 + x_2 + x_3 = \lambda^2$. Solving for x_3 gives us $x_3 = \lambda^2 - x_1 - x_2$. We know $y = \lambda x + v$, and $v = y_1 - \lambda x_1$, so:

$$y = \lambda x_3 + v$$
$$\Rightarrow y = \lambda x_3 + y_1 - \lambda x_1$$
$$\Rightarrow y = \lambda (x_3 - x_1) + y_1.$$

Then, we reflect the point over the x-axis, so we get $-y$ so that $y_3 = \lambda(x_1 - x_3) - y_1$. So, the point $P \oplus Q = (x_3, y_3)$ where $x_3 = \lambda^2 - x_1 - x_2$ and $y_3 = \lambda(x_1 - x_3) - y_1$, as desired.

We now discuss the commutative property: we need to show that for all points $P, Q \in E$, it is true that $P \oplus Q = Q \oplus P$. The proof we give here can be found in [9]. Let $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ on the elliptic curve E. The slope of the line through P and Q, is going to be the same as the slope through Q and P. Since they have the same slope and we can choose either of the points to plug into the point-slope formula to obtain the equation of the line, the line through P and Q is the same as the line through Q and P. Therefore, since the two lines are the same, this line will intersect the curve E at a third point R. Reflecting this point over the x-axis returns the same point $P \oplus Q$ or $Q \oplus P$ no matter which point we started with.

Now that the addition of points on elliptic curves over the field of real numbers has been defined, we can look at the identity in more detail. The identity property states that for every $P \in E$ there exists a point, O, such that $P \oplus O = O \oplus P = P$. We want to show that taking O to be the point at infinity is a good choice. So, consider $P \oplus O$. We see that the line passing through the point, $P = (x, y)$ and

Figure 4

O is the vertical line passing through P. This line intersects the curve at a third point, $P' = (x, -y)$, which is the reflection over the x-axis of the point P. (Or, if $y = 0$, this third point, P', will be P.) Since $P \oplus O$ is the reflection of P' over the x-axis, $P \oplus O = P$, as desired and shown in figure 4. Therefore, O must be the point at infinity. ∎
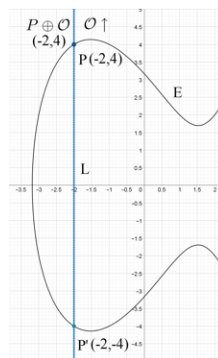
The Inverse

Consider the additive inverse of a point $P = (x, y)$. We can see that the inverse is the reflection of the point P. Therefore, the inverse is $P' = (x, -y)$. The line through P and P' is vertical so, the third point on that line is going to be O. If this point O is reflected over the x-axis, it will again be O, since a vertical line can only intersect one point at infinity, shown in figure 5.
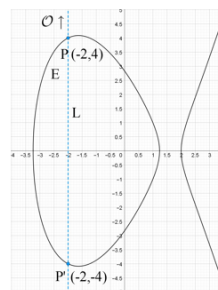
Figure 5

Again, this can be proven more rigorously using projective geometry. Therefore, $-P$ is the inverse of P, as desired. A more in-depth explanation of this can be found in Avner Ash's book, *Elliptic Tales: Curves, Counting, and Number Theory* [1]. The associative property can be proven as well, but it is more complicated. This proof can be found in Silverman's book *Rational Points on Elliptic Curves* [9].

It is now clear that the set of rational points on an elliptic curve forms an abelian group. This is because if (x_1, y_1) and (x_2, y_2) are rational points then (x_3, y_3) will be also. This is true because λ, the slope between these points, must be a rational number. Therefore, this leads to x_3, and y_3 being rational numbers as well by the equations that were proved above:

$x_3 = \lambda^2 - x_1 - x_2$, and $y_3 = \lambda(x_1 - x_3) - y_1$.

Addition Examples

The following examples demonstrate the previously mentioned formulas.

1. Adding Two Different Points: Given the elliptic curve:

$$y^2 = x^3 - 2x + 8$$

with points $P = (-2, 2)$ and $Q = (-1, 3)$ on that curve, find $P \oplus Q$.

Step 1: Find the slope of the line through points P and Q. The slope of the line is:

$$\lambda = \frac{3 - 2}{-1 - (-2)} = 1.$$

Step 2: Find point $P \oplus Q = (x_3, y_3)$. We know from above that $x_3 = \lambda^2 - x_1 - x_2$. So, using $\lambda = 1$, we get:

$$x_3 = 1^2 - (-2) - (-1) = 4.$$

Then $y_3 = \lambda(x_1 - x_3) - y_1$. So, using $\lambda = 1$ and $x_3 = 4$, we get:

$$y_1 = 1(-2 - 4) - 2 = -8.$$

Therefore, $P \oplus Q = (4, -8)$. Figure 6 is a graph of the $P \oplus Q$ addition as shown with the geometric description above.

Figure 6
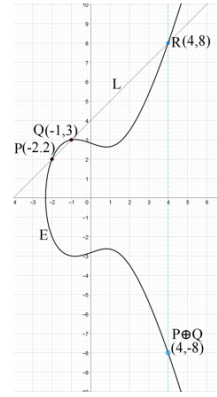
2. Adding a Point to Itself: In this case we count the point P as two points. Given the curve $E : y^2 = x^3 - 8x + 4$ with point $P = (0, 2)$ on that curve, and $P \oplus P$.

Step 1: Find the slope of the line through P tangent to E. From above we know that when $P = Q$:

$$\lambda = \frac{3x_1^2 + A}{2y_1}.$$

By substitution:

$$\lambda = \frac{3(0)^2 + (-8)}{2(2)} = -2.$$

Step 2: Find point $P \oplus P = (x_3, y_3)$ We know
$x_3 = \lambda^2 - x_1 - x_2$, and
$y_3 = \lambda(x_1 - x_3) - y_1$. Therefore,

$$x_3 = (-2)^2 - 0 - 0 = 4.$$

Then

$$y_3 = (-2)(0 - 4) - 2 = 6.$$

Therefore, the point $P \oplus P = (4, 6)$. Figure 7 is a graph of the $P \oplus P$ addition.
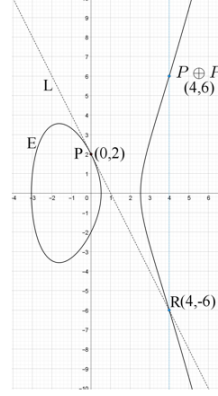
Figure 7

Scalar Multiplication

In our application of cryptography, we will also utilize a technique called scalar multiplication to make addition easier. In Elliptic Curve Cryptography, points like nP are needed, where n is an integer. It is possible to add P to itself n times, but it is more efficient to use an algorithm called the double-and-add algorithm. The steps of this algorithm are as follows [2]:

Step 1: Convert n to base 2.
Step 2: Write n as a sum of powers of 2.
Step 3: Double P until the highest power of 2 is reached.
Step 4: Add all $2kP$ that are multiplied by 1 in the base 2 form of n, where $\{k \in \mathbb{Z} | k > 0\}$.

Example: Given the elliptic curve $E : y^2 = x^3 - 8x + 4$ that contains point $P = (4, 6)$, find $18P$.

Step 1: Convert 18 to base 2: $18 = (10010)_2$.

Step 2: Write 18 as a sum of powers of 2: $18 = 2^1 + 2^4$.

Step 3: Double P until $2^4 P$ is reached:

$P = (4, 6)$
$2P = \left(\frac{28}{9}, -\frac{82}{27}\right)$
$2^2 P = \left(\frac{87340}{15129}, \frac{22807394}{1860867}\right)$
$2^3 P = \left(\frac{4987592101538286604}{19674402943966682361}, -\frac{2924583948822154691155564354}{2759639436895539304360899891}\right)$

and $2^4 P =$

$$\left(\frac{11893295501031752452052335739356492140786928939387218490172779561639948346 8}{42069732893939892763059238838804390921087107369516200950060286771459576 9}, \right.$$

$$\frac{12970391854011094707095123758257211842567263142544238507600946033482022644508241558042057935003066416054569409922}{8628884024661123220472128195788674818456268479252882637327134079159088997155171138675860576043728994363603} \left. \right)$$

Step 4: Add $2^1 P + 2^4 P$: $18P =$

$$\left(\frac{138467443867469134195998829239643209021824056084948080707838076492735963522236321848781380582 5}{4282868755902882833181031190787249431663350317379230574463642884175552222729384731913735579 04} \right.$$

$$\frac{-3061352400272384859804702732675611796239115578842922671074386532666660918058585046053852893078752935597 7}{88634367345926944266132521924973769294796607590248424849572646404120945682585502607476096516560871742 13} \cdots$$

$$\left. \frac{3696572864371502374993973849075790 09}{7118495670100304697750705312897662 08} \right)$$

Points like $Q = nP$ are complicated numbers which is good for cryptography because it is hard to guess the original P used and undo the addition.

Elliptic Curves over Finite Fields

In Elliptic Curve Cryptography, the curves that are used are curves over the finite field, $\mathbb{F}_p$. The solutions, (x, y), are taken mod p, where p is a very large prime number. Since the solutions are taken mod p, the curve is reduced to the finite plane. An example of a large prime that is used is 6277101735386680763835789423207666416083908700390324961279 [11]. For curves over the finite field, the equation of the elliptic curve will still have the same general structure, but it will now be mod p:

$$y^2 \equiv x^3 + Ax + B \pmod{p}$$

where $(x, y) \in (\mathbb{F}_p)^2$, p is a large prime and, A and B satisfy the following:

$$4A^3 + 27B^2 \not\equiv 0 \pmod{p}$$

This equation along with the point at infinity is what defines a curve over the finite field. The following is a graph of the curve $y^2 \equiv x^3 - 92x + 67 \pmod{103}$, with a line to indicate the symmetry.
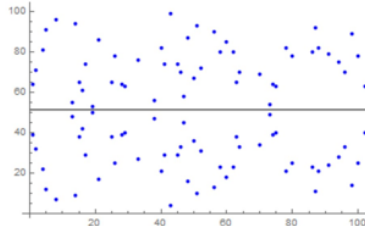

Figure 8

To find points on the elliptic curve E that is over the finite plane, all possible values for x, which are $0, 1, \ldots, (p-1)$, are plugged into the equation. If that value is a square mod p, then it creates two points on the curve. The question of how many points are on an elliptic curve is answered by the following theorem.

Theorem 2 *(The Theorem of Hasse) Let E be an elliptic curve over $\mathbb{F}_p$. Then*

$$\#E\left(\mathbb{F}_p\right) = p + 1 - t_p \text{ with } |t_p| \leqslant 2\sqrt{p}$$

This theorem says that the number of points on an elliptic curve on a finite plane is equal to $\#E\left(\mathbb{F}_p\right) = p + 1$, "up to random fluctuations" [6].

   Example: Given the curve $E : y^2 \equiv x^3 - 2x + 4 (\mathrm{mod}\, 13)$, find all possible points on the curve. We plug in $0, 1, \ldots, 12$ to the equation and find that the following are squares mod 13:

$$(4)^2 \equiv (1)^3 - 2(1) + 4 (\mathrm{mod}\, 13) \qquad (9)^2 \equiv (1)^3 - 2(1) + 4 (\mathrm{mod}\, 13)$$
$$(5)^2 \equiv (3)^3 - 2(3) + 4 (\mathrm{mod}\, 13) \qquad (8)^2 \equiv (3)^3 - 2(3) + 4 (\mathrm{mod}\, 13)$$
$$(3)^2 \equiv (10)^3 - 2(10) + 4 (\mathrm{mod}\, 13) \quad (10)^2 \equiv (10)^3 - 2(10) + 4 (\mathrm{mod}\, 13)$$

and $x = 2, 4, 5, 6, 7, 8, 9, 11, 12$, are not squares mod 13. The following is the graph showing the elliptic curve.
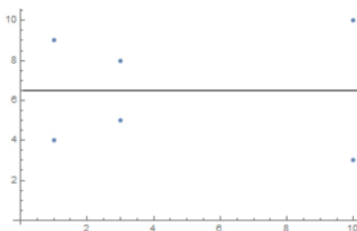

Figure 9

As p is increased, the number of points on the finite curve will increase. So, for the curve $E : y^2 \equiv x^3 - 107x + 90 (\mathrm{mod}\, 54629)$, the graph will be as follows:
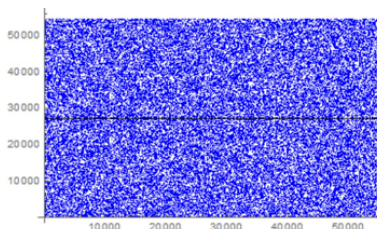

Figure 10

The line shows that the symmetry is still present.

Adding Points on a Finite Curve

As with the elliptic curve over the field of real numbers, the closure, associative, commutative, identity and inverse properties hold for the finite curve as well, that is, the points of the finite curve form an abelian group. This is because the algebraic version of point addition can be adapted to mod p calculations [4].

The equations used for adding points on an elliptic curve over the finite field are the same, but now the coefficients of the line through the two points, x_3 and y_3 will be reduced mod p. So, they will be as follows:

1. If $P = O$, then $P \oplus Q = Q = Q \oplus P$.
2. Let $P = (x_1, y_1)$ and $Q = (x_2, y_2)$.
    a) If $x_1 = x_2$ and $y_1 = -y_2$, then $P \oplus Q = O = Q \oplus P$.
    b) Generally, the point $P \oplus Q = (x_3, y_3)$, where
       $x_3 = \lambda^2 - x_1 - x_2 (\mathrm{mod} p)$ and
       $y_3 = \lambda(x_1 - x_3) - y_1 (\mathrm{mod} p)$ and λ is defined by:

$$\lambda = \begin{cases} \dfrac{y_2 - y_1}{x_2 - x_1}(\mathrm{mod} p) \; for \; P \neq Q \\[2ex] \dfrac{3x_1^2 + A}{2y_1}(\mathrm{mod} p) \; for \; P = Q \end{cases}.$$

3. $P \oplus Q = Q \oplus P$.

Example of Adding Two Different Points Given the elliptic curve $E : y^2 \equiv x^3 - 92x + 67(\mathrm{mod}\, 103)$, containing the points $P = (2, 32)$ and $Q = (8, 7)$, find $P \oplus Q$.

Step 1: Find the slope of the line passing through points P and Q.

$$\lambda = \frac{7 - 32}{8 - 2}(\mathrm{mod} 103)$$
$$= \frac{-25}{6}(\mathrm{mod} 103)$$
$$= 13.$$

Here $\frac{1}{6}$ means the multiplicative inverse of 6 mod 103, that is, 86.

Step 2: Find $P \oplus Q = (x_3, y_3)$. We know $x_3 = \lambda^2 - x_1 - x_2(\mathrm{mod} p)$ and $y_3 = \lambda(x_1 - x_3) - y_1(\mathrm{mod} p)$. So:

$$x_3 = (13^2 - 2 - 8)(\mathrm{mod} 103)$$
$$= 56.$$

and
$$y_3 = (13(2 - 56) - 32)(\mathrm{mod}\,103)$$
$$= -734(\mathrm{mod}\,103)$$
$$= 90.$$

Therefore, the point $P \oplus Q = (56, 90)$. The following is a graph of the $P \oplus Q$ addition where the dotted line is found by considering the Cartesian line of slope 13 through P and then identifying points $(x, 0)$ with $(x, 103)$. Likewise, points $(0, y)$ and $(103, y)$ are identified.
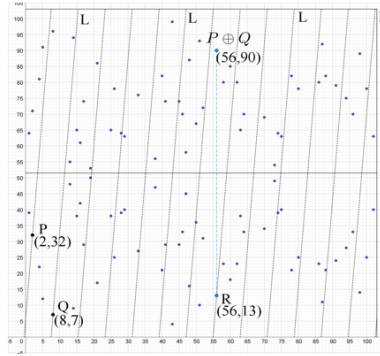
Figure 11

Example of Adding a Point to Itself Given the curve
$$E : y^2 \equiv x^3 - 5x + 7(\mathrm{mod}\,11)$$

containing the point $P = (1, 6)$, find $P \oplus P$. Analogous to the process of adding a point to itself for the curve over the field of real numbers, we count P as two points.

 Step1: Find the slope of the line tangent to the curve E. From above we

know:
$$\lambda = \frac{3x_1^2 + A}{2y_1}(\mathrm{mod}\,p)$$
$$= \frac{3(1)^2 + (-5)}{2(6)}(\mathrm{mod}\,11)$$
$$= -\frac{1}{6}(\mathrm{mod}\,11)$$
$$= 9.$$

 Step 2: Find $P \oplus P = (x_3, y_3)$. We know $x^3 = \lambda^2 - x_1 - x_2(\mathrm{mod}\,p)$

and $y_3 = \lambda(x_1 - x_3) - y_1 (\mathrm{mod} p)$. So:

$$x_3 = (9^2 - 1 - 1)(\mathrm{mod} 11)$$
$$= 79(\mathrm{mod} 11)$$
$$= 2,$$

and

$$y_3 = (9(1 - 2) - 6)(\mathrm{mod} 11)$$
$$= -15(\mathrm{mod} 11)$$
$$= 7.$$

Therefore, the point $P \oplus P = (2, 7)$. The following is a graph of the $P \oplus P$ addition.
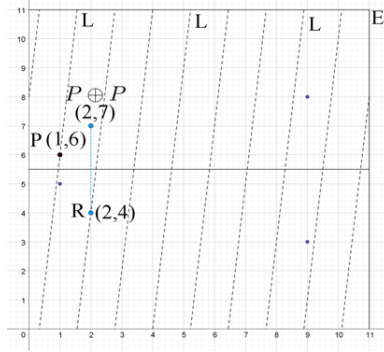
Figure 12

Encryption

To encrypt a message, either DES or AES is used, which are both a form of symmetric encryption. Symmetric encryption needs one secret key in order to decrypt and encrypt a message [5]. So, this key needs to get from Alice to Bob, in the presence of an eavesdropper, Eve, without having to hand deliver it. This is accomplished using a Diffie-Hellman key exchange.

To begin, Alice and Bob both agree on an elliptic curve, a point on that curve, and a finite field they wish to use to obtain the secret key. Then, Alice chooses a secret integer, a, and computes $A = aP$. Bob does the same, he chooses a secret integer, b, and computes $B = bP$. Then, they exchange A and B. Alice then computes $aB = abP$ and Bob computes $bA = baP$. Since the elliptic curve is an abelian group and has the commutative property, $abP = baP$. Therefore, they both end up with the same point. The x-coordinate of that point will be the secret key [12]. See Figure 13 below.

The security of Elliptic Curve Cryptography is determined by the curve,

and the finite field that is used. There are some curves that cannot be used because they are easily cracked [3]. In order for Eve to be able to find the secret key, she would have to find abP with what she is given; the elliptic curve, the finite field, the point P, A and B. This is called the Elliptic Curve Discrete Logarithmic Problem (ECDLP) [12]. If p is small, then it is a lot easier to determine the point abP, but as p grows it becomes increasingly more difficult. According to Hoffstein, "the fastest known algorithm to solve ECDLP in $E(\mathbb{F}_p)$ takes approximately $\sqrt{(p)}$ steps." [6]

Applications

Elliptic Curve Cryptography is used in many things. One use of it is in the secure exchange of bitcoins. Bitcoins are "owned" by people if they can give it to someone else. They must be able to show this exchange by having a record of it created on a ledger which is called the block chain [8]. According to Rykwalder, in order to have this ability, the person must have a private and public key. Another example of where this type of cryptography is used is in Microsoft's digital rights management. An example is shown below of how large the numbers used in this type of cryptography can be [7]:

p (prime number)

$= 785963102379428822376694789446897396207498568951$

$A = 317689081251325503476317476413827693272746955927$

$B = 790528966078787587181205720257185354321006651934$

P (point)

$= (771507216262649826170648268565579889907769254176,$

$390157510246556628525279459266514995562533196655)$

This example also shows how difficult it would be for Eve to discover what the secret key is. Above, Hoffstein noted that the fastest Eve would be able to figure out the secret key is in about $\sqrt{p}$ steps. That would mean, in this case, it would be about 8.865×10^{23} steps. This means, on a computer that can do 2.8 billion operations per second, which is a normal computer, it would take 100400 centuries to figure out the secret key.

Example of Diffie-Hellman Key Exchange

In Front of Eve

Alice and Bob agree on the elliptic curve
$E : y^2 \equiv x^3 - 5x + 12$ and a finite field
$$p = 859$$
that contains point
$$P = (85, 246)$$

In Private

Alice chooses a number $a = 592$
and computes $A = 592P$.
So $A = (116, 655)$.

Bob chooses a number $b = 341$
and computes $B = 341P$.
So, $B = (161, 419)$.

In Front of Eve

Alice sends A to Bob.
Bob sends B to Alice.

In Private

Alice computes $aB = (680, 256)$

Bob computes $bA = (680, 256)$.

Then they both have the point $abP = (680, 256)$ and the secret key 680
and can use this key to securely encrypt and decrypt all of their messages.

Figure 13

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Expressing Natural Numbers Using Ones

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Abstract

We consider the configuration of n ones and any number of additions and multiplications that will result in the largest value.

Introduction

Problem F26 in [1] asks the following question: For a given natural number n, what is the smallest number $f(n)$ of ones that can be used to represent n using ones and any number of $+$ and $\times$ signs and parentheses. It can be shown that

$$80 = (1 + 1 + 1 + 1 + 1) \times (1 + 1 + 1 + 1) \times (1 + 1 + 1 + 1)$$

requires the fewest number of ones and so $f(80) = 13$. Some work on this problem appeared in [2].

In this note, we ask a related question: Given n ones, what configuration will give the largest value?

The Solution

We begin by simplifying the notation. If $n = 8$ one possible configuration would be

$$(1 + 1 + 1 + 1 + 1) \times (1 + 1 + 1)$$

which we shall represent by $\pi_8(5,3) = 5 \cdot 3 = 15$. In general $\pi_n(a_1, a_2, \ldots, a_k) = a_1 \cdot a_2 \cdots a_k$ represents

$$\underbrace{(1 + 1 + \cdots + 1)}_{a_1 \text{ ones}} \times \underbrace{(1 + 1 + \cdots + 1)}_{a_2 \text{ ones}} \times \cdots \times \underbrace{(1 + 1 + \cdots + 1)}_{a_k \text{ ones}}$$

where $n = a_1 + a_2 + \cdots + a_k$. Let P_n be the maximum value of $\pi_n(a_1, a_2, \ldots, a_k)$.

Proposition 1 *The following gives values for P_n where $n \leqslant 5$.*

n	P_n	$\pi_n(a_1, a_2, \ldots, a_k)$
1	1	$\pi_1(1)$
2	2	$\pi_2(2)$
3	3	$\pi_3(3)$
4	4	$\pi_4(2,2)$
5	6	$\pi_5(3,2)$

Note that $\pi_4(4) = \pi_4(2,2)$. In the results that follow $(1+1+1+1)$ will be written as $(1+1) \times (1+1)$. Also note that $n = 5$ is the first value where $P_n > \pi_n(n)$. The next proposition shows how certain initial configurations can be realigned to form a configuration with a larger product.

Proposition 2 *Let n be a natural number.*
(a) For natural numbers $m \geq 5$,
$$\pi_n(m, a_2, \ldots, a_k) < \pi_n(3, m-3, a_2, \ldots, a_k);$$
(b) $\pi_n(1, a_2, \ldots, a_{k-1}, a_k) < \pi_n(a_2, \ldots, a_{k-1}, (a_k+1))$;
(c) $\pi_n(2,2,2, a_4, \ldots, a_k) < \pi_n(3,3, a_4, \ldots, a_k)$.

Proof. (a) Assume $m \in \mathbb{N}$ and $m \geq 5$. Then
$$\frac{9}{2} < m$$
$$\Rightarrow 9 < 2m$$
$$\Rightarrow m < 3m - 9$$
$$\Rightarrow m < 3(m-3).$$

(b) Since $a_i > 0$, then
$$\pi_n(1, a_2, \ldots, a_{k-1}, a_k) = a_2 a_3 \cdots a_{k-1} a_k$$
$$< a_2 a_3 \cdots a_{k-1} a_k + a_2 a_3 \cdots a_{k-1}$$
$$= a_2 a_3 \cdots a_{k-1}(a_k + 1)$$
$$= \pi_n(a_2, \ldots, a_{k-1}, (a_k+1)).$$

(c) We have
$$\pi_n(2,2,2, a_4, \ldots, a_k) = 8 a_4 \cdots a_{k-1} a_k$$
$$< 9 a_4 \cdots a_{k-1} a_k$$
$$= \pi_n(3,3, a_4, \ldots, a_k).$$

∎

The results in Proposition 2 show that P_n must arise from a configuration of sums where each summand contains at least two ones (by part (b))

and no more than four ones (by part (a)). Also, note that since we always choose to write $(1 + 1 + 1 + 1)$ as $(1 + 1) \times (1 + 1)$ we can say that each summand contains either two or three ones. In addition, by part (c), P_n arises from a configuration where at most two summands contain two ones. That, together with the fact that replacing any $\times$ with a $+$ will result in a smaller value of π_n, gives the classification in Theorem 1.

Theorem 3 *Let n be the number of ones and $1 < k$. We have three cases:*

(a) $n = 3k$ $\qquad\qquad P_n = 3^k = \pi_n \underbrace{(3, 3, \ldots, 3)}_{k \text{ threes}};$

(b) $n = 3k + 1$ $\qquad P_n = 3^{k-1} \cdot 2^2 = \pi_n \underbrace{(3, 3, \ldots, 3}_{k-1 \text{ threes}}, 2, 2);$

(c) $n = 3k + 2$ $\qquad P_n = 3^{k-1} \cdot 2 = \pi_n \underbrace{(3, 3, \ldots, 3}_{k \text{ threes}}, 2).$

Proof. The results for (a) and (c) follow directly from Proposition 2. For (b), as n is one greater than a multiple of three, we can arrange $3k + 1$ ones as k summands, each of the form $1 + 1 + 1$, plus a final 1 left over. That one is added to a summand containing three ones giving a summand containing four ones, which is then split into the product of two summands each containing two ones and the result follows. ∎

Note: Keene State College hosts a STEM Day for 5th Graders in May and this question was given as an exploration. Through trial and error, many groups were eventually able to describe in rough terms the general results.

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The Problem Corner

Edited by Pat Costello

The Problem Corner invites questions of interest to undergraduate students. As a rule, the solution should not demand any tools beyond calculus and linear algebra. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following new problems should be submitted on separate sheets before March 15, 2020. Solutions received after this will be considered up to the time when copy is prepared for publication. The solutions received will be published in the Spring 2020 issue of The Pentagon. Preference will be given to correct student solutions. Affirmation of student status and school should be included with solutions. New problems and solutions to problems in this issue should be sent to Pat Costello, Department of Mathematics and Statistics, Eastern Kentucky University, 521 Lancaster Avenue, Richmond, KY 40475-3102 (e-mail: pat.costello@eku.edu, fax: (859) 622-3051)

NEW PROBLEMS 840 - 848

Problem 840. *Proposed by the editor.*

Consider the sequence $a_0 = 1$, $a_1 = 1$, $a_n = 2a_{n-1} + a_{n-2}$ which is 1,1,3,7,17,41,99, ...
a) Prove that no term in the sequence ends in 5.
   [If you get this, send it.]
b) Prove that if p is prime, then $a_p \equiv 1 \,(\mathrm{mod}\,p)$.

Problem 841. *Proposed by José Luis Díaz-Barrero, School of Civil Engineering, Barcelona Tech - UPC, Barcelona, Spain.*

Let a, b, c be the roots of the equation $x^3 - x^2 - 2x - 3 = 0$. Find the value of $a^5 + b^5 + c^5$.

Problem 842. *Proposed by José Luis Díaz-Barrero, School of Civil Engineering, Barcelona Tech - UPC, Barcelona, Spain.*

Find all functions $f : \mathbb{R} \to \mathbb{R}$ such that

$$4^{-(x+y)} \le \frac{f(x)f(y)}{(x^4+1)(y^4+1)} \le \frac{f(x+y)}{(x+y)^4+1}$$

for all $x, y \in \mathbb{R}$.

Problem 843. *Proposed by Daniel Sitaru, "Theodor Costescu" National Economic College, Drobeta Turnu – Severin, Mehedinti, Romania.*

Prove that in $\triangle\ ABC$ you have

$$\sqrt{(2^{h_a} + 2^{h_b} + 2^{h_c})(2^{m_a} + 2^{m_b} + 2^{m_c})} < 2^a + 3^b + 4^c.$$

Problem 844. *Proposed by Daniel Sitaru, "Theodor Costescu" National Economic College, Drobeta Turnu – Severin, Mehedinti, Romania.*

Prove that if $0 < a < b < c < 1$, then

$$2\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a\ln a & b\ln b & c\ln c \end{vmatrix} >$$

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ (a-1)\ln(a^2+1) & (b-1)\ln(b^2+1) & (c-1)\ln(c^2+1) \end{vmatrix}.$$

Problem 845. *Proposed by Daniel Sitaru, "Theodor Costescu" National Economic College, Drobeta Turnu – Severin, Mehedinti, Romania.*

If $a, b, c \in [0, 1)$, then

$$8\int_0^a \left(\int_0^b \left(\int_0^c \frac{\sin^{-1}x \cdot \sin^{-1}y \cdot \sin^{-1}z}{(1+\sin^{-1}x)(1+\sin^{-1})(1+\sin^{-1}z)} dz \right) dy \right) dx$$
$$\leq a^2 b^2 c^2.$$

Problem 846. *Proposed by Pedro H.O. Pantoja, Natal/RN, Brazil.*

Evaluate $\int_0^{\pi/4} \cos^2(x) \cdot \ln(1 + \cos(4x)) dx$.

Problem 847. *Proposed by D.M. Bătinetu-Giurgiu, "Matei Basarab" National College, Bucharest, Romania and Neculai Stanciu, "George Emil Palade" School, Buzău, Romania.*

Let $(x_n), (y_n)$ be positive sequences of real numbers such that $\lim\limits_{n\to\infty} \frac{x_n}{n} = x$ and $\lim\limits_{n\to\infty} (y_{n+1} - y_n) = y$. Evaluate $\lim\limits_{n\to\infty} \left(\frac{y_{n+1}}{y_n} \right)^{x_n}$.

Problem 848. *Proposed by D.M. Bătinetu-Giurgiu, "Matei Basarab" National College, Bucharest, Romania and Neculai Stanciu, "George Emil Palade" School, Buzău, Romania.*

If $m \in (1, \infty)$, n an integer greater than 1, a_k positive reals and $\sum_{k=1}^{n} a_k = a$, then

$$\sum_{k=1}^{n} \left(\frac{a_k}{a_{k+1}} + \frac{1}{a_k} \right)^{1/m} \geq \left(\frac{2}{\sqrt{a}} \right)^{1/m} n^{1 + \frac{1}{2m}},$$

where $a_{n+1} = a_1$.

SOLUTIONS TO PROBLEMS 820 - 828

Problem 820. *Proposed by the editor*

Find a 4-digit positive integer $N = abcd$ which is divisible by 11 and $N/11 = b^2 + c^2 + d^2$.

Solution *by the Missouri State University Problem Solving Group, Springfield, MO.*

Since $10 \equiv -1 \pmod{11}$, a number $N = 10^3 a + 10^2 b + 10c + d$ is divisible by 11 if and only if $-a + b - c + d = 0 \pmod{11}$. Since $b^2 + c^2 + d^2 \leq 9^2 + 9^2 + 9^2 = 243$, we have

$$N = 11(b^2 + c^2 + d^2) \leq 11(243) = 2673.$$

Therefore, $a = 1$ or $a = 2$.

Suppose that $a = 2$. Then $bcd \leq 673$. Hence $0 \leq b \leq 6$. But then

$$N = 11(b^2 + c^2 + d^2) \leq 11(6^2 + 9^2 + 9^2) = 2178.$$

This shows $0 \leq b \leq 1$. But then

$$N = 11(b^2 + c^2 + d^2) \leq 11(1^2 + 9^2 + 9^2) = 1793,$$

a contradiction to $a = 2$. We have shown $a = 1$, and now we will try to find b.

Suppose $b = 0$. Then $-1 + 0 - c + d \equiv 0 \pmod{11}$ by the divisibility condition. But since c and d are digits, we have $d = c + 1$. From

$$N = 11(b^2 + c^2 + d^2),$$

we have $10^3 + 10c + (c + 1) = 11(0^2 + c^2 + (c + 1)^2)$ which simplifies to $22c^2 + 11c - 990 = 0$. This quadratic equation has no integer solutions.

So $b = 0$ is not a possibility.

Suppose $b = 1$. We would have $d = c$. Then

$$10^3 + 10^2 + 10c + c = 11(1^2 + c^2 + c^2)$$

which simplifies to $22c^2 + 11c - 1089 = 0$. This quadratic has no integer solutions. So $b = 1$ is not a possibility.

Suppose $b = 2$. We would have $d = c - 1$. Then

$$10^3 + 2 \cdot 10^2 + 10c + (c - 1) = 11(2^2 + c^2 + (c - 1)^2)$$

which simplifies to $11(2c^2 - 3c - 104) = 0$ and this has integer solution $c = 8$ which makes $d = 7$. The number $N = 1287$ is the only solution.

Also solved by Brad Meyer (student), Missouri State University, Springfield, MO; John Zerger, Catawba College, Salisbury, NC; Ioannis Sfikas, Athens, Greece; Carl Libis, Columbia Southern University, Orange Beach, AL; Abhijit Bhattacharjee (student), Banaras Hindu University, India; Cal Poly Pomona Problem Solving Group, Pomona, CA; Jeremiah Bartz, University of North Dakota, Grand Forks, ND; and the proposer.

Problem 821. *Proposed by Daniel Sitaru, "Theodor Costescu" National Economic College, Drobeta Turnu – Severin, Mehedinti, Romania.*

Prove that if $a, b, c \in \mathbb{R}$ then $4 \sum\limits_{cyclic} a|b(1 - b^2)| \leq \sum\limits_{cyclic} a(1 + b^2)^2$.

Solution *by Nicusor Zlota, "Traian Vuia" Technical College, Focsani, Romania.*

We have

$$
\begin{aligned}
4a\left|b(1 - b^2)\right| \leq a(1 + b^2)^2 &\Leftrightarrow 4a\sqrt{(b(1 - b^2))^2} \leq a(1 + b^2)^2 \\
&\Leftrightarrow 16a^2b^2(1 - b^2)^2 \leq a^2(1 + b^2)^4 \\
&\Leftrightarrow a^2(b^8 - 12b^6 + 38b^4 - 12b^2 + 1) \geq 0 \\
&\Leftrightarrow a^2(b^2 + 2b - 1)^2(b^2 - 2b - 1)^2 \geq 0.
\end{aligned}
$$

The last inequality is true so the first is true and then summing, we get the desired result.

Also solved by Almas Bebirov, Baku State University, Lerik, Azerbaijan; Ioannis Sfikas, Athens, Greece; and the proposer.

Problem 822. *Proposed by Daniel Sitaru, "Theodor Costescu" National Economic College, Drobeta Turnu – Severin, Mehedinti, Romania.*

Prove that in any acute-angled $\triangle ABC$ you have

$$2 \sum_{cyclic} \tan^3 A \geq \sum_{cyclic} \sqrt{\frac{\tan^6 A + \tan^6 B}{2}} + 3(\tan A + \tan B + \tan C).$$

Solution *by the proposer.*

Lemma. If $a, b \in (0, 1)$ then $a + b \geq \sqrt{\frac{a^2 + b^2}{2}} + \sqrt{ab}$.

Proof. Denote $\begin{cases} x = \sqrt{\frac{a^2 + b^2}{2}} \\ y = \sqrt{ab} \end{cases} \Rightarrow \begin{cases} a^2 + b^2 = 2x^2 \\ ab = y^2 \end{cases}$ Then

$$a + b \geq x + y$$
$$\Leftrightarrow (a + b)^2 \geq (x + y)^2$$
$$\Leftrightarrow 2x^2 + 2y^2 \geq (x + y)^2$$
$$\Leftrightarrow 2x^2 + 2y^2 \geq x^2 + 2xy + y^2$$
$$\Leftrightarrow x^2 - 2xy + y^2 \geq 0$$
$$\Leftrightarrow (x - y)^2 \geq 0,$$

which is true. Now replace a and b in the Lemma with $a = \tan^3 A; b = \tan^3 B$ and get

$$\tan^3 A + \tan^3 B \geq \sqrt{\frac{\tan^6 A + \tan^6 B}{2}} + \sqrt{\tan^3 A \tan^3 B}$$

so that

$$\sum \tan^3 A + \tan^3 B \geq$$
$$\sum \sqrt{\frac{\tan^6 A + \tan^6 B}{2}}$$
$$+ \sum \tan A \tan B \sqrt{\tan A \tan B}$$

and by the AM-GM

$$2\sum \tan^3 A \geq \sum \sqrt{\frac{\tan^6 A + \tan^6 B}{2}} + 3\sqrt[3]{\tan^3 A \tan^3 B \tan^3 C}$$

$$= \sum \sqrt{\frac{\tan^6 A + \tan^6 B}{2}} + 3(\tan A + \tan B + \tan C).$$

Also solved by Ioannis Sfikas, Athens, Greece; Nicusor Zlota, "Rraian Vuia" Technical College, Focsani, Romania; Soumava Chakraborty, Soft-Web Technologies, Kolkata, India; and Tran Hong-Dong Thap (student), Vietnam.

Problem 823. *Proposed by Pedro H.O. Pantoja, University of Campina Grande, Brazil.*

Let x, y, z be positive real numbers. Prove that

$$\frac{1}{xy + yz + zx} \leq \frac{3x}{(y+2z)^3} + \frac{3y}{(z+2x)^3} + \frac{3z}{(x+2y)^3} \leq \frac{x^3 y + y^3 z + z^3 x}{(3xyz)^2}.$$

Solution *by Ioannis Sfikas, Athens, Greece.*

Since $y + 2z = y + z + z \geq 3\sqrt[3]{yz^2}$ we have $(y + 2z)^3 \geq 27yz^2$. This means that

$$\frac{1}{(y+2z)^3} \leq \frac{1}{27yz^2} \Rightarrow \frac{3x}{(y+2z)^3} \leq \frac{x}{9yz^2}.$$

So we have

$$\frac{3x}{(y+2z)^3} + \frac{3y}{(z+2x)^3} + \frac{3z}{(x+2y)^3} \leq \frac{x}{9yz^2} + \frac{y}{9zx^2} + \frac{z}{9xy^2}$$

$$= \frac{x^3 y + y^3 z + z^3 x}{(3xyz)^2}.$$

Furthermore, we have

$$\frac{3x}{(y+2z)^3} + \frac{3y}{(z+2x)^3} + \frac{3z}{(x+2y)^3} = 3\left[\frac{\frac{x^2}{(y+2z)^3}}{x^2} + \frac{\frac{y^2}{(z+2x)^3}}{y^2} + \frac{\frac{z^2}{(x+2y)^3}}{z^2}\right]$$

$$\geq \frac{3\left(\frac{x}{(y+2z)} + \frac{y}{(z+2x)} + \frac{z}{x+2y}\right)^3}{3(x^2+y^2+z^2)}$$

$$\geq \frac{\left[\frac{(x+y+z)^2}{3(xy+xz+zx)}\right]^3}{(x^2+y^2+z^2)}$$

$$\geq \frac{1}{xy+xz+zx}.$$

Also solved by Almas Bebirov, Baku State University, Lerik, Azerbaijan; Titu Zvonaru, Comanesti, Romania; Sanong Huayrerai, Nakon Pathom, Thailand; Tran Hong-Dong Thap (student), Vietnam; Nicusor Zlota, "Traian Vuia" Technical College, Focsani, Romania; and the proposer.

Problem 824. *Proposed by Pedro H.O. Pantoja, University of Campina Grande, Brazil.*

Find all positive integers a, b, c where a and b are prime numbers with $a \not\equiv 0 \pmod{c}$ such that $51a + 7ab + bc^2 = abc^2$.

Solution *by Brian Beasley, Presbyterian College, Clinton, SC.*

Since b is prime and b divides $51a$, either b divides 51 or b divides a. Since a is also prime, we have either $b = a, b = 3$, or $b = 17$.

If $b = a$, then $51 + 7a + c^2 = ac^2$, so $a = \frac{(51+c^2)}{(c^2-7)}$. Since a is positive, $c > 2$. Since $a \geq 2$, we also have $51 + c^2 \geq 2(c^2 - 7)$ and thus $c < 9$. For $c \in 3, 4, 5, 6, 7, 8$, only $c = 6$ produces a prime number for a, namely $a = 3$. Hence this case yields $(a, b, c) = (3, 3, 6)$.

If $b = 3$, then $72a + 3c^2 = 3ac^2$, so $a = \frac{c^2}{(c^2-24)}$. This yields $4 < c < 7$, but only $c = 6$ produces a prime number for a. Once again, we conclude $(a, b, c) = (3, 3, 6)$.

If $b = 17$, then $170a + 17c^2 = 17ac^2$, so $a = \frac{c^2}{(c^2-10)}$. This yields $3 < c < 5$, but $c = 4$ does not produce an integral value for a.

Therefore the unique solution is $(a, b, c) = (3, 3, 6)$.

Also solved by Ioannis Sfikas, Athens, Greece; Cal Poly Pomona Problem Solving Group, Pomona, CA; John Zerger, Catawba College, Salisbury, NC; and the proposer.

Problem 825. *Proposed by Ovidiu Furdui and Alina Sintamarian, Technical University of Cluj–Napoca, Cluj–Napoca, Romania.*

Let $k \geq 0$ be an integer. Calculate

$$\sum_{n=1}^{\infty} \left[\left(\frac{1}{n^2} + \frac{1}{(n+1)^2} + \ldots \right) - \frac{1}{n+k} \right].$$

Solution *by Cal Poly Pomona Problem Solving Group, Pomona, CA.*

Consider the partial sum

$$S_j = \sum_{n=1}^{j} \left[\left(\frac{1}{n^2} + \frac{1}{(n+1)^2} + \ldots \right) - \frac{1}{n+k} \right]$$

$$= 1 + 2 \cdot \frac{1}{4} + 3 \cdot \frac{1}{9} + \ldots + j \cdot \frac{1}{j^2} + j \left(\frac{1}{(j+1)^2} + \ldots \right)$$

$$- \left(\frac{1}{1+k} - \ldots - \frac{1}{j+k} \right).$$

If $j > k$, then we have

$$S_j = \left(1 + \frac{1}{2} + \ldots + \frac{1}{k} \right) - \left(\frac{1}{j+1} + \ldots + \frac{1}{j+k} \right) + j \left(\frac{1}{(j+1)^2} + \ldots \right).$$

Since $\left(\frac{1}{j+1} + \ldots + \frac{1}{j+k} \right)$ is a finite sum, the limit as j goes to infinity is 0. and it is known that $\lim_{j \to \infty} j \left(\frac{1}{(j+1)^2} + \frac{1}{(j+2)^2} + \ldots \right) = 1$. Thus

$\lim_{j \to \infty} S_j = \left(1 + \frac{1}{2} + \ldots + \frac{1}{k} \right) - 0 + 1$

When $k = 0$, the first parenthesis is empty so the sum is 1.

When $k > 0$, the sum is $1 + \left(1 + \frac{1}{2} + \ldots + \frac{1}{k} \right)$.

Also solved by Ioannis Sfikas, Athens, Greece; and the proposers.

Problem 826. *Proposed by D.M. Batinetu–Giurgiu, "Matei Basarab" National College, Bucharest, Romania and Neculai Stanciu, "George Emil Palade" School, Buzau, Romania.*

Let F_n and L_n be the nth Fibonacci and Lucas numbers defined by

$F_1 = F_2 = 1$ and $F_n = F_{n+1} + F_{n+2}$ for $n \geq 3$ and by $L_1 = 1, L_2 = 3$ and $L_n = L_{n+1} + L_{n+2}$ for $n \geq 3$. Let k be a positive integer and

$$F(k) = \begin{pmatrix} F_k^2 & F_{k+1}^2 \\ F_{k+1}^2 & F_k^2 \end{pmatrix} \begin{pmatrix} L_{k+1} & L_k \\ L_k & L_{k+1} \end{pmatrix}.$$

Evaluate $\prod_{k=1}^{n} F(k)$ as a multiple of the matrix $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$.

Solution *by the proposers.*

We have

$$F(1) = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} L_2 & L_1 \\ L_1 & L_2 \end{pmatrix} = (L_1 + L_2) \cdot E = L_3 \cdot E$$

where $E = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$. Also

$$\begin{aligned} E \cdot F(k) &= \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} F_k^2 & F_{k+1}^2 \\ F_{k+1}^2 & F_k^2 \end{pmatrix} \begin{pmatrix} L_{k+1} & L_k \\ L_k & L_{k+1} \end{pmatrix} \\ &= (F_k^2 + F_{k+1}^2)(L_k + L_{k+1}) \cdot E \\ &= F_{2k+1} L_{k+2} E. \end{aligned}$$

when we use the known identity $F_k^2 + F_{k+1}^2 = F_{2k+1}$. Therefore

$$\begin{aligned} \prod_{k=1}^{n} F(k) &= F(1) \prod_{k=2}^{n} F(k) \\ &= L_3 \cdot (E \cdot F(2)) \cdot F(3)... \cdot F(n) \\ &= L_3 \cdot F_5 \cdot L_3 \cdot (E \cdot F(3)) \cdot F(4) \cdot ... \cdot F(n) \\ ... &= L_3 \cdot F_5 \cdot L_3 \cdot F_7 \cdot L_5 \cdot F_9 \cdot L_7...(E \cdot F(n-1)) \cdot F(n) \\ &= L_3 \cdot F_5 \cdot L_3 \cdot F_7 \cdot L_5 \cdot F_9 \cdot L_7...L_{n+1} \cdot F_{2n-1} \cdot E \cdot F(n) \\ &= L_3 \cdot F_5 \cdot L_3 \cdot F_7 \cdot L_5 \cdot F_9...L_{n+1} \cdot F_{2n-1} \cdot L_{n+2} \cdot F_{2n+1} \cdot E \\ &= L_3 \cdot \prod_{k=3}^{n+2} L_k \cdot \prod_{k=2}^{n} F_{2k+1} \cdot E. \end{aligned}$$

Also solved by Ioannis Sfikas, Athens, Greece.

Problem 827. *Proposed by D.M. Batinetu–Giurgiu, "Matei Basarab"*
National College, Bucharest, Romania and Neculai Stanciu, "George
Emil Palade" School, Buzau, Romania.

Let (a_n) be a sequence of positive real numbers such that
$\lim\limits_{n \to \infty} \frac{a_n}{n!} = a > 0$. Find

$$\lim_{n \to \infty} \left(\frac{(n+1)^2}{\sqrt[n+1]{a_{n+1}}} - \frac{n^2}{\sqrt[n]{a_n}} \right).$$

Solution *by Marian Ursărescu, Roman Vodă College, Roman City, Romania*

$$
\begin{aligned}
L &= \lim_{n \to \infty} \frac{(n+1)^2}{\sqrt[n+1]{a_n + 1}} - \frac{n^2}{\sqrt[n]{a_n}} \\
&= \lim_{n \to \infty} e^{\ln \frac{(n+1)^2}{\sqrt[n+1]{a_n+1}}} - e^{\ln \frac{n^2}{\sqrt[n]{a_n}}} \\
&= \lim_{n \to \infty} e^{\ln \frac{n^2}{\sqrt[n]{a_n}}} \left(e^{\ln \frac{(n+1)^2}{\sqrt[n+1]{a_n+1}} - \ln \frac{n^2}{\sqrt[n]{a_n}}} - 1 \right) \\
&= \lim_{n \to \infty} \frac{n}{\sqrt[n]{a_n}} \cdot n \left(e^{\ln \frac{(n+1)^2}{\sqrt[n+1]{a_n+1}} \cdot \frac{\sqrt[n]{a_n}}{n^2}} - 1 \right). \qquad (1)
\end{aligned}
$$

Now

$$
\begin{aligned}
\lim_{n \to \infty} \frac{n}{\sqrt[n]{a_n}} &= \lim_{n \to \infty} \sqrt[n]{\frac{n^n}{a_n}} \\
&= \lim_{n \to \infty} \frac{(n+1)^{n+1}}{a_{n+1}} \cdot \frac{a_n}{n^n} \\
&= \lim_{n \to \infty} \frac{(n+1)!}{a_{n+1}} \cdot \frac{a_n}{n!} \cdot \frac{n!}{(n+1)!} \cdot \frac{(n+1)^{n+1}}{n^n} \\
&= \lim_{n \to \infty} \frac{1}{a} \cdot a \cdot \left(\frac{n+1}{n} \right)^n = e.
\end{aligned}
$$

Let $x_n = \frac{(n+1)^2}{\sqrt[n+1]{a_{n+1}}} \cdot \frac{\sqrt[n]{a_n}}{n^2}$ and note $\lim\limits_{n \to \infty} \frac{n+1}{\sqrt[n+1]{a_{n+1}}} \cdot \frac{\sqrt[n]{a_n}}{n} \cdot \frac{n+1}{n} = e \cdot \frac{1}{e} \cdot 1 = 1.$

Thus

$$\lim_{n\to\infty} n(e^{\ln x_n} - 1) = \lim_{n\to\infty} n\frac{(e^{\ln x_n} - 1)}{\ln x_n} \cdot \ln x_n$$

$$= \lim_{n\to\infty} n \ln x_n = \lim_{n\to\infty} \ln x_n^n$$

$$= \lim_{n\to\infty} \ln\left[\left(1 + \frac{1}{n}\right)^{2n} \cdot \frac{a_n}{a_{n+1}} \cdot \sqrt[n]{a_{n+1}}\right]$$

$$= \lim_{n\to\infty} \ln\left[e^2 \cdot \frac{a_n}{n!} \cdot \frac{(n+1)!}{a_{n+1}} \cdot \frac{n!}{(n+1)!} \cdot \sqrt[n]{a_{n+1}}\right]$$

$$= \lim_{n\to\infty} \ln\left(e^2 \cdot a \cdot \frac{1}{a} \cdot \frac{1}{e}\right) = \ln e = 1. \qquad (2)$$

From (1) and (2) we get $L = e$.

Also solved by Ioannis Sfikas, Athens, Greece; Shafiqur Rahman, Bangladesh; Remus Florin Stanca, Ramnicu, Valcea City, Romania; and the proposers.

Problem 828. *Proposed by D.M. Batinetu–Giurgiu, "Matei Basarab" National College, Bucharest, Romania and Neculai Stanciu, "George Emil Palade" School, Buzau, Romania.*

Determine all injective functions $f : \mathbb{R} \to \mathbb{R}$ with $f(0) \neq 1/b$ and
$$f(f(x)y^3) + ax^9y^9 = bf(x^3)f(y^3)$$
for all $x, y \in \mathbb{R}$, where $a > 0$, $b > 0$.

Solution *by Cal Poly Pomona Problem Solving Group, Pomona, CA.*

Let $x, y = 0$ and then $f(0) = bf(0)$ or $f(0)(1 - bf(0)) = 0$. Since $f(0) \neq \frac{1}{b}$, we must have $f(0) = 0$. Since the equality holds for all reals, we have

$$f\left(f(x)y^3\right) + ax^9y^9 = bf(x^3)f(y^3)$$

and

$$f\left(f(y)x^3\right) + ax^9y^9 = bf(x^3)f(y^3).$$

So
$$f(f(x)y^3) + ax^9y^9 = f(f(y)x^3) + ax^9y^9.$$
Then $f(f(x)y^3) = f(f(y)x^3)$. Since f is injective, $f(x)y^3 = f(y)x^3$. Let $y = 1$. Then $f(x) = cx^3$ where where $c = f(1)$. Substituting into the original equality yields $c^4x^9y^9 + ax^9y^9 = bc^2x^9y^9$ or $c^4 - bc^2 + a = 0$. The

quadratic formula gives us $c^2 = \frac{b \pm \sqrt{b^2 - 4a}}{2}$. Because f is a real function, there are no solutions if $a > \frac{b^2}{4}$. If $a = \frac{b^2}{4}$, there are 2 solutions. Finally, suppose $a < \frac{b^2}{4}$. Since both a and b are positive, $b^2 > b^2 - 4a$ and by taking square roots $b > \sqrt{b^2 - 4a}$ and there are 4 solutions.

Also solved by Remus Florin Stanca, Ramnicu Valcea City, Romania; Ioannis Sfikas, Athens, Greece; and the proposers.

Kappa Mu Epsilon News

Edited by Cynthia Huffman, Historian
Updated information as of June 2019

News of chapter activities and other noteworthy KME events should be sent to

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KAPPA MU EPSILON
Installation Report
Kentucky Gamma, Bellarmine University
Louisville, Kentucky

The Kentucky Gamma Chapter of Kappa Mu Epsilon was installed at 5:00 p.m. on April 23, 2019 at a ceremony held on the campus of Bellarmine University in Louisville, Kentucky. The meeting was conducted by current KME member, Dr. Anne Raymond. KME national president, Dr. Brian Hollenbeck, served as the installing officer.

Six students and four faculty members were initiated as the charter members of the Kentucky Gamma Chapter. The four faculty are Assistant Professor of Mathematics Dr. Jen Miller, Associate Professors of Mathematics, Dr. Susan White and Dr. Daylene Zielinski, and Professor of Mathematics, Dr. Mike Ackerman. The six students are Megan Alexander, Jamey VanDyke, Philip Golden, Emily Barrett, Alyssa Wyman, and Andrea Howard. The first officers of the chapter were installed: Megan Alexander, President; Jamey VanDyke, Vice President; Philip Golden, Secretary; Emily Barrett, Treasurer; Jen Miller, Corresponding Secretary; and Anne Raymond, faculty sponsor.

Following the installation ceremony, Dr. Hollenbeck presented a talk titled, When the "Best" Strategy Fails, during which attendees played and analyzed the dice game, Kyboi. The event concluded with some refreshments.

Kentucky Gamma

Chapter News

AL Gamma – University of Montevello
Chapter President – Gray Lackey; 17 Current Members; 10 New Members
Other Spring 2019 Officers: Darius Burns, Vice President; Faith Kendrick,
Secretary; Dr. Scott Varagona, Corresponding Secretary and Faculty Spon-
sor.
AL Zeta – Birmingham-Southern College
Corresponding Secretary – Dr. Allie Ray; 5 New Members
New Initiates – Mac DeLay, Annie Dial, Conde Ireland, Dr. Allie Ray, and Christopher
Sucic
AL Eta – The University of West Alabama
Chapter President – Kaitlyn Strachan; 14 Current Members; 4 New Mem-
bers
Other Spring 2019 Officers: Samantha Day, Vice President; Dr. Jay Smith,
Corresponding Secretary and Faculty Sponsor.
The Alabama Eta chapter of Kappa Mu Epsilon initiated 4 new members
the spring semester of 2019.
AL Theta – Jacksonville State University
Chapter President – Marcus Shell; 50 Current Members; 18 New Members
Other Spring 2019 Officers: Benjamin Junkins, Vice President; Sabin Ban-
jara, Secretary; Leeanne Powell, Treasurer; and Dr. David Dempsey, Cor-

responding Secretary and Faculty Sponsor.

The Alabama Theta chapter met biweekly this semester, having periodic pizza/game nights, as well as other events, including a bowling outing and a talk by a professional actuary. On March 1, 2019, the Alabama Theta chapter initiated 18 new members (17 students and 1 new faculty member). New members received their certificates, pins, and honor cords in a ceremony held in room 355 Ayers Hall. In April, five student members attended the KME National Convention. Four students gave presentations: Benjamin Junkins, Marcus Shell, Joel Branham, and Holly Sparkman; we are proud of Ben Junkins for winning one of the top paper awards! (Also, Leeanne Powell assisted KME Treasurer David Dempsey by writing 40+ convention checks!) At our last meeting in April, new officers were elected for the 2019-2020 academic year. We are looking forward to another great year and a regional convention.

New Initiates – Adam Wesley Alcorn, Sabin Banjara, George Philips Beason III, Victoria Anne Beck, Chloe Gabrielle Borders, Travis Norris Cotney, Hannah Catherine Davis, Hali Michelle Dickeson, Michael Green, Kirstin Nicole Honeycutt, Terri Abigail Kennedy, Austin David McDaniel, Lauren Lynn Padgett, Tyler William Pressnell, Kazi A. Rahman, Katelynn Nicole Shadoan, Riley Kathleen Sitton, and Rebecca Ulrich.

CA Epsilon – California Baptist University

Corresponding Secretary – James Buchholz; 20 New Members

New Initiates – Anthony Anderson, Jacob Barraza, Ryan Berardelli, Hannah Bernal, Rebekah Bernal, Shannon Bivens, Matthew Gillis, Jared Gilster, Emmanuel Gutierrez, Sandy Huerta-Cruz, Noah Jackowitz, Caitlyn James, Rachel Koch, Angela Ku, Noah Lay, Kayley Maynard, Brandy Moore, Jennifer Ramirez, Sara Stites, and Seth Truitt.

CT Beta – Eastern Connecticut State University

Corresponding Secretary – Dr. Mehdi Khorami; 475 Current Members; 14 New Members

New Initiates – Kathryn Barrett, Shelby Bellamy, Arianna Biales, Nicholas Corentin, Anthony Coss, Zoe Czerenda, Joyce Gao, Stephanie Hogan, Haley Knox, Jeffrey Mathieu, Katherine ORourke, Rachael Scullion, Rebecca Szruba, and Lauren. Zabilansky

CT Gamma – Central Connecticut State University

Chapter President – Nicholas Sabia; 70 Current Members; 17 New Members

Other Spring 2019 Officers: Jonathan Maldonado, Vice President; Alyssa Mercaldi, Secretary; Sabrina Doolgar, Treasurer; Dr. Leah Frazee, Corresponding Secretary; and Dr. Marian Anton, Faculty Sponsor

We held our 2018-2019 Initiation Ceremony and Banquet on April 5, 2019, elected a new president, elected a new vice president, and the new corresponding secretary was initiated.

New Initiates – Moriah Aronson, Katie Barnicle, Amy Carlson, Leah Frazee, Seth Gagnon,

Miranda Gazaferi, Joshua Glynn, Shannon Guiheen, Robin Kalder, Rebecca Lunt, Jonathan Maldonado, Geoffrey May, Lydia Murphy, Emily Negron, Nicholas Sabia, Dajana Sejdiraj, and Brittany Sitler.

FL Gamma – Southeastern University

Chapter President – Shane Coris; 63 Current Members; 3 New Members
Other Spring 2019 Officers: Kaitlyn Brett, Vice President; Elizabeth Berna-towicz, Secretary; and Dr. Berhane Ghaim, Corresponding Secretary and Faculty Sponsor.

Florida Gamma held its initiation ceremony on April 4, 2019. There were three new initiates and with this the total membership of the chapter is 63.

GA Beta – Georgia College & State University

Corresponding Secretary – Rodica Cazacu; 7 New Members

New Initiates – Hanwen Chen, Justin Hockey, Emily Howe, Byung Rok Kim, Madison Link, Courtney McDonald, and Virginia Oliver.

GA Epsilon – Wesleyan College

Corresponding Secretary – Dr. Joe Iskra; 4 New Members

New Initiates – Elizabeth F. Bender, Kali Elizabeth Goding, Melissa Jazmine Rey, and Akshayaa Shrestha.

HI Alpha – Hawaii Pacific University

Chapter President – Petra Keiser; 20 Current Members; 13 New Members
Other Spring 2019 Officers: Alexxis De Lamere, Vice President; Tara Davis, Corresponding Secretary and Faculty Sponsor.

We had an initiation dinner in March to initiate the 13 new members.

New Initiates – Gioia R. Aquino, Jeffery H. Bourna, Tara Davis, Jonathan DiVietro, Mhary Grace C. de Francia, Maxime Gayte, Andrea Langeland, Daniel O. Mendoza Hermosillo, Alicia Nawatani-Dias, Saskia Rechin, Mina Hany Sokar, Vince Sushima, and Michelle Tubolino.

IA Alpha – University of Northern Iowa

Chapter President – Isaac Neppel; 25 Current Members; 2 New Members
Other Spring 2019 Officers: Jaclyn Miller, Vice President; Christopher Merck, Secretary; Stephanie Peiffer, Treasurer; and Dr. Mark D. Ecker, Corresponding Secretary and Faculty Sponsor.

Our first spring KME meeting was held on February 28, 2019 in Wright Hall, where student member Christopher Merck presented his paper entitled "State Domestic Product: What Actually Influences a State's GDP'. Our second meeting was held on April 4, 2019 in Wright Hall, where student member Lincoln Fuhs talked about his undergraduate research paper entitled "A Statistical Analysis of NBA Players: What Role Does Position Play?". Student member Hannah Larsen addressed the spring initiation banquet on April 30, 2019 with her senior seminar project entitled "Factors that Affect Home Runs in MLB". Our banquet was held at Godfather's

Pizza in Cedar Falls, where four new members were initiated.

New Initiates – Rachel Liercke and Staci Schmeling.

IL Zeta – Dominican University

Chapter President – Frankie Greco; 21 Current Members; 2 New Members

Other Spring 2019 Officers: Dr. Aliza Steurer, Corresponding Secretary and Faculty Sponsor.

The Illinois Zeta Chapter at Dominican University was excited to initiate two new members this spring, Michael Cross and Zayra Solis. Michael Cross is a mathematics and computer science double major. Zayra Solis is a mathematics major who is obtaining licensure to teach high school mathematics. Family and friends of Michael and Zayra, fellow classmates, and faculty and students from nearby schools took part in the initiation ceremony on April 11, 2019. We were excited to welcome James Tanton as the plenary speaker, who gave the talk "How Many Degrees Are in a Martian Circle?"

New Initiates – Michael Cross and Zayra Solis.

IL Eta – Western Illinois University

Corresponding Secretary – Amy Ekanayake; 1 New Member

New Initiates – Jordan Hughes.

IL Theta – Benedictine University

Corresponding Secretary – Dr. Thomas Wangler; 8 New Members

New Initiates – Katelyn Beamish, Stefano Chiaradonna, Fernando Elesterio, Kathryn (Katie) Farb, Rishabh (Krish) Jakhariya, Mariam Khan, Emma Roberts, and Alaina Thompson.

IN Beta – Butler University

Corresponding Secretary – Chris Wilson; 13 New Members

New Initiates – Richard Clark, Alexandra Gossett, Ryan Heumann, Joseph Hoover, Daniel Mallerdino, Grace Maynard, Haley Niemann, Tyler Pollard, Isabelle Ramey, Brianna Sorenson, Lauren Turnbull, Troy Wiegand, and Max Connor Wiley.

KS Alpha – Pittsburg State University

Chapter President – Peyton Burlingame; 20 Current Members; 9 New Members

Other Spring 2019 Officers: Rylie Bertels, Vice President; Paul Modlin, Secretary; James Beyer, Treasurer; Dr. Tim Flood, Corresponding Secretary; and Dr. Scott Van Thuong, Faculty Sponsor.

The Kansas Alpha chapter had an active spring, with many new members joining our ranks. At our first meeting, we gained experience in problem solving in preparation for the Kansas Collegiate Math Competition (in which a team from Pitt State placed 1st, tying with KU!) We concluded the semester by playing the card game SET over pizza and beverages. Drs. Childers and Huffman represented Kansas Alpha at the KME

national meeting this year. We look forward to an active Fall semester as well!

New Initiates –Cali Byrn, Sarah Case, Andrew Chesney, Tyler Clark, Rylee Dennis, Sloan Geddry, Skyler Hausback, Morgan Panovich, and Morgan Singletary.

KS Beta – Emporia State University

Chapter President – Katie Beckley; 45 Current Members; 4 New Members
Other Spring 2019 Officers: Alec Bergeron, Vice President; Elisabeth Evans, Secretary; Amber Innes, Treasurer; Tom Mahoney, Corresponding Secretary; and Brian Hollenbeck, Faculty Sponsor.

Our local chapter assisted in the Math Family Game Night at our local high school, as well as held a bake sale on campus for our math and science open house. We had 5 students and one faculty attend the National Convention in Maryland where 1 student gave a presentation. Our KME lounge has been extremely popular among all students as a social hangout and studying lounge. We have also held events such as movie nights and game nights during the year.

New Initiates – Samuel Ewusi Dadzie, Elisabeth Evans, Amber Innes, and Yuki Yomogita.

KS Delta – Washburn University

Chapter President – Laura Crosswhite; 20 Current Members; 6 New Members
Other Spring 2019 Officers: Jacob Talkin, Vice President; Mary Greene, Secretary; Benjamin Nelson, Treasurer; Kevin Charlwood, Corresponding Secretary and Faculty Sponsor.

The Kansas Delta chapter of KME met once each month over lunch throughout the spring semester with our math club. We hosted a graduate student from UNL who spoke on Kruski's algorithm form graph theory, a statistician from the Kansas Department of Labor, an actuary from BCBS of Kansas, and one of our faculty spoke on square roots as periodic continued fractions. We held our annual spring initiation ceremony in March, initiating 5 new student members and 1 new faculty member.

New Initiates – Abigail Beliel, Michaela Brown, Paul Enneking, Madison Henley, Yuta Tsutsumi, and Todd Cooksey (faculty).

KY Beta – University of the Cumberlands

Chapter President – Matthew Prather; 22 Current Members; 4 New Members
Other Spring 2019 Officers: Abby Walker, Vice President; Dustin Hensley, Secretary; Joshua Isenhoff, Treasurer; Dr. Jonathan Ramey, Corresponding Secretary and Faculty Sponsor.

On April 11, members assisted in hosting a regional high school math contest, held annually at the University of the Cumberlands. On April 29, 2019, the Kentucky Beta chapter held an initiation at the Correll Science

Complex and a dinner at the Torres Steakhouse. Kappa Mu Epsilon initiated four new student members at the ceremony, presided over by outgoing president, Matthew Prather. As an additional feature, senior awards were given by the department at the banquet.

KY Gamma – Bellarmine University

Corresponding Secretary – Jen Miller; 19 New Members

New Initiates – Michael Ackerman, Megan Alexander, Emily Barrett, Brennan Bauer, Maxwell Cartor, Abigail Finch, Philip Golden, Jerod Hopson, Andrea Howard, Joseph Isakson, Gregory Kelsey, Brandon Messex, Jennifer Miller, Jordan Reinhart, Bekkah Trachtenburg, Jamey Van Dyke, Susan White, Alyssa Wyman, and Daylene Zielinski.

MD Alpha – Notre Dame of Maryland University

Chapter President – Chinwendu Nwokeabia; 28 Current Members; 8 New Members

Other Spring 2019 Officers: Bhavya Bhardwaj, Vice President; Justice Walrath, Secretary; Hannah Woodworth, Treasurer; Charles Buehrle, Corresponding Secretary and Faculty Sponsor.

Below are items from MD Alpha.

New Initiates – Amanda Ashton, Aisha Azhar, Hannah Campbell, Emily Garzon, Sarah Hasnain, Hunter King, Keira Mull, and Uloaku Onwumelu.

MD Delta – Frostburg State University

Chapter President – Demetrick McDonald; 27 Current Members; 14 New Members

Other Spring 2019 Officers: Braden Ebersole, Vice President; Jordan Thomas, Secretary; Zach Kline, Treasurer; Mark Hughes, Corresponding

Secretary and Faculty Sponsor; and Frank Barnet, Faculty Sponsor.
Spring Semester 2019 was especially active for Maryland Delta Chapter. After an organizational meeting in February our Initiation Ceremony was held on March 10. Fourteen new members were welcomed to the chapter which is the largest group we've had in quite a few years. The ceremony included a presentation by faculty sponsor Dr. Mark Hughes on some interesting applications of Cavalieri's Principle due to Roberval and Torricelli. A few days later we held our annual Pi-Day Bake Sale which was quite successful. The month of April was very special in that Maryland Delta Chapter had the honor of hosting Kappa Mu Epsilon's 42nd Biennial National Convention. It was a wonderful experience to meet students and faculty representing some 18 KME chapters coming from 11 different states. Among the highlights of the convention were the twelve student presentations where a lot of talent was on display! Thanks go to the students of Maryland Delta Chapter who helped make this a successful event, in particular, our chapter secretary Jordan Thomas. During our late April meeting, new officers were elected for the upcoming school year. Jordan Thomas will serve as President, with Katelynn Suesse as Vice President and Bailey Brewer as Secretary. We finished the semester with a picnic on our "reading day" just before the start of final exams. We were fortunate to have great weather (unlike last year!) and a lot of fun was had by all. We offer our best wishes to graduating chapter members, namely, Devon Zollinhofer, Erikas Laignel and chapter officers Demetrick McDonald, Braden Ebersole and Zach Kline.

New Initiates – Perry Beachum, Matthew Beall, Philip Bernstein, Bailey Brewer, Ryan Esteban, Katie Hamilton, Ian Macomber, Bryan May, Ryan Miller, Jesse Miltenberger, Deion Okonkwo, Griffith Speis, Sophia Staggers, and Devin Williams.

MD Epsilon – Stevenson University
Corresponding Secretary – Dr. Benjamin Wilson; 10 New Members

New Initiates – Caroline Dangel, Zachary DeMunda, Henry Fleishman, Meytal Guller, Michael Hall, Cassidy Hankins, Thomas Holcomb, Katie Leftridge, Sarah Riggin, and Evan Williams.

MI Alpha – Albion College
Corresponding Secretary – Mark Bolman; 4 New Members

New Initiates – Subeedei Barkhasbadi, Kendra Cook, Claire Mitchell, and Samantha White.

MI Beta – Central Michigan University
Chapter President – Natalie DeVos; 15 Current Members; 6 New Members
Other Spring 2019 Officers: MinJeong Kim, Vice President; Emily Naegelin, Secretary; Evan Miller, Treasurer; Camilla Madacki, Public Relations; and Dr. Ben Salisbury, Corresponding Secretary and Faculty Sponsor.
The first meeting of the Spring 2019 semester was on January 14. The

meeting conducted some ice breakers to get acquainted with new attendees and played the game MindTrap. KME held a book sale in from January 15 until January 17 to raise funds for their activities. On February 4, KME held a mathematically inspired escape room. The game was organized by KME board member Evan Miller. KME and Gamma Iota Sigma had been in discussion for some time about a joint panel intended to give professional advice to those majoring in mathematical sciences. This panel came to fruition on February 19. The panel consisted of a mathematics professor, a high school mathematics teacher, an actuary, two data scientists, a current Ph.D. student, and the current mayor of Mount Pleasant (who graduated with a mathematics major from CMU). There was a Sage-Math/Python tutorial at the February 25 meeting. KME held a competition amongst its members for a Pi Day T-shirt design. The winning design was submitted by Jean Han. Shirts were sold in the department from March 12 until March 14. By the completion of the sale, the shirts were sold out. On March 18, Olivia Walch (University of Michigan) gave a lecture entitled "Math, Sleep, and Our Phones." In her talk, Dr. Walch discussed the mathematics of sleep and circadian rhythms. On April Fools' Day, KME hosted a meeting on mathematics and magic. KME members raised money to support the "Special Olympics" event on CMU campus by selling medallions in April 2018. The overall campaign took place from March 19 until May 5 in preparation for the games held May 31 to June 2. KME had a table set up in the department selling medallions from April 9 until April 12. KME held its 2019 initiation ceremony on April 14. Six new members were initiated into the society. Jordan Watts, Assistant Professor of Mathematics at CMU, gave a talk entitled "Game of Bones: A Song of Aid and Exploration". Mathematical Jeopardy! was the name of the game on April 15. Math-a-palooza was again co-organized by KME with the AMS Graduate Student Chapter. The Spring 2019 event was held on April 26.

New Initiates – Lauren Hutter, Luke Kupari, Wenjie Li, Evan Miller, Bridget Parker, and Antonia Sundeen.

MI Delta – Hillsdale College

Chapter President – Gill West; 62 Current Members; 13 New Members
Other Spring 2019 Officers: Abigail Trouwborst, Vice President; Emma Clifton, Secretary; Ben Becker, Treasurer; and Dr. David Gaebler, Corresponding Secretary and Faculty Sponsor.

The Michigan Delta chapter inaugurated 13 members on February 26, 2019. The initiation ceremony was accompanied by a math talk by Dr. David Gaebler on "Generatingfunctionology: Bridging the Gap Between the Continuous and the Discrete."

MI Epsilon – Kettering University

Chapter President – Danny Boyle; 32 Current Members; 17 New Members
Other Spring 2019 Officers: Rebecca Abbott McCune, Vice President; Makayla Carpenter, Secretary; Lindsey Malson, Treasurer; Boyan Dimitrov, Corresponding Secretary; and Dr. Matt Causley, Faculty Sponsor.

Kettering has now successfully re-established both its A and B-section chapters of KME. We were very pleased to add 17 members to B-section

last fall, and downright elated to initiate new 32 members this spring! Future events that are planned include social mixers, tutoring high school students in the greater Flint community, and perhaps a T-shirt design to raise funds for the society. The Winter and Spring terms brought lots of excitement into our Mathematics life. First of all, our Actuarial program was granted the Advanced Curriculum designation by the Society of Actuaries thanks to the work of our colleague Hee Seok Nam. He joined Kettering 4 years ago, and his job is getting the highest ratings. Next I would note the inclusion of the "Islands" virtual environment in our Statistics education program. This program is supported by Terrell grant through our Center of Excellence in Teaching and Learning (CETL) and was initiated by our Department Head, Dr. Leszek Gawarecki). The "Islands" offer a holistic experience for students: they transition through all stages of a real statistical study, from planning, data collection and organization, to formulating and testing hypotheses and drawing conclusions and making recommendations. Final reports are in the form of class presentations. Students are excited about the new "toy". We intend to present our results at the next Lilly conference, the QuadPod Symposium and maybe at some of the KME traditional sessions. Another important and significant outcome is the nomination of our student and KME President, Peter Morell, for the Presidential Medal at the upcoming Graduation, June 15. Peter is completing his Kettering education with remarkable results: he received the Applied Mathematics Student Research Award in 2015 as a sophomore for his research on fast approximations of the Euler Constant "e", in collaboration with our current faculty sponsor, Dr. Matt Causley. Peter is the President of our KME Kettering chapter of Section B. We are glad for his achievements. Wish him a good luck in his future professional life. Sorry to see Peter leaving. Another deserving news is that our colleague and longtime KME member, professor Ilya Kudish retired. He is Professor Emeritus at Kettering, one of the best in Tribology research, the NFL grant recipient years ago, on the wall of fame posted and valuable applied mathematics member. We are sorry to see him leaving, but wish Ilya a happy retirement and further best in his research endeavors. I finish my report with a photo of our B-section Kettering KME Michigan Epsilon student members.

On the floor are the Corresponding Secretary, Professor Dr. Boyan
Dimitrov, and the Faculty Sponsor, Professor Dr. Matt Causley

New Initiates – Andrea Allen, Kareem Alsarraj, Khary Atchinson, Isabella Barrett, Karen
Bennett, Larissa Buckingham, Sophia Dancel, Andrew Gomez, Jordan Howell, Joyce He,
Amy Jachim, Chandler Larimore, Nolan Martin, Peter Morell, Leah Stock, Molly Wittig,
and Rachel Wyatt.

MO Beta – University of Central Missouri

Chapter President – Matthew Enlow; 22 Current Members; 6 New Members

Other Spring 2019 Officers: Derek Noe, Vice President; Jonathan Hunt, Secretary; Yah Lee Chua, Treasurer; Alec McClendon, Historian; Rhonda McKee, Corresponding Secretary; and Steve Shattuck, Faculty Sponsor.

The Missouri Beta Chapter enjoyed several mathematical activities during
their spring semester monthly meetings. A highlight of the semester was a
road trip to the KME National Convention in Silver Spring, Maryland. Our
chapter was represented at the convention by 11 students and three faculty
members. Our president Matthew Enlow presented his research and was
listed among the top four papers.

New Initiates – Brittany Lynn Fuller, Victoria Hracovschi, Julia Johnson, Alec McClendon,
Riley Kathleen Meyer, and Rachelle Williams.

MO Gamma – William Jewell College

Chapter President – Katie Bird; 4 Current Members; 4 New Members
Other Spring 2019 Officers: Abby Christensen, Vice President; and Erin Martin, Corresponding Secretary and Faculty Sponsor.

We met and heard about the research project of our senior math majors.

MO Epsilon – Central Methodist University

Corresponding Secretary – Pam Gordy; 10 New Members

New Initiates – Cassandra Armentrout, Kody William Hackman, Hayden Hackman, Jensen

Lake, Janie Leathers, Sabrina Otzenberger, Samuel Priddy, Petra Schaffer, Abbey Sorrell, and Bailey Stevens.

MO Theta – Evangel University

Chapter President – Heather Culbertson; 17 Current Members; 5 New Members

Other Spring 2019 Officers: Jacob Crews, Vice President; and Don Tosh, Corresponding Secretary and Faculty Sponsor.

Meetings were held monthly. In February we initiated 5 new members and elected new officers. In April Dr. Tosh, Dianne Twigger and five students attended the national convention at Frostburg State University in Maryland, where one student, Rebekah Chase, won an award for the paper she presented. Also in April we had our end-of-year meeting, where honor cords were given to graduating members.

New Initiates – Jonathan Dasal, Cammi Kjetland, Riley Nelson, Trey Shupp, and Jonas Taylor.

MO Iota – Missouri Southern State University

Corresponding Secretary – Amila Appuhamy; 4 New Members

New Initiates – Trevor Hailey, Jacob Lett, Kim Richardson, and Justin Sanders.

MO Kappa – Drury University

Corresponding Secretary – Dr. Carol Browning; 8 New Members

New Initiates – Claire Allen, Brendan Bena, Lily Brockus, Riley Gatten, Benjamin High, Sarah Lester, Michelle Medina, and Mark Wood.

MO Nu— Columbia College

Corresponding Secretary – Kenny Felts; 3 Current Members; 5 New Members

New Initiates – Karissa Brickey, Mark Dierker, Daymond Dollens, Bibek Poudel, and Alix Ray.

MO Xi – William Woods University

Chapter President – Briley Browning; 9 Current Members; 4 New Members

Other Spring 2019 Officers: Mikayla Laburay, Vice President; and Dr. Chris Schneider, Corresponding Secretary and Faculty Sponsor.

The Missouri Xi chapter initiated four new members during the Spring 2019 ceremony: Mackenzie Hawkins, Aurora Henriksen, Connor Poulson, and Maggie Wieberg. They were joined by returning members Briley Browning and Mikayla Laburay, as well as faculty members Chris Schneider (sponsor), Julie Davenport, and Raymond Hune. Refreshments and fellowship were enjoyed by all after the ceremony.

New Initiates – Mackenzie Breanna Hawkins, Aurora Lucie Henriksen, Connor William

Poulson, and Maggie Marie Wieberg.

MS Alpha – Mississippi University for Women

Chapter President – Ishan Lamichhang; 13 Current Members; 6 New Members

Other Spring 2019 Officers: Ashruti Pant, Vice President; Anush Aryal, Secretary; and Dr. Joshua Hanes, Corresponding Secretary and Faculty Sponsor.

New Initiates – Anush Aryal, Ishan Lamichhang, Sonji Lamichhane, Ashruti Pant, Resha Rajkarmkar, and Sandesh Timilsina.

MS Gamma – The University of Southern Mississippi

Chapter President – Yumi Mahajan; 15 Current Members; 3 New Members

Other Spring 2019 Officers: Hamas Tahir, Vice President; Gokul Bhusal, Secretary; Amit Tripathi, Treasurer; Zhifu Xie, Corresponding Secretary; and Ana Wan, Faculty Sponsor.

New Initiates – Lindsey Gomez, Shradha Bhatta, and Sanjan Adhikari.

NC Epsilon – North Carolina Wesleyan College

Corresponding Secretary – Gail Stafford; 8 New Members

New Initiates – Angela Nicole Bird, Payton Coleman, David Austin Mooney, Chinwe Cynthia Obi, Julia Anne Trimmer, Samantha G. Valeros, Jackie N. Webb, and Abigail Wooten.

NC Zeta – Catawba College

Chapter President – Marcia-Mariel Erhart; 16 Current Members; 7 New Members

Other Spring 2019 Officers: Matt Hefner, Vice President; Cody Bennett, Secretary; Branden Davis, Treasurer; and Dr. Doug Brown, Corresponding Secretary and Faculty Sponsor.

The NC Zeta Chapter sent two students and two faculty to the biennial national convention in Frostburg, MD. There Marcia-Mariel Erhart presented a paper for which she won an award. The chapter initiated 7 new members on April 30 and said farewell to 7 graduating seniors. A cookies and ice-cream social followed the initiation ceremony.

New Initiates – Allison Baker, Kaitlin Koons, Rachel Martin, Luca Picci, Kelly Sandoval, Amber White, and Nathan Workman.

NC Eta – Johnson C. Smith University

Corresponding Secretary – Brian Hunt; 8 New Members

New Initiates – Taneva Bush, Miranda Davis, Zabdiel Escalona, Charmaine Jeff, Brenda Montanez, Jania Rodriguez, Kaitlin Santos, and Jessica Stills.

NE Beta – University of Nebraska Kearney

Chapter President – Tiffany Collins; 10 Current Members; 2 New Members

Other Spring 2019 Officers: Kyle Glandt, Vice President; Evan Olson,

Secretary; Joshua Garcia, Treasurer; and Dr. Katherine Kime, Corresponding Secretary and Faculty Sponsor.

Our KME chapter sponsored a campus showing of the movie Good Will Hunting, in conjunction with the History Honor Society, Phi Alpha Theta. We had one graduate, Vice President Kyle Glandt, who will attend graduate school in computer science in the fall at Kansas State. Kyle developed the format for a database with the names of all our chapter KME members (over 900 as of this writing). The database now has entries up through Spring 2009, with KME student members sharing the task of entering names. We are sad to report that Dr. Richard Barlow, UNK professor and long-time KME faculty sponsor, passed away March 25, 2019. He was also an undergraduate at UNK and the first member of our chapter, initiated in December,1959. He often taught statistics and calculus and was a highly enthusiastic and inspired lecturer until his retirement in 2007. His good cheer and voluble nature made a positive contribution to our department.

New Initiates – – Erin Green and Tatiana Moore.

NE Delta – Nebraska Wesleyan University
Corresponding Secretary – Melissa Erdmann; 6 New Members

New Initiates – Lily Craig, Caitlynn Croner, Andreana Cuka, Jackson Maschman, Macey Wiebe, and Adrianna Vinton.

NE Gamma – Chadron State College
Corresponding Secretary – Gregory Moses; 8 New Members

New Initiates – Miles Chasek, Viraj Faria, Noah Fisher, Alyssa Geist, Caleb Haskell, Andrew Hultquist, Tochukwu Mbanugo, and William Morgan.

NY Iota – Wagner College
Corresponding Secretary – Marisa Scarpa; 7 New Members

New Initiates – Delaney Beck, Noel-Aman DelCerro Vilalta, Alexandra Dmytrow, Matthew Louis Greiss, Regina Ismaili, Allison Krauza, and Jacqueline Sjogren.

NY Kappa – Pace University
Corresponding Secretary – Shamita Dutta Gupta; 20 Current Members; 1 New Member

Our initiation ceremony was held on May 9. Typically Mathematics faculty from both campuses come together to celebrate the candidates who are initiated over a luncheon, a lecture and initiation ceremony proceedings. During the school year we host a number of student activities, such as hosting the AMC 8, 10 and 12 contest. We also host school trips with a fun day of math activities and campus visit for their college readiness programs. This year Ryan Middle School visited us on March 8.

NY Lambda – LIU Post
Chapter President – Clifford Clark; 23 Current Members; 13 New Mem-

bers

Other Spring 2019 Officers: Alessandro Tripoli, Vice President; Amun Patel, Secretary; Megan Frey and Paul Tredwell, Co-Treasurers; and Dr. Corbett Redden, Corresponding Secretary and Faculty Sponsor.

Two members of the NY Lambda chapter attended this year's KME National Conference, and one of the students (Sandy Hussain) gave a talk. Our chapter also held its annual banquet and initiation ceremony on April 7, 2019.

New Initiates – Abdullah Alatawi, Trky Alhsmy, Fatimah Alshahrani, Alexa Armenti, Charlotte Beshers, Toby Guan, Sandy Hussain, Dilek Su Kucuk, Thomas Reid, Dana Reynolds, Chad Tomassetti, Courtney VanRoten, and Jiarui Zhang.

NY Mu – St. Thomas Aquinas College

Corresponding Secretary – Dr. Heather A. Rave; 7 New Members

New Initiates – Julia Marie Foto, Thomas John Holmes, Abagaile Therese Kimbrell, Rebecca MacLeod, Alessandro J Molinas, Morgan Helena Santora, and Chae H. Yu.

NY Nu – Hartwick College

Corresponding Secretary – Gerald Hunsberger; 1 New Member

New Initiates – Jack Philp.

NY Xi – Buffalo State College

Corresponding Secretary – Jane Cushman; 17 Members

New Initiates – Ahmed Akbar, Karissa Allen, Stephen Bewlay, Veronique Budd-Nicholas, Zachary Colson, Breanna Cook, Abdulkhaliq Edan, Alexander Furtney, Hannah Harshberger, Kyle Hunn, Alan Pomietlarz, Victoria Rigler, David Simmons, Michele Schultz, Carmen Terrell, Allison Torsey, amd Marcus Webb.

NY Pi – Mount Saint Mary College

Corresponding Secretary – Lee Fothergill; 4 New Members

New Initiates – Charles Benfer, Stephanie Ann Brown, Alexa Garbarino, and Sophia M. Reinhardt.

NY Rho – Molloy College

Corresponding Secretary – Manyiu Tse and Deborah Upton; 13 New Members

New Initiates – Christopher Arata, Kelsey Burke, Samantha Fischetto, Kathryn Gibson, Christina Labita, Alissa Leone, Heather McAllister, John Pascuzzi, Mia Picone, Frank Rossetti, John Rotchford, Jamie Sindicic, and Sophia Valkiv.

NY Sigma – College of Mount Saint Vincent

Corresponding Secretary – Peter Luthy; 15 New Members

New Initiates – Maria Aceituno, Daniel Amarante, Keith Carreon, Nicole Desouza, Savannah George, Weily Lang, Peter Luthy, Jose Mendoza, Andrea Minei, Victor Miroshnikov, Amir Niknejad, Danielle Quaranto, Sean Rice, Janet Rollins, and Lauren Tate.

OH Gamma – Baldwin Wallace University

Chapter President – Corrinne Horvath; 50 Current Members; 15 New

Members
Other Spring 2019 Officers: Katherine Wohlford, Vice President; Thomas Rector, Secretary; David Calvis, Corresponding Secretary and Faculty Sponsor.

On March 4, we received 15 new initiates into membership. Our officers for 2019-20 will be President: Sydney Leither, Vice-President: Jessica Blakley, and Secretary: Jeremiah Allis.

New Initiates – Miriam Alramzi, Jessica Blakley, Christopher Bradshaw, Zachary Egler, Sujatha Gaddamanugu, Cameron Krivanek, Sydney Leither, Sarah Massey, Mayre Newcomb, Samantha Pruett, Harrison Rouse, Mary Schentur, Ethan Slutzky, Hannah Timony, and Devon Tingley.

OH Zeta – Muskingum University

Corresponding Secretary – Richard Daquila; 5 New Members

New Initiates – Caitlyn Christman, Emily Ellyson, Jocelyn McBurney-Buell, Paige Morrow, and Gabriel Weeldreyer.

OH Eta – Ohio Northern University

Corresponding Secretary – Ryan Rahrig; 1 New Member

New Initiates – Bryan Peck.

OH Theta – Capital University

Chapter President – Krista Taulker; 15 Current Members; 6 New Members
Other Spring 2019 Officers: Kathryn Stahl, Vice President; Taylor Manivanh, Secretary; Jarrett Williams, Treasurer; Paula Federico, Corresponding Secretary; and Jon Stadler, Faculty Sponsor.

This semester our Chapter of KME hosted a campus wide celebration for Pi Day on March 14, 2019. The event consisted of giving mini pies and coffee to students entering our Science Building and talk given by Dr. Stadler about Math and Juggling. Students also designed a Pi Day t-shirt for the event. Funding for this event was provided by the University President's office.

This year's initiation ceremony took place on Sunday, April 7th and we welcomed 6 new members. Dr. Judy Holdener from Kenyon College give

a talk entitled "Mathematical Patterns in Digital Art." She created a piece of art incorporating graphics from different areas and shared a little bit of the mathematical background that led to some parts of the art composition; it was very interesting and engaged our diverse audience of students, families, and faculty. During the ceremony, we also distributed KME cords to the graduating seniors.

At the end of April, students met and selected officers for the next academic year. The officers for next academic year will be: Taylor Manivanh – President, Jarrett Williams – Treasurer, Colin Swisher – Vice President, and Josh Wissman – Secretary.

New Initiates – Emma Cary, Meredith Cox, Sonia Ezenwajiaku, Emily Singer, Colin Swisher, and Joshua Wissman.

PA Alpha – Westminster College
Corresponding Secretary – Pamela Richardson; 7 New Members

New Initiates – Jessica Booher, Andrius Burnelis, Elaina Chapnell, Hannah Fell, Natalie Horstman, Mackenzie Smith, and Emily Wilson.

PA Epsilon – Kutztown University
Corresponding Secretary – Dr. Lyn McQuaid; 13 New Members

New Initiates – Safal Raut Chhetri, Angela De Cicco, Laura Durdon, Zachary Fike, Christine Holland, Karissa Jean, Jenna Kanyak, Georgia Lobb, Vanessa Maybruck, Abby Moyer, Isaac Reiter, Alexander Swift, and Xinyi To.

PA Theta – Susquehanna University
Corresponding Secretary – Kenneth Brakke; 14 New Members

New Initiates – Sana Ahmed, Jessica Bolger, Milo J. Brooking, Jordan E. Dreibelbis, Amanda E. Grosz, Jakob H. Hutter, Kyle C. Liedtka, Bryce D. Natter, Tyler J. Ogurcak, Christopher J. Petraskie, Abigail L. Pihanich, Ian H. Reish, Paige Sherman, and Christian A. Wirt.

PA Iota – Shippensburg University
Chapter President – Rebecca Feaser; 761 Current Members; 1 New Mem-

ber

Other Spring 2019 Officers: Josue Murillo, Vice President; Conner Chapman, Secretary; Crystal Evans, Treasurer; Dr. Paul Taylor, Corresponding Secretary and Faculty Sponsor.

PA Kappa – Holy Family University

Chapter Co-Presidents – Melissa Cahill & Tuyen Le; 4 Current Members; 1 New Member

Other Spring 2019 Officer: Sister Marcella Louise Wallowicz CSFN, PhD, Corresponding Secretary and Faculty Sponsor.

The PA Kappa Chapter held its annual initation ceremony on Friday, April 5, 2019. Kaleen Harrigan, a middle-level science and math major, was initiated as the newest member of the chapter.

New Initiates – Kaleen Harrigan.

PA Lambda -– Bloomsburg University

Corresponding Secretary – Eric B. Kahn; 6 New Members

New Initiates – Kaleb Lansberry, Jordan Marlow, Saliou Ndiaye, Walter Nelson, Taylor Runkle, and Zachary Swanger.

PA Mu – Saint Francis University

Chapter President – Arlan (AJ) Zelenky; 61 Current Members; 11 New Members

Other Spring 2019 Officers: Vanessa Valovage, Vice President; Ry Gallagher, Secretary; Hannah Boyd, Treasurer; Dr. Brendon LaBuz, Corresponding Secretary and Faculty Sponsor.

The Pennsylvania Mu Chapter of Kappa Mu Epsilon held their initiation ceremony on Tuesday, February 26, 2019. The evening began with a welcome from outgoing corresponding secretary Dr. Peter Skoner followed by a prayer by chapter vice president Vanessa Valovage. After dinner, Faculty Sponsor Dr. Brendon LaBuz presented "To Be Irrational" which culminated with Geoffrey C. Berresford's simple proof that the square root of any integer that is not a perfect square is irrational. The highlight of the evening was the initiation of our eleven new members. As usual our KME chapter served free pie to the campus community on π day. Two faculty members and two student members attended the National Convention at Frostburg State University.

New Initiates – Ian Gabig, Michael Gallagher, Nicholas Hofer, Danielle Humbert, Kari Lagan, Nathan Moore, Taylor Nesmith, Mackenna Ray, Teresa Reid, Nathan Thompson,

and Anthony Vassalotti.

PA Nu – Ursinus College

Corresponding Secretary – Nicholas Scoville; 3 New Members

New Initiates – Thomas Cleary, Zachary Goranson and Jacob O'Neill.

PA Xi – Cedar Crest College

Corresponding Secretary – Dr. Joshua Harrington; 6 New Members

New Initiates – ETCCora Bankert, Emily Bianchini, Rebecca Fox, Grace Hurd, Madeline Kohutka, and Lois Polashenski.

PA Rho – Thiel College

Chapter President – Josh Evjene; 7 Current Members; 2 New Members
Other Spring 2019 Officers: Taylor Guth, Vice President; Courtney Harriman, Secretary; Brett Eckroate, Treasurer; Dr. Jie Wu, Faculty Sponsor; and Dr. Russell Richins, Corresponding Secretary.

Kappa Mu Epsilon at Thiel College held another charity fundraiser and Challenge 24 tournament in February. Josh Evjene, the chapter president, arranged several free tutoring events toward the end of the semester. On Pi Day, the chapter members sold pi themed snacks and helped students find their birthdays in the digits of pi. We held our initiation ceremony and dinner in April and held elections shortly thereafter.

PA Tau – DeSales University

Chapter President – Brendan G. Graham; 6 Current Members; 7 New Members
Other Spring 2019 Officers: Alexander N. Cominsky, Vice President; Emily M. Bennett, Secretary/Treasurer; and Brother Daniel P. Wisniewski, O.S.F.S., Corresponding Secretary and Faculty Sponsor.

On Sunday, April 28, 2019, the PA Tau Chapter of Kappa Mu Epsilon at DeSales University (DSU) initiated seven new KME members. The event included a presentation entitled "Applications of Math in Pharmaceutical Quality" by Ms. Colleen Caffarelli, Regulatory Compliance Specialist at Alcami Corporation (Edison, NJ), who received her B.S. in mathematics and chemistry in 2012 from DSU. Ms. Caffarelli was a member of the inaugural cohort of KME initiates of the PA Tau Chapter in 2012. In attendance were family and friends of the new and current KME members, as well as several KME alumni.

New Initiates – Alexander D. Accardi, Caroline M. Accurso, Nicole A. Cruts, Nikki E. Heckermann, Jacob W. Jones, Tyler D. Sarge, and Marissa J. Shores.

PA Upsilon – Cedar Crest College

Corresponding Secretary – Dr. Jared Burns; 5 New Members

New Initiates – Austin Braendel, Timothy Deyell, Rieley Hoopes, Patrick O'Connor, and

Evan Suter.

RI Beta – Bryant University

Chapter President – Danica Butler; 26 Current Members; 16 New Members

Other Spring 2019 Officers: Tyler Talbot, Vice President; Jacob Mingola, Secretary; Anthony Distefano, Treasurer; John Quinn, Corresponding Secretary; and Alan Olinsky, Faculty Sponsor.

We held our annual Math Honors ceremony at Bryant University on Thursday, April 26, 2019. We initiated 16 new members into the RI Beta Chapter of KME including 15 students and 1 faculty member.

SC Gamma – Winthrop University

Corresponding Secretary – Dr. Jessie Hamm; 3 New Members

New Initiates – Claire Berchtold, Darien Nguyen, and Nolan Worthy.

SC Epsilon – Francis Marion University

Corresponding Secretary – Dr. Daniel Scofield; 14 New Members

New Initiates – Jonathan G. Britt, Sierra R. Cartano, Gregory Ivan Dungan, Johanna Hoffmeyer Gibson, Michelle R. Greene, Victoria A. Hicks, Edward L. Hubbard, Colby J. Jackson, Charles E. Jeffcoat, III, Johnathan S. Larymore, Kaitlyn E. Lowry, Zachary R. Middleton, Taylor D. Watson, and Corbin E. Witt.

TN Alpha – Tennessee Tech University

Corresponding Secretary – Barry Elliott; 2 New Members

New Initiates – Cory Stephenson and Yixing Wang.

TN Beta – East Tennessee State University

Corresponding Secretary – Rodney Keaton; 14 New Members

New Initiates – Amelia Bernard, Brianna Cardenal, Logan Carmody, Rebecca Eubanks, Taylor Hutchinsonk, Scarlett Knott, Richard McDonald, Amanda McKinney, Ethan Nussio, Eyan Nussio, Anna Salyer, Hannah Schmidt, Mariah Taylor, and Bailey Yearout.

TN Gamma – Union University

Chapter President – Yoo Jin (Ashley) Moon; 5 New Members

Other Spring 2019 Officers: Cole LeMahieu, Vice President; Jenna Dula, Secretary and Treasurer; Josie Carrier, Webmaster and Historian; Bryan Dawson, Corresponding Secretary; and Matt Lunsford, Faculty Sponsor.

TN Gamma held its annual initiation banquet at the Old Country Store on April 29, 2019. Former chapter member Vicki Searl, currently a graduate student at the University of Louisville, was the speaker.

New Initiates – Emory Craft, Michael Drury, Spencer Alan McCloy, Stuart Milam, and Ben Noland.

TN Delta – Carson-Newman University

Chapter President – Branda Moore; 14 Current Members; 3 New Members

Other Spring 2019 Officers: Kaylib Powell, Vice President; and Kenneth

Massey, Corresponding Secretary and Faculty Sponsor.

Highlights of the year include a 3D rendered video about statistics, a drone flight, board game nights, and lawn games at the Starnes Estate.

New Initiates – Morgan Coffey, Thomas Fodor, and Zackary Norwood.

TN Zeta – Lee University

Corresponding Secretary – Caroline Maher-Boulis; 7 New Members

New Initiates – – Abigail Baucom, Blayne Carroll Jr., Ava Brooke Chaffin, Lauren Noel Holley, Madison Mabe, Jennifer Minutolo, and Jennifer Rose.

TX Eta – Hardin-Simmons University

Corresponding Secretary – Jessica Rieger; 6 New Members

New Initiates – Samantha Atkins, Jeremy Bowen, Roy Dittfurth, Johnathan Laws, James Timmons, and James Williams.

TX Iota – McMurry University

Corresponding Secretary – Dr. Kelly L. McCoun; 7 New Members

New Initiates – Kaci Builta, Kaleb Dunks, Emily Fisher, Neah Garza, Muhanad Hawsawi, Cody Kovar, and Martin Mercado.

TX Lambda – Trinity University

Chapter President – Nicole Spooner; 290 Current Members; 10 New Members

Other Spring 2019 Officers: Frances Stone, Vice President; Bryan Granizo, Secretary; Dr. Hoa Nguyen, Corresponding Secretary and Faculty Sponsor.

TX Mu – Schreiner University

Corresponding Secretary – Clint Coles; 12 New Members

New Initiates – Joseph Anthony Albor, Caleb Brandon, Camden G. Camacho, Cassandra Nicole Fuller, Mayte Gonzalez, Courtland Gully, Addison Leigh Hutchings, Dibenhi Y. Miramontes, Hannah Ortiz, Julia N. Saavedra, Katelyn E. Smith, and Rebecca Vairin.

VA Beta – Radford University

Chapter President – Morgyn Church; 16 Current Members; 4 New Members

Other Spring 2019 Officer: Eric P. Choate, Corresponding Secretary and Faculty Sponsor.

VA Delta – Marymount University

Chapter President – Theresa Buscemi; 6 Current Members, 4 New Members

Other Spring 2019 Officers: Prakriti Deuja, Vice President; Sarah McDonald, Secretary and Treasurer; Jacquelyn Rische, Corresponding Secretary and Faculty Sponsor.

Some of our members visited a mathematical art exhibit called "Mathematical Beauty" on April 22, 2019. The exhibit was at the American Association for the Advancement of Science (AAAS) in Washington, DC.

We met with one of the artists, Margaret Kepner, who gave us a personalized tour of the exhibit and talked about the mathematics behind some of her pieces there. On May 4, 2019, some of our members attended the National Math Festival in Washington, DC. We volunteered with the Association for Women in Mathematics, The Bridges Organization, and Ithaca College's Geometric Balloon Bending.

WI Alpha – Mount Mary University

Corresponding Secretary – Jane F. St. Peter; 4 Current Members; 3 New Members

The WI Alpha chapter will have a new corresponding secretary, Sherrie Serros, starting in the fall, due to the retirement of Jane St. Peter.

New Initiates – Jeannette Ingabire, Judith Ifarinde-Lay, and Sherrie Serros.

WV Alpha – Bethany College

Chapter President – Jacob D. Stemmerich; 16 Current Members; 4 New Members

Other Spring 2019 Officers: Rachel E. Gantzer, Vice President; Joseph S. Sawyer, Secretary/Treasurer; and Dr. Adam C. Fletcher, Corresponding Secretary and Faculty Sponsor.

It has been a busy academic year for the West Virginia Alpha chapter! In the fall semester, several members of the chapter (including two of its officers) traveled with the Mathematics and Computer Science Club and the "HackBethany" cybersecurity club to SecureWV/Hack3rcon in Charleston, West Virginia. The chapter helped the club host the thirteenth annual Math/Science Day on campus this spring, and sponsored Bethany College's initiation ceremony of another new member into the Upsilon Pi Epsilon international computing sciences honor society in April, as well as welcoming four new members into its own ranks. One member of the chapter accompanied three members of the Math/Computer Science Club to the national biennial convention in Frostburg, Maryland, where members of the chapter served on the Awards and Resolutions committees.

Active Chapters of Kappa Mu Epsilon

Listed by date of installation

Chapter	Location	Installation Date
OK Alpha	Northeastern State University, Tahlequah	18 Apr 1931
IA Alpha	University of Northern Iowa, Cedar Falls	27 May 1931
KS Alpha	Pittsburg State University, Pittsburg	30 Jan 1932
MO Alpha	Missouri State University, Springfield	20 May 1932
MS Alpha	Mississippi University for Women, Columbus	30 May 1932
NE Alpha	Wayne State College, Wayne	17 Jan 1933
KS Beta	Emporia State University, Emporia	12 May 1934
AL Alpha	Athens State University, Athens	5 Mar 1935
NM Alpha	University of New Mexico, Albuquerque	28 Mar 1935
IL Beta	Eastern Illinois University, Charleston	11 Apr 1935
AL Beta	University of North Alabama, Florence	20 May 1935
AL Gamma	University of Montevallo, Montevallo	24 Apr 1937
OH Alpha	Bowling Green State University, Bowling Green	24 Apr 1937
MI Alpha	Albion College, Albion	29 May 1937
MO Beta	University of Central Missouri, Warrensburg	10 Jun 1938
TX Alpha	Texas Tech University, Lubbock	10 May 1940
KS Gamma	Benedictine College, Atchison	26 May 1940
IA Beta	Drake University, Des Moines	27 May 1940
TN Alpha	Tennessee Technological University, Cookeville	5 Jun 1941
MI Beta	Central Michigan University, Mount Pleasant	25 Apr 1942
NJ Beta	Montclair State University, Upper Montclair	21 Apr 1944
IL Delta	University of St. Francis, Joliet	21 May 1945
KS Delta	Washburn University, Topeka	29 Mar 1947
MO Gamma	William Jewell College, Liberty	7 May 1947
TX Gamma	Texas Woman's University, Denton	7 May 1947
WI Alpha	Mount Mary College, Milwaukee	11 May 1947
OH Gamma	Baldwin-Wallace College, Berea	6 Jun 1947
MO Epsilon	Central Methodist College, Fayette	18 May 1949
MS Gamma	University of Southern Mississippi, Hattiesburg	21 May 1949
IN Alpha	Manchester College, North Manchester	16 May 1950
PA Alpha	Westminster College, New Wilmington	17 May 1950
IN Beta	Butler University, Indianapolis	16 May 1952
KS Epsilon	Fort Hays State University, Hays	6 Dec 1952
PA Beta	LaSalle University, Philadelphia	19 May 1953
VA Alpha	Virginia State University, Petersburg	29 Jan 1955
IN Gamma	Anderson University, Anderson	5 Apr 1957
CA Gamma	California Polytechnic State University, San Luis Obispo	23 May 1958
TN Beta	East Tennessee State University, Johnson City	22 May 1959
PA Gamma	Waynesburg College, Waynesburg	23 May 1959
VA Beta	Radford University, Radford	12 Nov 1959
NE Beta	University of Nebraska—Kearney, Kearney	11 Dec 1959
IN Delta	University of Evansville, Evansville	27 May 1960
OH Epsilon	Marietta College, Marietta	29 Oct 1960
MO Zeta	University of Missouri—Rolla, Rolla	19 May 1961

NE Gamma	Chadron State College, Chadron	19 May 1962
MD Alpha	College of Notre Dame of Maryland, Baltimore	22 May 1963
CA Delta	California State Polytechnic University, Pomona	5 Nov 1964
PA Delta	Marywood University, Scranton	8 Nov 1964
PA Epsilon	Kutztown University of Pennsylvania, Kutztown	3 Apr 1965
AL Epsilon	Huntingdon College, Montgomery	15 Apr 1965
PA Zeta	Indiana University of Pennsylvania, Indiana	6 May 1965
TN Gamma	Union University, Jackson	24 May 1965
IA Gamma	Morningside College, Sioux City	25 May 1965
MD Beta	McDaniel College, Westminster	30 May 1965
IL Zeta	Dominican University, River Forest	26 Feb 1967
SC Beta	South Carolina State College, Orangeburg	6 May 1967
PA Eta	Grove City College, Grove City	13 May 1967
NY Eta	Niagara University, Niagara University	18 May 1968
MA Alpha	Assumption College, Worcester	19 Nov 1968
MO Eta	Truman State University, Kirksville	7 Dec 1968
IL Eta	Western Illinois University, Macomb	9 May 1969
OH Zeta	Muskingum College, New Concord	17 May 1969
PA Theta	Susquehanna University, Selinsgrove	26 May 1969
PA Iota	Shippensburg University of Pennsylvania, Shippensburg	1 Nov 1969
MS Delta	William Carey College, Hattiesburg	17 Dec 1970
MO Theta	Evangel University, Springfield	12 Jan 1971
PA Kappa	Holy Family College, Philadelphia	23 Jan 1971
CO Beta	Colorado School of Mines, Golden	4 Mar 1971
KY Alpha	Eastern Kentucky University, Richmond	27 Mar 1971
TN Delta	Carson-Newman College, Jefferson City	15 May 1971
NY Iota	Wagner College, Staten Island	19 May 1971
SC Gamma	Winthrop University, Rock Hill	3 Nov 1972
IA Delta	Wartburg College, Waverly	6 Apr 1973
PA Lambda	Bloomsburg University of Pennsylvania, Bloomsburg	17 Oct 1973
OK Gamma	Southwestern Oklahoma State University, Weatherford	1 May 1973
NY Kappa	Pace University, New York	24 Apr 1974
TX Eta	Hardin-Simmons University, Abilene	3 May 1975
MO Iota	Missouri Southern State University, Joplin	8 May 1975
GA Alpha	State University of West Georgia, Carrollton	21 May 1975
WV Alpha	Bethany College, Bethany	21 May 1975
FL Beta	Florida Southern College, Lakeland	31 Oct 1976
WI Gamma	University of Wisconsin—Eau Claire, Eau Claire	4 Feb 1978
MD Delta	Frostburg State University, Frostburg	17 Sep 1978
IL Theta	Benedictine University, Lisle	18 May 1979
PA Mu	St. Francis University, Loretto	14 Sep 1979
AL Zeta	Birmingham-Southern College, Birmingham	18 Feb 1981
CT Beta	Eastern Connecticut State University, Willimantic	2 May 1981
NY Lambda	C.W. Post Campus of Long Island University, Brookville	2 May 1983
MO Kappa	Drury University, Springfield	30 Nov 1984
CO Gamma	Fort Lewis College, Durango	29 Mar 1985
NE Delta	Nebraska Wesleyan University, Lincoln	18 Apr 1986
TX Iota	McMurry University, Abilene	25 Apr 1987
PA Nu	Ursinus College, Collegeville	28 Apr 1987
VA Gamma	Liberty University, Lynchburg	30 Apr 1987

NY Mu	St. Thomas Aquinas College, Sparkill	14 May 1987
OH Eta	Ohio Northern University, Ada	15 Dec 1987
OK Delta	Oral Roberts University, Tulsa	10 Apr 1990
CO Delta	Mesa State College, Grand Junction	27 Apr 1990
PA Xi	Cedar Crest College, Allentown	30 Oct 1990
MO Lambda	Missouri Western State College, St. Joseph	10 Feb 1991
TX Kappa	University of Mary Hardin-Baylor, Belton	21 Feb 1991
SC Delta	Erskine College, Due West	28 Apr 1991
NY Nu	Hartwick College, Oneonta	14 May 1992
NH Alpha	Keene State College, Keene	16 Feb 1993
LA Gamma	Northwestern State University, Natchitoches	24 Mar 1993
KY Beta	Cumberland College, Williamsburg	3 May 1993
MS Epsilon	Delta State University, Cleveland	19 Nov 1994
PA Omicron	University of Pittsburgh at Johnstown, Johnstown	10 Apr 1997
MI Delta	Hillsdale College, Hillsdale	30 Apr 1997
MI Epsilon	Kettering University, Flint	28 Mar 1998
MO Mu	Harris-Stowe College, St. Louis	25 Apr 1998
GA Beta	Georgia College and State University, Milledgeville	25 Apr 1998
AL Eta	University of West Alabama, Livingston	4 May 1998
PA Pi	Slippery Rock University, Slippery Rock	19 Apr 1999
TX Lambda	Trinity University, San Antonio	22 Nov 1999
GA Gamma	Piedmont College, Demorest	7 Apr 2000
LA Delta	University of Louisiana, Monroe	11 Feb 2001
GA Delta	Berry College, Mount Berry	21 Apr 2001
TX Mu	Schreiner University, Kerrville	28 Apr 2001
CA Epsilon	California Baptist University, Riverside	21 Apr 2003
PA Rho	Thiel College, Greenville	13 Feb 2004
VA Delta	Marymount University, Arlington	26 Mar 2004
NY Omicron	St. Joseph's College, Patchogue	1 May 2004
IL Iota	Lewis University, Romeoville	26 Feb 2005
WV Beta	Wheeling Jesuit University, Wheeling	11 Mar 2005
SC Epsilon	Francis Marion University, Florence	18 Mar 2005
PA Sigma	Lycoming College, Williamsport	1 Apr 2005
MO Nu	Columbia College, Columbia	29 Apr 2005
MD Epsilon	Stevenson University, Stevenson	3 Dec 2005
NJ Delta	Centenary College, Hackettstown	1 Dec 2006
NY Pi	Mount Saint Mary College, Newburgh	20 Mar 2007
OK Epsilon	Oklahoma Christian University, Oklahoma City	20 Apr 2007
HA Alpha	Hawaii Pacific University, Waipahu	22 Oct 2007
NC Epsilon	North Carolina Wesleyan College, Rocky Mount	24 Mar 2008
NY Rho	Molloy College, Rockville Center	21 Apr 2009
NC Zeta	Catawba College, Salisbury	17 Sep 2009
RI Alpha	Roger Williams University, Bristol	13 Nov 2009
NJ Epsilon	New Jersey City University, Jersey City	22 Feb 2010
NC Eta	Johnson C. Smith University, Charlotte	18 Mar 2010
AL Theta	Jacksonville State University, Jacksonville	29 Mar 2010
GA Epsilon	Wesleyan College, Macon	30 Mar 2010
FL Gamma	Southeastern University, Lakeland	31 Mar 2010
MA Beta	Stonehill College, Easton	8 Apr 2011
AR Beta	Henderson State University, Arkadelphia	10 Oct 2011

PA Tau	DeSales University, Center Valley	29 Apr 2012
TN Zeta	Lee University, Cleveland	5 Nov 2012
RI Beta	Bryant University, Smithfield	3 Apr 2013
SD Beta	Black Hills State University, Spearfish	20 Sept 2013
FL Delta	Embry-Riddle Aeronautical University, Daytona Beach	22 Apr 2014
IA Epsilon	Central College, Pella	30 Apr 2014
CA Eta	Fresno Pacific University, Fresno	24 Mar 2015
OH Theta	Capital University, Bexley	24 Apr 2015
GA Zeta	Georgia Gwinnett College, Lawrenceville	28 Apr 2015
MO Xi	William Woods University, Fulton	17 Feb 2016
IL Kappa	Aurora University, Aurora	3 May 2016
GA Eta	Atlanta Metropolitan University, Atlanta	1 Jan 2017
CT Gamma	Central Connecticut University, New Britan	24 Mar 2017
KS Eta	Sterling College, Sterling	30 Nov 2017
NY Sigma	College of Mount Saint Vincent, The Bronx	4 Apr 2018
PA Upsilon	Seton Hill University, Greensburg	5 May 2018
KY Gamma	Bellarmine University, Louisville	23 April 2019