THE PENTAGON

A Mathematics Magazine for Students

Volume 80 Number 2 Spring 2021

Contents

Typeset in WinEdt. Printed in the United States of America.

The Pentagon (ISSN 0031-4870) is published semiannually in December and May by Kappa Mu Epsilon. No responsibility is assumed for opinions expressed by individual authors. Papers written by undergraduate mathematics students for undergraduate mathematics students are solicited. Papers written by graduate students or faculty will be considered on a space-available basis. Submissions should be made by means of an attachment to an e-mail sent to the editor. Either a TeX file or Word document is acceptable. An additional copy of the article as a pdf file is desirable. Standard notational conventions should be respected. Graphs, tables, or other materials taken from copyrighted works MUST be accompanied by an appropriate release form the copyright holder permitting their further reproduction. Student authors should include the names and addresses of their faculty advisors. Contributors to The Problem Corner or Kappa Mu Epsilon News are invited to correspond directly with the appropriate Associate Editor.

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Magic Squares, Part 1

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Abstract

Magic squares originated over 3,000 years ago from a legend about a Divine Turtle found in the ancient Chinese book *Yih King*. According to the legend, it is mathematically possible to form $n \times n$ squares where each row, column, and diagonal add up to the same magic constant or value using the numbers 1 through n^2. Over time, mathematicians have developed different methods to compose extraordinary magic squares, other than the original Lo-Shu 3×3 magic square. My research goes in-depth with the composition of the three main types of magic squares, odd, singly-even, and doubly-even, which can be created utilizing the Siamese, Lozenge, Singly-Even, and Doubly-Even Methods.

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1. Introduction

Magic squares originated over 3,000 years ago from the legend of the Divine Turtle found in an Ancient Chinese book *Yih King*. According to the legend, each year the king made sacrifices to the river god for the

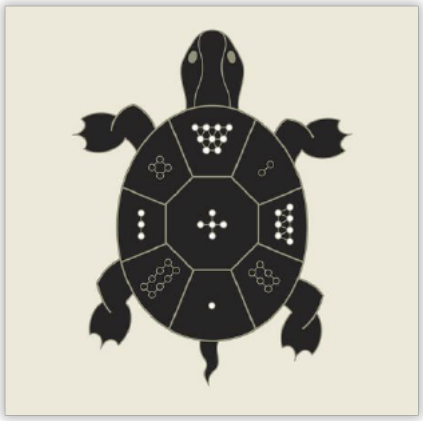

Figure 1: (Thoth, 2018)

flooded river and each year, they continued to see a turtle appear during the sacrifice. Then one year, the people noticed the turtle had dots on its shell. Looking at Figure 1, the people realized that each row, column, and diagonal added up to 15. This was the number of days in each of the 24 cycles of the Chinese Solar year. From then on, the king gave 15 sacrifices per year thanks to the Divine Turtle. Ever since that day, mathematicians have been analyzing and developing magic squares. [5][3]

One familiar creator of magic squares is Benjamin Franklin. Franklin is known for creating two unique magic squares, the 8×8 and 16×16, with the most notable being the 16×16. The 16×16 was created during a single night in the year 1767. Franklin was the clerk at the Pennsylvania Assembly meetings, and he grew very bored during these times. His friend, Mr. Logan, gave him a book on magic squares to help him pass the time. During one tedious meeting, Franklin generated his unique 16×16 magic square. He then sent this to his friends stating how magic squares were his new way to pass time. However, it was discovered years after Franklin published his magic squares that he failed to have the sums of the diagonals equal the sums of the rows and sums of the columns, so technically he did not create true magic squares. [4]

1.1 Magic Square Description

This research depends on understanding what a magic square is. A magic square is an $n \times n$ square whose rows, columns, and diagonals all add up to the same value. This research only involved normal magic squares, which are a type of magic square that only contains the integers 1 to n^2 The variable n represents the number of rows and columns in the magic square. Referring to Table 1, known as the Lo-Shu magic square, one can see this is a 3×3, since there are 3 rows and 3 columns in the square. Adding up each row, column, and diagonal yields a sum of 15. This sum of each row, column, and diagonal will be referred to as the *magic constant*. This number is not generated randomly; rather, the formula for calculating the magic constant is $\frac{n(n^2+1)}{2}$. For example, when finding the magic constant for the Lo-Shu magic square, the first step is determining what the value of n is going to be in our equation. Since we are dealing with a 3×3,

the value for n is going to be 3. Plugging this into our equation gives $\frac{3(3^2+1)}{2} = \frac{3(10)}{2} = 15$. For the rest of this study, we will find the magic constant using the formula in this manner.

8	1	6
3	5	7
4	9	2

Table 1: Lo-Shu Magic Square

2. Composition

Understanding the composition of magic squares is crucial for determining any property of them. The composition of a magic square describes how a normal magic square is created or established. Every individual box inside the magic square will be referred to as a *cell*. The rows will be numbered from top to bottom, and the columns will be numbered from left to right. Each one is made up with a particular set of steps to produce a unique type of magic square. The three main types of magic squares are *odd*, *singly-even*, and *doubly-even*. To assemble the main types, we utilize different methods: the Siamese or Lozenge Method will produce an odd type, the Singly- Even Method will produce a singly-even type, and the Doubly-Even Method will produce a doubly-even type. Comprehending the composition will prepare the way for more intricate details on magic squares in the future.

2.1 Siamese Method

The *Siamese Method* is used for creating an $n \times n$ magic square where n is odd. After completing my research, I believe this method is the foundation for all magic squares because this method is used in other composition methods and properties that will be discussed later. It is also the method that was used to create the Lo-Shu Magic Square shown in the introduction (Table 1), which is the first known magic square to date. To learn the Siamese Method, 3×3 and 5×5 magic squares will be produced utilizing the steps in the method.

2.1.1 3×3 Magic Square

When forming a 3×3 magic square using the Siamese Method, one will produce the Lo-Shu Magic Square. To start this process, we must find the magic constant. Since we are dealing with a 3×3, our n value is going to be 3, meaning the numbers to place in the magic square are going to be 1 through $n^2 = 3^2 = 9$. As we saw earlier, plugging this n value into the magic constant formula gives us $\frac{3(3^2+1)}{2} = \frac{3(10)}{2} = 15$. This means that each row, column, and diagonal has to equal 15.

For this method, placing the numbers in the cells happens sequentially, so 1 is placed first followed by 2 and so forth. For each magic square created using this method, the number 1 gets placed in the top row in the middle column as shown in Table 2.

	1	

Table 2: 3×3 Magic Square (Incomplete)

The next sequential number gets placed in the cell that is one column to the right and one row above the cell which the last number was placed. This step will occur n times; that is, until the next number to be placed would occur in an occupied cell. In this case, the step will occur 3 times until the next cell will be occupied. However, when looking at Table 3, one can see that 2 is floating above the confines of the magic square. This will happen throughout this method and there are two rules to follow when this happens:

1. If after moving one column to the right and one row up, the number is above the magic square, place the number in the bottom row in the same column.

2. If after moving one column to the right and one row up, the number is to the right of the magic square, place the number in the leftmost column in the same row.

		2
	1	

Table 3: 3×3 Magic Square (Incomplete)

Following those steps, the cell that number 2 should be placed in is the bottom row in the 3rd column. When placing 3, the same problem occurs as with placing 2, except that it floats to the right of the magic square. Following the special steps leads to 3 being placed in the leftmost column in the 2nd row. Both of those steps are shown in Table 4.

	1	
3		
		2

Table 4: 3×3 Magic Square (Incomplete)

The next issue occurs when attempting to place the 4 because moving one cell to the right and up one cell is an already occupied cell, which was expected because the main step was already repeated three times. When this happens, the next sequential number gets placed in the cell directly below the last cell filled in. Table 5 shows that 4 should be placed directly below 3, since that was the last number placed. Continuing the main step two more times yields placing 5 and 6 in the cells shown in Table 5 until reaching the occupied cell where 4 resides.

	1	6
3	5	
4		2

Table 5: 3×3 Magic Square (Incomplete)

When placing 7, the desired cell is occupied so 7 gets placed under the last number filled in, 6. Following the step two more times completes the magic square. The final3×3 produced with the Siamese method is presented in Table 6.

8	1	6
3	5	7
4	9	2

Table 6: 3×3 Magic Square (Complete)

Looking at the final product and adding up every row, column, and diagonal, we see that they all do equal the magic constant of 15.

2.1.2 5×5 Magic Square

To better understand the Siamese Method, another example is explained thoroughly. To start the process of developing a 5×5 magic square, the n value is 5, so the numbers to be placed are going to be 1 through $n^2 = 5^2 = 25$. Now plugging 5 into the magic constant formula gives $\frac{5(5^2+1)}{2} = \frac{5(26)}{2} = 65$. The first number to place is 1, which is to be placed in the top row in the middle column, which can be seen in Table 7.

		1		

Table 7: 5×5 Magic Square (Incomplete)

Since the value for n is 5, this indicates that five placements can be made until reaching an occupied cell. The next sequential number to be placed is 2 by going right one column and up one row. Note here that as seen in Table 8, as in the 3×3, the 2 is floating above the magic square. This illustrates an interesting point that whenever using the Siamese Method and placing the number 2 in a cell, 2 is always going to float above the magic square.

			2	
		1		

Table 8: 5×5 Magic Square (Incomplete)

Following the special rules for the 3×3 case, the 2 is going to get placed in the bottom row, but in the same column, which is the 4th. Placing the 3 happens without any inconvenience. Both of these steps are displayed in Table 9.

		1		
				3
			2	

Table 9: 5×5 Magic Square (Incomplete)

The inconvenience comes again when trying to place the 4 because when going over one cell to the right and up one cell, the 4 floats to the right of the magic square. This is similar to when the 3 floated on the right of the magic square in the previous example. This invokes the special rules which places the 4 in the leftmost column in the middle row. The 5 is able to be placed without an issue. Both of these placements are shown in Table 10.

		1		
	5			
4				
				3
			2	

Table 10: 5×5 Magic Square (Incomplete)

When trying to move one cell to the right and up one, we find that the cell is occupied by the number 1. As in the previous example, the next sequential number, 6, is to be placed under the last cell that was filled in, which in this case is the cell with the 5 in it. Placement continues four more times until an occupied cell is reached yet again. These five placements are displayed in Table 11.

		1	8	
	5	7		
4	6			
10				3
			2	9

Table 11: 5×5 Magic Square (Incomplete)

The steps of the Siamese Method are continued until every cell in the magic square is occupied. Table 12 represents the final 5×5 magic square formed by the Siamese Method. Note that in any odd order magic square greater than 3 that is formed by the Siamese Method, it has one diagonal that is in sequential order. When looking at Table 12, that diagonal is represented by the maroon numbers. In any of these type of magic squares, this sequential diagonal can be found in those related diagonal cells.

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

Table 12: 5×5 Magic Square (Complete)

2.2 Lozenge Method

The *Lozenge Method* is used to create an $n \times n$ magic square where n is odd. This is another method for creating odd ordered magic squares similar to the Siamese Method. However, a different magic square will be produced using the Lozenge Method in that the numbers will be placed in different cells, but with the same magic constant. To learn the Lozenge Method, 5×5 and 7×7 magic squares will be produced utilizing the steps in this method.

2.2.1 5×5 Magic Square

Similar to the Siamese Method, the first step in the Lozenge Method is to calculate the magic constant. The first magic square that is going to be produced using this method is a 5×5 magic square. Since the magic square is a 5×5, the value for the variable n is going to be 5. Referring to the Siamese 5×5 magic square already created, we know the magic constant should be 65, and we know that the numbers that will be used are going to be 1 through 25. This method can be a little more difficult than the Siamese Method.

After finding the magic constant, a diamond shape must be drawn on our magic square. The four corners of the diamond get placed as followed: the top corner of the diamond is placed in the top row in the middle column, the right corner is placed in the right column in the middle row, the bottom corner is placed in the bottom row in the middle column, and the left corner is placed in the left column in the middle row. These corners are then all connected by four lines. An example of how this diamond should be drawn is shown in Table 13.

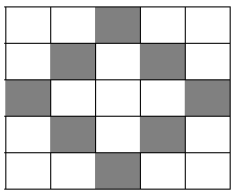

Table 13: 5×5 Magic Square (Diamond Pattern)

Now the odd numbers from 1 through n^2 are going to be placed sequentially, filling in the diamond. That is, the diamond is going to get filled in diagonally from the bottom left to the top right of each diagonal on the border and inside of the diamond. A diagonal is formed by the starting number and going one cell to the right and up one, just like in the Siamese Method. When looking at the diamond, the first diagonal starts with the left corner and goes up to the top corner. There are three cells in this diagonal so the numbers 1, 3, and 5 would get placed in these cells in that order. This diagonal is shown in Table 14.

		5		
	3			
1				

Table 14: 5×5 Magic Square (Incomplete)

The next numbers to be entered into the diamond go with the two interior cells on the next diagonal to the right. We start one cell to the right of 1. The second cell is next to 3 and below 5. These two cells get filled in with 7 and 9, as Table 15 details.

		5		
	3	9		
1	7			

Table 15: 5×5 Magic Square (Incomplete)

The rest of the diamond is filled in using a similar method as the first two diagonal examples, with the last cell filled in being the right corner of the diamond. After filling in all of the diagonals using the odd numbers from 1 through 25, one gets the figure displayed in Table 16.

		5		
	3	9	15	
1	7	13	19	25
	11	17	23	
		21		

Table 16: 5×5 Magic Square (Incomplete)

The next step involves drawing different colored diagonal lines on the full magic square, not just the diamond cells. Each diagonal starts with the numbers that started the diagonals in the previous step and is going to be five cells long. Thus, the starting numbers will be 1, 7, 11, 17, and 21. When each diagonal is made, the starting number will be highlighted in yellow. The diagonal starts and ends with the starting number. To fully understand this step, take a look at the starting number 1. Starting at 1, a diagonal gets produced but does not stop at 5, it continues until it reaches 1 again. When the next cell to be colored in floats above or next to the magic square the same special steps are followed as in the Siamese Method. Thus, the following diagonal line colored in with blue is shown in Table 17.

		5		
	3	9	15	
1	7	13	19	25
	11	17	23	
		21		

Table 17: 5×5 Magic Square (Incomplete)

The next diagonal is produced very similarly, except the starting number is 7. This means that the last colored-in cell with this diagonal is going to be the cell under 1. This diagonal will be displayed in Table 18 with the color green.

		5		
	3	9	15	
1	7	13	19	25
	11	17	23	
		21		

Table 18: 5×5 Magic Square (Incomplete)

This process is continued with the 11 diagonal being colored with red, the 17 diagonal being colored with orange, and the 21 diagonal being colored with maroon. This is all shown in Table 19.

		5		
	3	9	15	
1	7	13	19	25
	11	17	23	
		21		

Table 19: 5×5 Magic Square (Incomplete)

Finally, the even numbers from 2 through 24 have to be placed in cells. To place these numbers, we revisit the diagonals in the order they were just created and place the numbers according to that order. Let's start with the blue diagonal that starts with 1. When looking at the blue diagonal, one can see that there are two open cells. The two numbers that get filled in here are 2 and 4. The order they get filled in is the order you colored in the diagonal. After cell 5, the next cell colored in was the cell next to 21 and under 23. This is the cell that gets 2, and the cell next to 23 and under 25 gets 4 placed in it. The completed blue diagonal is displayed in Table 20.

		5		
	3	9	15	
1	7	13	19	25
	11	17	23	4
		21	2	

Table 20: 5×5 Magic Square (Incomplete)

The green diagonal starting with 7 is going to look a little different. First of all, that diagonal has three empty cells, which means the numbers to be placed are going to be 6, 8, and 10. At first glance, it looks like the first cell to be filled in would be the cell below 1 and next to 11, but remember the coloring started at cell 7, so the cell below 1 and next to 11 was actually the last cell colored in, meaning that 10 would go here, not 6. The first empty cell colored in was the cell next to 5 and above 15, so that is where 6 gets placed. The next empty cell to get colored in was the cell next to 2 and below 4, so this is where 8 goes. The complete green diagonal is exhibited in Table 21.

		5	6	
	3	9	15	
1	7	13	19	25
10	11	17	23	4
		21	2	8

Table 21: 5×5 Magic Square (Incomplete)

Following this process until all the even values from 2 to 24 are placed in the empty cells, yields the final magic square shown in Table 22. Notice that every row, column, and diagonal add up to the desired magic constant of 65, but the numbers are not placed in the same cells as the Siamese Method produced.

18	24	5	6	12
22	3	9	15	16
1	7	13	19	25
10	11	17	23	4
14	20	21	2	8

Table 22: 5×5 Magic Square (Complete)

2.2.2 7×7 Magic Square

To understand the Lozenge Method more concretely, another magic square will be produced. A 7×7 will be produced to enhance the steps shown in the previous example. To begin this magic square, we find the magic constant by plugging n in as 7. This gives $\frac{7(7^2+1)}{2} = \frac{7(50)}{2} = 175$. Also using 7 as the n value means that the numbers to be placed in the cells are 1 through $n^2 = 49$.

The next step is to draw the diamond on the magic square. Remember that the top corner is in the top row in the middle column, the right corner is in the right column in the middle row, the bottom corner is in the bottom row middle column, and the left corner is in the left column in the middle row. Connecting all the corners together produces the diamond shown in Table 23.

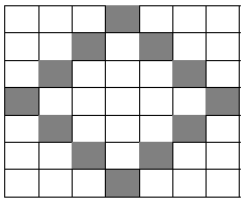

Table 23: 7×7 Magic Square (Diamond Pattern)

Remember the diamond is to be filled in with the odd numbers from 1 through 49 in sequential order diagonally. Once again the 1 is going to be placed in the left corner and start the first diagonal by going up one cell and to the right one ending at the top corner. Since there are four cells in this diagonal the numbers 1, 3, 5, and 7 will be placed as exhibited in Table 24.

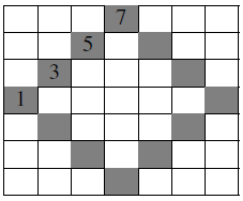

Table 24: 7×7 Magic Square (Incomplete)

The next diagonal will start in the cell next to 1 and below 3 and continue until it reaches the cell next to 5 and under 7. One can now see that the next diagonal will always start in the cell next to 1 and under 3. These three empty cells will be filled in with the numbers 9, 11, and 13, and displayed in Table 25.

			7			
		5	13			
	3	11				
1	9					

Table 25: 7×7 Magic Square (Incomplete)

Continue to fill in the odd numbers from 1 through 49 diagonally until you reach the right corner, which should be filled in with 49. Please note that the left corner will always be filled in with 1 and the right corner will always be filled in with n^2. All of the odd numbers placed in their cells are shown in Table 26.

			7			
		5	13	21		
	3	11	19	27	35	
1	9	17	25	33	41	49
	15	23	31	39	47	
		29	37	45		
			43			

Table 26: 7×7 Magic Square (Incomplete)

Now the even numbers from 2 through 48 must be placed in the empty cells. Again, we have to create the different colored diagonals starting with the numbers that started the diagonals in the diamond. Each diagonal will consist of seven cells and the seven yellow numbers this time will be 1, 9, 15, 23, 29, 37, and 43. The pattern here is that the number of cells in each diagonal and the number of yellow numbers will always be the same as the n value. When creating the colored diagonal leading with 1, the numbers 1, 3, 5, and 7 will be colored along with the empty cells to the right of 43, 45, and 47. The diagonal starting with 1 is colored in blue in Table 27.

			7			
		5	13	21		
	3	11	19	27	35	
1	9	17	25	33	41	49
	15	23	31	39	47	
		29	37	45		
			43			

Table 27: 7×7 Magic Square (Incomplete)

Continue on to the next diagonal, which is produced the same way except that it starts with the number 9 instead of 1. Note here, that as with the second colored diagonal in the 5×5, the last colored cell in this diagonal is under 1. The diagonal starting with 9 will be colored in green and displayed in Table 28.

			7			
		5	13	21		
	3	11	19	27	35	
1	9	17	25	33	41	49
	15	23	31	39	47	
		29	37	45		
			43			

Table 28: 7×7 Magic Square (Incomplete)

This process is continued with the 15 diagonal being colored in red, the 23 diagonal being colored in orange, the 29 diagonal being colored in maroon, the 37 diagonal being colored in purple, and the 43 diagonal being colored in pink. The finished red, orange, maroon, purple, and pink diagonals are displayed in Table 29.

			7			
		5	13	21		
	3	11	19	27	35	
1	9	17	25	33	41	49
	15	23	31	39	47	
		29	37	45		
			43			

Table 29: 7×7 Magic Square (Incomplete)

The even numbers from 2 through 48 can now be placed in their appropriate cells. To determine the appropriate cells, we start with the first colored diagonal and continue filling in the even numbers in sequential order until that diagonal is completed, and then the next colored diagonal gets filled in. This continues until all colored diagonals are filled in. For this example, the first color to fill in is blue, with the last color being pink.

Looking for the first empty cell in the blue diagonal, one sees that it is the cell next to 43 and below 45. Note that the pattern here is that the

first even number 2 will always be placed to the right of the bottom corner of the diamond. Since there are three empty cells in the blue diagonal, the next empty cell will be 4, with the last empty cell being 6. These three completed cells are displayed in Table 30.

			7			
		5	13	21		
	3	11	19	27	35	
1	9	17	25	33	41	49
	15	23	31	39	47	6
		29	37	45	4	
			43	2		

Table 30: 7×7 Magic Square (Incomplete)

The next colored diagonal is the green one, which has 4 empty cells that will contain the numbers 8, 10, 12, and 14. When filling in the empty cells for this diagonal, the same guidelines must be followed as in the 5×5 example. When going through the diagonal, the starting cell has to be the yellow number. This makes the first empty cell to be above 21 and next to 7. Thus, 8 gets placed in this cell. The last cell to get filled in for this diagonal is the cell under 1 and next to 15. The number to get entered here is 14. The completed green diagonal is exhibited in Table 31.

			7	8		
		5	13	21		
	3	11	19	27	35	
1	9	17	25	33	41	49
14	15	23	31	39	47	6
		29	37	45	4	12
			43	2	10	

Table 31: 7×7 Magic Square (Incomplete)

Continue this process until 48 is placed in the last cell of the pink diagonal. The completed 7×7 magic square produced by the Lozenge Method is shown in Table 32. Note that each row, column, and diagonal adds up to the desired magic constant value of 175.

32	40	48	7	8	16	24
38	46	5	13	21	22	30
44	3	11	19	27	35	36
1	9	17	25	33	41	49
14	15	23	31	39	47	6
20	28	29	37	45	4	12
26	34	42	43	2	10	18

Table 32: 7×7 Magic Square (Complete)

2.3 Singly-Even Method

The *Singly-Even Method* is for creating an $n \times n$ magic square, where n is singly-even, meaning $n = 4m + 2$, where $m \in \mathbb{Z}^+$, so n is divisible by 2 but not 4. Examples of what n could equal are 6, 10, 14 ... because they all can be divided by 2 but not 4. After researching the four methods, I tend to believe this is the most complicated one. However, if the Siamese Method is mastered before attempting this method, it makes the Singly-Even Method much easier because the Siamese Method is used intensely in this process. To learn the Singly-Even Method, 6×6 and 10×10 magic squares will be produced utilizing the steps in the method.

2.3.1 6×6 Magic Square

As in every method, the first step is to find the magic constant. When dealing with a 6×6, the n value is going to be 6. From the n value, we know that the numbers to be used will be 1 through $n^2 = 36$, and the magic constant will be $\frac{6(62+1)}{2} = \frac{6(37)}{2} = 111$.

Next, the magic square gets split up into four smaller magic squares that are of equal size. Each smaller magic square is going to have size $\frac{n}{2} \times \frac{n}{2}$. Thus, for a 6×6, each smaller magic square is going to be $\frac{6}{2} \times \frac{6}{2} = 3 \times 3$. We color in each smaller magic square, so it is easy to distinguish the different smaller magic squares inside the main one. The top left smaller magic square will be blue, the top right will be green, the bottom left will be red, and the bottom right will be orange.

Now that each smaller magic square is colored, each one gets labeled with a letter. The blue is labeled A, green is labeled C, red is labeled D, and orange is labeled B. It is very important that those colors always get labeled with those specific letters in that order because if they are not, the rest of the method will not work properly. These steps are illustrated in Table 33.

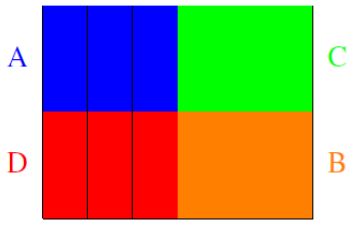

Table 33: 6×6 Magic Square (Color Scheme)

The numbers 1 through 64 have to be split up exactly throughout these four smaller magic squares. To do this, each group consists of $\frac{n^2}{4}$ values. Here, $\frac{6^2}{4} = \frac{36}{4} = 9$, so each group will have nine numbers in it. The groups occur in sequential order. Thus, 1 through 9 is Group 1, 10 through 18

is Group 2, 19 through 27 is Group 3, and 28 through 36 is Group 4. Assigning the groups to each smaller magic square is done alphabetically. Group 1 goes to A, Group 2 goes to B, Group 3 goes to C, and Group 4 goes to D.

Each smaller magic square gets treated like its own magic square and is formed using the Siamese Method. First let us take a look at A. Since this one involves Group 1, the magic square will look identical to the 3×3 produced in the Siamese Method section. The same process will be used for B, C, and D, except B will start with 10, C will start with 19, and D will start with 28 instead of starting with 1 like a normal magic square formed by the Siamese Method. All four smaller completely filled-in magic squares are provided in Table 34.

	8	1	6	26	19	24	
A	3	5	7	21	23	25	C
	4	9	2	22	27	20	
	35	28	33	17	10	15	
D	30	32	34	12	14	16	B
	31	36	29	13	18	11	

Table 34: 6×6 Magic Square (Preliminary Step)

The rows, columns, and diagonals do not yet add up to the magic constant because different cells have to be swapped first. The cells that have to be swapped are categorized by highlighted areas that are going to be produced next. In A, three highlighted areas are going to be made; A-1, A-2, and A-3.

To create A-1, highlight the first row in A until the middle column in A is reached. Do not include the cell in the middle column; only highlight up until this cell. In this example, only the first cell will be highlighted because A only has three cells in that row. Thus, the middle column would be the 2nd. To complete this highlight, make a square with the cells that are highlighted in the first row. For example, if the first row would have three cells highlighted, the highlighted area would have to be a 3×3, so the first three rows would have to be highlighted all the way to the 4th column. However, for this example, only one cell was highlighted so the area needed is only a 1×1. Therefore, only one cell has to be highlighted.

A-2 is created utilizing what was just done in A-1. The row directly below A-1 is where A-2 occurs. (For this example, A-2 would occur on row 2, but in the hypothetical example above, A-2 would occur on row 4.) A-2 skips the first cell in that row, but then highlights as many cells as were highlighted in row 1. For the 6×6, only one cell is going to be

highlighted since only one cell was highlighted in row 1. Only one row ever gets highlighted for A-2.

Making A-3 is identical to A-1, except it is placed in the bottom left corner of A instead of the top left corner. So for this example, row 3 column 1 is the only cell that is going to be highlighted. The A-1, A-2 and A-3 highlighted areas are illustrated in Table 35.

	A-1	8	1	6	26	19	24	
A	A-2	3	5	7	21	23	25	C
	A-3	4	9	2	22	27	20	
		35	28	33	17	10	15	
D		30	32	34	12	14	16	B
		31	36	29	13	18	11	

Table 35: 6×6 Magic Square (Preliminary Step)

The next highlighted areas to create are in D, which are going to be labeled D-1, D-2, and D-3. These three highlighted labels are identical to A-1, A-2, and A-3, except they are in D. The same steps that are completed for the highlights in A are repeated and D-1, D-2, and D-3 are formed as shown in Table 36.

	A-1	8	1	6	26	19	24	
A	A-2	3	5	7	21	23	25	C
	A-3	4	9	2	22	27	20	
	D-1	35	28	33	17	10	15	
D	D-2	30	32	34	12	14	16	B
	D-3	31	36	29	13	18	11	

Table 36: 6×6 Magic Square (Preliminary Step)

The last highlighted areas to make are in B and C, and each smaller magic square only has one highlighted area each. To make B-1 and C-1 the number of cells colored in for the 1st row of A-1 has to be considered. Subtract one from that amount and that is the number of columns that get highlighted from right to left in B and C. For a 6×6 it is hard to see this done because row 1 in A-1 only has one cell highlighted, so subtracting 1 from that leaves 0, meaning that no columns get highlighted for B-1 and

C-1. In the next example, a 10×10, there will be columns to highlight in B-1 and C-1.

Now that all the highlighted areas are completed, the swapping of the areas must occur. The swapping is always done the same way. The highlighted areas in A are always swapped with the highlighted areas in D, and the highlighted areas in C are always swapped with the highlighted areas in B. They are also swapped based on the hyphenated numbers. Thus, the value in the cells of A-1 are swapped with D-1, similarly A-2 with D-2 and A-3 with D-3. The swapped cells are illustrated in Table 37 with the darker and lighter yellow highlights being switched.

D-1	35	1	6	26	19	24
D-2	3	32	7	21	23	25
D-3	31	9	2	22	27	20
A-1	8	28	33	17	10	15
A-2	30	5	34	12	14	16
A-3	4	36	29	13	18	11

Table 37: 6×6 Magic Square (Complete)

Each row, column, and diagonal now adds up to the magic constant of 111. Therefore, Table 37 is the final 6×6 magic square produced using the Singly-Even Method.

2.3.2 10×10 Magic Square

Creating a 10×10 magic square using this method will help to reinforce the steps in the previous example and display an actual swap between B-1 and C-1. The n value is 10, which means the numbers to be used are 1 through $n^2 = 10^2 = 100$, and the magic constant is $\frac{10(10^2+1)}{2} = \frac{10(101)}{2} = 505$.

The size of the four smaller magic squares must be determined. Plugging 10 into the formula gives $\frac{10}{2} \times \frac{10}{2} = 5 \times 5$. Like the 6×6, the top left smaller magic square is colored with blue, the top right is colored in green, the bottom left will be red, and the bottom right will be orange. The four smaller magic squares colored in are depicted in Table 38.

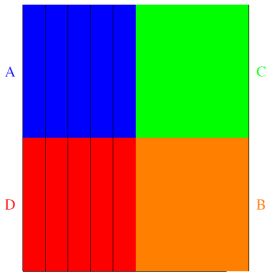

Table 38: 10×10 Magic Square (Color Scheme)

The smaller magic squares get labeled with the letters A, B, C, and D associated with the same colors as before. However, the numbers that get placed in each letter square are going to be different. To determine the size of each of our four number groups, plug 10 into the formula to get $\frac{10^2}{4} = \frac{100}{4} = 25$. This means that Group 1 contains 1 through 25, Group 2 contains 26 through 50, Group 3 contains 51 through 75, and Group 4 contains 76 through 100. No matter what numbers are in each group, the groups still get assigned to the same letters. Therefore, Group 1 goes to A, Group 2 goes to B, Group 3 goes to C, and Group 4 goes to D.

To fill in each smaller magic square, the Siamese Method will be used, meaning that A will look identical to the 5×5 magic square produced in the Siamese Method section. B will be created the same way except it will start with 26, C will start with 51, and D will start with 76 instead of 1. Each completed smaller magic square is provided in Table 39.

	17	24	1	8	15	67	74	51	58	65	
	23	5	7	14	16	73	55	57	64	66	
A	4	6	13	20	22	54	56	63	70	72	C
	10	12	19	21	3	60	62	69	71	53	
	11	18	25	2	9	61	68	75	52	59	
	92	99	76	83	90	42	49	26	33	40	
	98	80	82	89	91	48	30	32	39	41	
D	79	81	88	95	97	29	31	38	45	47	B
	85	87	94	96	78	35	37	44	46	28	
	86	93	100	77	84	36	43	50	27	34	

Table 39: 10×10 Magic Square (Preliminary Step)

The highlighted areas are more complicated in this example than the last. To start building A-1, the top row cells get highlighted until the center

column. Since A is a 5×5, the middle column is the 3rd, so the first two cells get highlighted. The highlighted area for A-1 has to be a square, so it must be turned into a 2×2 highlighted area. (In the previous example, nothing in row 2 got highlighted for A-1, because only one cell was highlighted in row 1. However, this time, two cells were highlighted, so this means row 2 also has to have the first two cells highlighted.)

Creating A-2 is done in a very similar manner. A-2 occurs on the row directly below A-1, so it occurs on row 3. The first column is always skipped and then the same number of cells highlighted in row 1 of A-1 gets highlighted for A-2. This means that two cells get highlighted for A-2.

A-3 is established exactly as in the previous example. It is replicated from A-1, except it is created in the bottom left corner of A instead of the top left corner. The A-1, A-2, and A-3 highlighted areas are shown in Table 40.

	A-1	17	24	1	8	15	67	74	51	58	65	
		23	5	7	14	16	73	55	57	64	66	
A	A-2	4	6	13	20	22	54	56	63	70	72	C
		10	12	19	21	3	60	62	69	71	53	
	A-3	11	18	25	2	9	61	68	75	52	59	
		92	99	76	83	90	42	49	26	33	40	
		98	80	82	89	91	48	30	32	39	41	
D		79	81	88	95	97	29	31	38	45	47	B
		85	87	94	96	78	35	37	44	46	28	
		86	93	100	77	84	36	43	50	27	34	

Table 40: 10×10 Magic Square (Preliminary Step)

Creating the highlighted areas in D involves the same process as in the first example, which means the highlighted areas in A are replicated in D. Thus, D-1 is a 2×2 in the top left corner, D-2 is a 1×2 in the middle row, and D-3 is a 2×2 in the bottom left corner. All three highlighted areas are shown in Table 41 with a lighter yellow.

	A-1	17	24	1	8	15	67	74	51	58	65	
		23	5	7	14	16	73	55	57	64	66	
A	A-2	4	6	13	20	22	54	56	63	70	72	C
		10	12	19	21	3	60	62	69	71	53	
	A-3	11	18	25	2	9	61	68	75	52	59	
	D-1	92	99	76	83	90	42	49	26	33	40	
		98	80	82	89	91	48	30	32	39	41	
D	D-2	79	81	88	95	97	29	31	38	45	47	B
		85	87	94	96	78	35	37	44	46	28	
	D-3	86	93	100	77	84	36	43	50	27	34	

Table 41: 10×10 Magic Square (Preliminary Step)

Establishing B-1 and C-1 highlighted areas are going to occur differently in this example than the previous one because they are nonempty. (In the 6×6 case, subtracting 1 from the 1st row in A-1 yielded an answer of 0, so no columns in B or C were highlighted.) In this case, subtracting 1 from the number of highlighted squares in row 1 in A-1 yields $2 - 1 = 1$. Therefore, one column in B and C has to be highlighted from right to left, meaning the last column in B and C has to be highlighted and labeled B-1 and C-1. This is depicted in Table 42.

	A-1	17	24	1	8	15	67	74	51	58	65		
		23	5	7	14	16	73	55	57	64	66		
A	A-2	4	6	13	20	22	54	56	63	70	72	C-1	C
		10	12	19	21	3	60	62	69	71	53		
	A-3	11	18	25	2	9	61	68	75	52	59		
	D-1	92	99	76	83	90	42	49	26	33	40		
		98	80	82	89	91	48	30	32	39	41		
D	D-2	79	81	88	95	97	29	31	38	45	47	B-1	B
		85	87	94	96	78	35	37	44	46	28		
	D-3	86	93	100	77	84	36	43	50	27	34		

Table 42: 10×10 Magic Square (Preliminary Step)

To finish creating the 10×10 magic square, the highlighted areas in A (A-1, A-2, and A-3) are swapped with the highlighted areas in D (D-1, D-2, and D-3) and the highlighted areas in C are swapped with the highlighted areas in B. In Table 43, the darker yellow highlighted areas have swapped places with the lighter colored yellow highlighted areas to better see the movement.

D-1	92	99	1	8	15	67	74	51	58	40	
	98	80	7	14	16	73	55	57	64	41	
D-2	4	81	88	20	22	54	56	63	70	47	B-1
	85	87	19	21	3	60	62	69	71	28	
D-3	86	93	25	2	9	61	68	75	52	34	
A-1	17	24	76	83	90	42	49	26	33	65	
	23	5	82	89	91	48	30	32	39	66	
A-2	79	6	13	95	97	29	31	38	45	72	C-1
	10	12	94	96	78	35	37	44	46	53	
A-3	11	18	100	77	84	36	43	50	27	59	

Table 43: 10×10 Magic Square (Complete)

The 10×10 is now complete with every row, column, and diagonal adding up to the desired magic constant of 505. After completing both examples, it is important to note that the 6×6 is the only magic square formed with the Singly-Even Method that does not highlight any cells in B and C. For every other magic square created using this method, there will be cells highlighted in B and C.

2.4 Doubly-Even Method

The *Doubly-Even Method* is used for creating an $n \times n$ magic square, where n is doubly-even means that $n = 4m$, where $m \in \mathbb{Z}^+$, so n is divisible by both 2 and 4. Examples of what n could equal are 4, 8, 12 ... because they all can be divided by both 2 and 4. This method is the final method out of the four researched during my study, and I believe that this is the easiest and quickest to do. To understand the Doubly-Even Method, 4×4 and 8×8 magic squares will be produced utilizing the steps in this method.

2.4.1 4×4 Magic Square

As with every other method, the magic constant is the first part to determine. Creating a 4×4 magic square means the n value is 4, so the magic

constant is $\frac{4(4^2+1)}{2} = \frac{4(17)}{2} = 34$. Also, since n equals 4, the numbers that will be used are 1 through $n^2 = 16$.

Similar to the Singly-Even Method, the main magic square gets broken into smaller highlighted squares. The first highlighted mini-squares created are going to consist of the 4 corner squares. In each corner of the main magic square, a mini-square is going to be created that is $\frac{n}{4} \times \frac{n}{4}$. For this example, the dimensions of the mini-squares are going to be $\frac{4}{4} \times \frac{4}{4} = 1 \times 1$. Thus, only the cell in each corner of the main magic square is going to be colored in with blue. The corner highlights are created in Table 44.

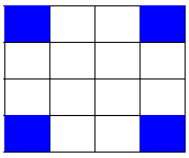

Table 44: 4×4 Magic Square (Corner Highlights)

The second area to be highlighted is called the main highlight. The dimensions of this highlight are $\frac{n}{2} \times \frac{n}{2}$ and it is placed in the center of the main magic square. For this example, the dimensions are $\frac{4}{2} \times \frac{4}{2} = 2 \times 2$. The main highlight should touch the corners of the four blue highlighted areas but should not overlap them. The main highlight is produced in Table 45.

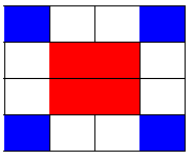

Table 45: 4×4 Magic Square (Corner and Center Highlights)

All of the highlighted areas are created, so the numbers can be placed in their correct cells. Only highlighted cells get filled in on the first round of placing numbers. The first round starts at the top left cell and continues row by row left to right, counting sequentially from 1 to 16. When reaching a cell that is highlighted, place the corresponding number in the cell. If the cell is not highlighted, skip that cell and move to the next, but continue counting. For instance, the first row has cells 1, 2, 3, and 4. 1 and 4 are counted in cells that are highlighted, so those numbers get placed. However, 2 and 3 are counted in cells that are white, so those cells get skipped without a number being placed in them. Once each row is completed, move down to the following row to continue the process. Placing numbers in each highlighted cell is depicted in Table 46.

1			4
	6	7	
	10	11	
13			16

Table 46: 4×4 Magic Square (Incomplete)

A similar process is used to fill in the white cells, except the first number counted is 16. Instead of counting up from 1 to 16 sequentially, this process starts at 16 and counts backwards to 1 in each row from left to right. Starting with the top row and working down to the bottom row, each number is placed when counted in a white cell, but not in a highlighted cell. For instance, the first row would have cells 16, 15, 14, and 13. 15 and 14 would be placed in the cells because they are white, but 16 and 13 would not be placed because those cells are highlighted and a number already exists in these cells. The white cell placements are illustrated in Table 47.

1	15	14	4
12	6	7	9
8	10	11	5
13	3	2	16

Table 47: 4×4 Magic Square (Complete)

After completing all of those steps, a doubly-even magic square is produced. Adding up each row, column, and diagonal proves that each add up to the desired magic constant of 34. Table 47 depicts a magic square produced by the Doubly-Even Method.

2.4.2 8×8 Magic Square

Creating an 8×8 magic square using the Doubly-Even Method is very similar to creating the 4×4 magic square, except the dimensions of the highlighted areas are going to be different. The value of n is 8, so the magic constant is $\frac{8(8^2+1)}{2} = \frac{8(65)}{2} = 260$, and the numbers to be placed are 1 through $n^2 = 64$.

The four highlighted corner mini-squares are going to have dimensions of $\frac{n}{4} \times \frac{n}{4} = \frac{8}{4} \times \frac{8}{4} = 2 \times 2$. For example, when creating the left corner with these dimensions, the first two cells in the top two rows will be highlighted. This same format will be repeated in each of the corners of the main magic square, which is shown in Table 48 in blue.

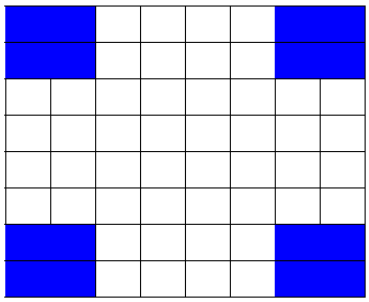

Table 48: 8×8 Magic Square (Corner Highlights)

The main highlight is created with the dimensions $\frac{n}{2} \times \frac{n}{2} = \frac{8}{2} \times \frac{8}{2} = 4 \times 4$ and is placed in the center of the main magic square. When this is highlighted in red, it is easier to see how the corners of the main highlight touch the corners of each corner highlight; however, they never overlap each other. The main highlight is portrayed in Table 49.

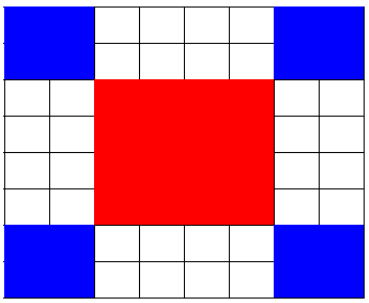

Table 49: 8×8 Magic Square (Corner and Center Highlights)

Placing the numbers in their respective cells occurs exactly as it occurred in the 4×4 example. The first cell to be counted is the top left cell and continues from left to right counting sequentially from 1 to 64. When a highlighted cell gets counted, the corresponding number gets placed in that cell. When a cell is white, then nothing gets placed in that cell at this time. When a row is completed, the same process continues on the following row until the last row is finished. For clarification, the first row would have cells 1, 2, 3, 4, 5, 6, 7, and 8, but only 1, 2, 7, and 8 would be placed because 3, 4, 5, and 6 are counted as white cells. The first round of number placements is pictured in Table 50.

1	2					7	8
9	10					15	16
		19	20	21	22		
		27	28	29	30		
		35	36	37	38		
		43	44	45	46		
49	50					55	56
57	58					63	64

Table 50: 8×8 Magic Square (Incomplete)

The white cells are completed by starting at the top left cell as 64 and counting backwards from 64 to 1 in order and placing numbers only into white cells. This is completed row by row starting at the top left and continuing down until the last row is completed. For example, the first row would have cells 64, 63, 62, 61, 60, 59, 58, and 57, but only the cells 62, 61, 60, and 59 get filled in because the 64, 63, 58, and 57 cells are highlighted and a number already exists in these cells. The completed white cells are found in Table 51.

1	2	62	61	60	59	7	8
9	10	54	53	52	51	15	16
48	47	19	20	21	22	42	41
40	39	27	28	29	30	34	33
32	31	35	36	37	38	26	25
24	23	43	44	45	46	18	17
49	50	14	13	12	11	55	56
57	58	6	5	4	3	63	64

Table 51: 8×8 Magic Square (Complete)

Table 51 is the complete8×8 magic square produced by the Doubly-Even Method. Each row, column, and diagonal adds up to the expected magic constant of 260.

3. Conclusion

The research conducted on magic squares proved that a simple concept of an $n \times n$ square having each row, column, and diagonal equaling the same value can be very complex when studied closely. To be able to understand any aspect of magic squares, comprehending the Siamese Method for composing odd magic squares is crucial. Other composition methods that are essential following the Siamese Method are the Lozenge Method (for odd magic squares), the Singly-Even Method (for singly-even magic

squares), and the Doubly-Even Method (for doubly-even magic squares). The composition of a magic square is truly beautiful, and it is intriguing to think that over 3,000 years ago a Divine Turtle brought a mystery to this world that is still being studied today and will be for years to come.

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Acknowledgements

I would like to thank my Thesis Advisor, Dr. Lyn McQuaid, for everything she has done for me during the past three semesters. To start, I had no idea what I wanted to do my Capstone on and she came to me and asked if I wanted to work with her on magic squares. I am so thankful she did because I have had so much fun researching and writing on this topic. Dr. McQuaid has been available to talk on Zoom or email whenever I needed her, during the school day, at night, on a weekend, or even during winter or summer breaks. When I was doing my semester of research she knew I needed a book and did not stop until I got it. She had the library get it on loan from a different university so I could finish my research. Dr. McQuaid has become my biggest role model of the type of math teacher I want to be for my students one day. In addition, I would like to thank my mother, Donna Moyer, for being by my side through this whole process. My mother has proofread every slide written for my presentations, every page in this paper, and checked to make sure all of my magic squares had rows, columns, and diagonals that equaled the magic constant. She has found countless mistakes that I would never have caught. When I was learning all the methods, she sat and I taught her every single method multiple times to make sure I had all the steps. My mother also sat and listened to me practice my presentations for more hours than I can count. When I was frustrated because a magic square, proof, or format was not what I wanted, she would be the first one to sit down and look at it with me. My mother was there for me every step of the way, whether she understood it or not, whether I was in a pleasant mood or not, or whether it was super late or early. So honestly, there would be no way I would have ever completed this project without her.

Editor's Note

Lindsey Moyer's work won third place for presentations at the (virtual) 43rd Biennial Convention hosted by the University of Central Missouri in Warrensburg, Missouri, April 15-17, 2021. This is the first of two parts, the second to appear in the Fall 2021 issue of *The Pentagon*.

Pursuit Curves for Accelerating Prey

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Abstract

We consider a model of pursuit curve where the target is moving with constant acceleration and the pursuer is moving with a constant speed. We derive the Bouguer-style differential equation for this scenario and explore some critical cases where capture is just barely possible. In particular, we derive a relationship between the parameters: a, v, and x_0 (the acceleration of the target, the velocity (speed) of the pursuer, and the initial displacement between them). We also include a numerical study of this equation using fourth order Runge-Kutta.

1. Introduction

The word "Radiodrome" comes from the Latin "Radius" and the Greek "Dromos", which refers to running or a racetrack. A radiodrome is the curve of pursuit, from one entity chasing another entity, with certain restrictions made on the tangent vectors of these entities. An example of a radiodrome is a curve traced by a dog when chasing a rabbit, where the dog does not try to anticipate the rabbit's movement, but always heads directly toward the rabbit. Radiodromes were first studied in the context of ordinary differential equations in 1732 by Pierre Bouguer, a French mathematician, geophysicist, geodesist, and astronomer. He is also known as "the father of naval architecture".

In this paper, we examine a variation on the Bouguer model in which the prey (target) has constant acceleration and the predator (pursuer) has constant velocity. We explore the relationship between the prey's acceleration a, the predator's velocity v and the initial displacement x_0 between the predator and the prey at the critical case where capture can barely happen. In order to arrive at the differential equation to model this scenario, we apply the arc length formula to compute the predator's path and make use of the fact that the predator is always heading toward the prey. We also propose and prove a theorem concerning the relationship between v, a and x_0 at the critical case. In order to further justify our theorem, we systematize our model, which is a second-order ordinary differential equation, and numerically solve it by applying fourth-order Runge-Kutta. We also examine the case in which both the target and the pursuer have constant accelerations.

2. Derivation of the model

We start by placing the predator at the origin and the prey at $(x_0, 0)$ on the x-axis. The prey is moving at a constant acceleration a without initial speed in the vertical direction, while the predator has constant velocity v and is always targeting the prey.

Let $(x(t), y(t))$ be the position of the predator at time t. The tangent vector to the predator's path is pointing to $(x_0, \frac{1}{2}at^2)$, which is the position of the prey at time t. The slope of this tangent line is:

$$\frac{dy}{dx} = \frac{y - \frac{1}{2}at^2}{x - x_0} = y'$$

So

$$t = \sqrt{\frac{2y - 2y'(x - x_0)}{a}} \tag{2.1}$$

On the other hand, the predator path can also be modeled using the arc-length formula. We also know the predator is moving at a constant speed, v.

$$vt = \int_0^x \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

By the Fundamental Theorem of Calculus,

$$v\frac{dt}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \sqrt{1 + y'^2} \tag{2.2}$$

From (2.1), take the derivative of both sides

$$\frac{dt}{dx} = -\frac{y''(x - x_0)}{a}\sqrt{\frac{a}{2y - 2y'(x - x_0)}} \tag{2.3}$$

From (2.2) and (2.3)

$$\sqrt{1 + y'^2} = -\frac{v \cdot y''(x - x_0)}{a}\sqrt{\frac{a}{2y - 2y'(x - x_0)}}$$

$$\frac{v^2}{2a}\left[y''(x - x_0)\right]^2 = \left[1 + y'^2\right]\left[y - y'(x - x_0)\right]$$

This is the Nonlinear Second-order Ordinary Differential Equation for the path of the predator.

3. Critical Captures and The Critical Line

Since the prey has acceleration a, it is conspicuous to see that the prey can only escape when its speed exceeds the predator's constant velocity v. In other words, the moment the prey's velocity equals v is the last possible moment of capture.

Definition

Definition 1 (Critical Captures)**.** *Critical captures occur when the predator catches the prey at the exact moment the prey's velocity reaches v.*

Let $v_{prey}(t)$ be the velocity of the prey in the vertical direction. At the moment of critical capture, t_c, $v_{prey} = at_c = v$. So

$$t_c = \frac{v}{a}$$

Therefore, critical capture must happen at

$$y_c = \frac{1}{2}at_c^2 = \frac{v^2}{2a}$$

Definition 2 (Critical Curve)**.** *Define a critical curve to be the curve traced by the predator when a critical capture occurs. In other words, a critical curve is the predator's path when it catches the prey at exactly* $y_c = \frac{v^2}{2a}$ *at time* $t_c = \frac{v}{a}$.

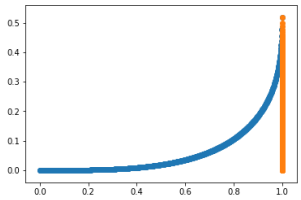


Figure 2: A critical capture

At the moment of capture, t_c, the predator's x and y values, x_c and y_c are related to each other by a multiplicative constant k.

$$y_c = kx_c$$

However, $y_c = \frac{v^2}{2a}$ and $x_C = x_0$, the prey's initial position. So

$$\frac{v^2}{2a} = kx_0$$

Theorem 3. *There exists a unique critical constant k that satisfies* $\frac{v^2}{2a} = kx_0$ *for all critical captures.*

Proof. We begin by examining a critical capture with the parameters x_0, a and v. We know that there always exists a constant k satisfying $\frac{v^2}{2a} = kx_0$. In order to prove the uniqueness of k, we scale a by a constant $q, q > 0$ and explore every case. We have:

$$\frac{dy}{dx} = \frac{y - \frac{1}{2}qat^2}{x - x_0} \tag{3.1}$$

We know if we only scale a and everything else remains the same, we no longer have a critical capture. Therefore, the first thing we explore is absorbing q into time t. Consider the critical capture happened at $\widetilde{t}$ and $\widetilde{a} = qa$:

$$\frac{dy}{dx} = \frac{y - \frac{1}{2}a(\sqrt{q}t)^2}{x - x_0}$$

Now $\widetilde{t} = \sqrt{q}t$. We know at the moment of critical capture, $t = \frac{v}{a}$. So:

$$\widetilde{t} = \frac{\widetilde{v}}{a} = \sqrt{q}t = \sqrt{q}\frac{v}{a}$$

So $\widetilde{v} = \sqrt{q}v$ and

$$\frac{\widetilde{v}^2}{2\widetilde{a}} = \frac{qv^2}{2qa} = \frac{v^2}{2a} = kx_0$$

So if we rescale a by q and t by $\sqrt{q}$, in order to remain critical, we rescale v by $\sqrt{q}$, and the relationship between predator's initial velocity, prey's acceleration and initial placement is maintained.

Next, we consider the case where we scale x, x_0 and y. From (3.1)

$$\frac{dy}{dx} = \frac{y - \frac{1}{2}qat^2}{x - x_0} = \frac{\frac{1}{q}y - \frac{1}{2}at^2}{\frac{1}{q}x - \frac{1}{q}x_0}$$

Now, let $\widetilde{x_0} = \frac{1}{q}x_0$ while keeping v the same. Since $\frac{v^2}{2a} = kx_0$ We have:

$$\frac{v^2}{2\widetilde{a}} = \frac{v^2}{2qa} = \frac{1}{q}kx_0 = k\widetilde{x_0}$$

The relationship is maintained when we scale a by q and x_0 by $\frac{1}{q}$ while keeping v fixed. Next, instead of keeping one of the parameters constant,

we scale all three of them by different values. Let $\widetilde{a} = qa$, $\widetilde{v} = pv$ and $\widetilde{x_0} = cx_0$.

When a is scaled by $q > 1$, the critical velocity ranges from v to $\sqrt{q}v$, and for $q < 1$, the critical velocity ranges from $\sqrt{q}v$ to v. Similarly, when $q > 1$, x_0 ranges from $\frac{1}{q}x_0$ to x_0 or from x_0 to $\frac{1}{q}x_0$ if $q < 1$. So to maintain a critical curve, for $q > 1$, we must have:

$$1 \leq p \leq \sqrt{q}$$
$$\frac{1}{q} \leq c \leq 1$$

And for $q < 1$.

$$\sqrt{q} \leq p \leq 1$$
$$1 \leq c \leq \frac{1}{q}$$

Consider the critical capture with v, a and x_0:

$$\frac{v^2}{2ax_0} = k$$

When v, a and x_0 are scaled:

$$\frac{\widetilde{v}^2}{2\widetilde{a}\widetilde{x_0}} = \frac{p^2v^2}{2qacx_0}$$

In order to show the uniqueness of k, we must show $\frac{p^2}{qc} = 1$. Since p, q, c are all constant, $\frac{p^2}{qc}$ is also a constant. Moreover, according to the previous cases, when v is scaled by $\sqrt{q}$, $p = \sqrt{q}$, the initial placement is maintained: $c = 1$, so $\frac{p^2}{qc} = 1$. At the other end, when x_0 is scaled by $\frac{1}{q}$, v is fixed, which means $p = 1$. Therefore, $\frac{p^2}{qc} = 1$. $\frac{p^2}{qc}$ is a constant bounded by 1 at both ends, which indicates $\frac{p^2}{qc} = 1$. Therefore:

$$\frac{\widetilde{v}^2}{2\widetilde{a}\widetilde{x_0}} = \frac{p^2v^2}{2qacx_0} = \frac{v^2}{2ax_0} = k$$

■

Corollary 4. *The family of critical curves are all simply rescalings of one another.*

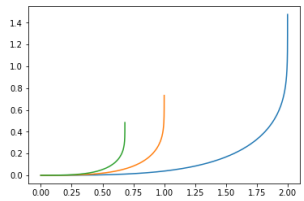


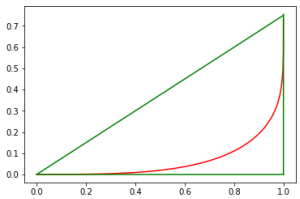
Figure 3: Some critical curves

This follows directly from theorem 1.

Lemma 5. *In the critical case, the predator always travels twice as much as the prey does.*

Proof. At the time of critical capture t_c, the predator travels $vt_c = \frac{v^2}{a}$ while the prey travels $y_c = \frac{v^2}{2a}$. ■

Theorem 6. *The critical constant k has an upper bound of* 1 *and a lower bound of* $\frac{1}{\sqrt{3}}$.



Proof. Consider the critical capture with x_0, v and a. By the time of capture, the prey travels $\frac{v^2}{2a} = kx_0$ while the predator travels $\frac{v^2}{a} = 2kx_0$.The path of the prey and the x-axis form a right triangle, whose hypotenuse is $h = \sqrt{k^2x_0^2 + x_0^2} = x_0\sqrt{k^2+1}$ long. This length has to be strictly shorter than the predator's path: $x_0\sqrt{k^2+1} < 2kx_0$, which means $\sqrt{k^2+1} < 2k$ so $k^2+1 < 4k^2$ and $1 < 3k^2$. Therefore, $k > \frac{1}{\sqrt{3}}$.

On the other hand, the predator's path has to be shorter than the sum of the legs: $2k < 1+k$ so $k < 1$. Therefore, $\frac{1}{\sqrt{3}} < k < 1$. ■

Definition 7 (Critical Line)**.** *Define the line $y = kx$ as the critical line.*

Theorem 8. *If capture is to occur, it must occur at or below the critical line. If the prey is able to surpass the critical line, the prey will escape.*

Proof. The moment the prey surpasses the critical line, $y_{prey} \geq kx_0$, so $\frac{1}{2}at^2 \geq \frac{v^2}{2a}$. However, as $v_{prey} = at$, $\frac{1}{2}v_{prey}t \geq \frac{v^2}{2a}$, so $atv_{prey} \geq v^2$, which means $v_{prey}^2 \geq v^2$. Therefore, $v_{prey} \geq v$ and the prey escapes.

■

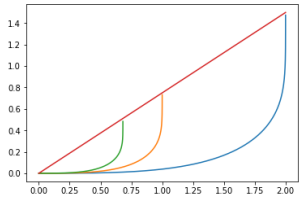


Figure 4: The critical line

This result agrees with the classical Bouguer model when both the prey and the predator have constant velocities. For the Bouguer model, captures can only occur when the predator's velocity is greater than the prey's velocity.

4. Simulations and Numerical Approach

We simulate the model using the classic forth-order Runge-Kutta. We begin by systematizing the differential equation for the path of the predator.

$$\frac{v^2}{2a}[y''(x-x_0)]^2 = [1+y'^2][y-y'(x-x_0)]$$

From (4), isolate y''

$$y'' = \frac{1}{x-x_0}\sqrt{\frac{a}{v^2}(1+y'^2)(2y-2(x-x_0)y')}$$

Let $u_1 = y$ and $u_2 = y'$. Notice $u_2' = y''$.

$$\begin{cases} u_1' = u_2 \\ u_2' = \frac{1}{x-x_0}\sqrt{\frac{a}{v^2}(1+(u_2)^2)(2u_1-2(x-x_0)u_2)} \end{cases}$$

This is a system of First Order Ordinary Differential Equations for the path of the predator. We apply the classic forth-order Runge-Kutta to numerically simulate our model. Implemented in Python 3, our simulation allows us to visualize the pursuit curve as well as take a closer look at the critical constant k.

From (5), we know the critical constant k decides the relationship of the parameters v, a and x_0. Using our simulation, we fix two of the parameters and try adjusting the last one to get a critical capture, and then compute k. We also use Bisection method to help getting the last parameter. We keep track of the distance between the actual capture and the theoretical critical capture and bisecting the last parameter's expected range until this

Step Size	x_0	k
10^{-4}	0.7062	0.7079
10^{-5}	0.6815	0.7336
10^{-6}	0.6686	0.7477
10^{-7}	0.65625	0.7601

distance goes to 0. We start with the step size of 10^{-4} and divide the step size by 10. For example, keep $a = v = 1$ and adjust x_0, we have:

According to the table above, we suggest that in this case $x_0 = 0.65 \pm 0.02$, while $k = 0.76 \pm 0.04$. We also apply the same process to other cases where we change v or a. We present a few other combination of v, a and x_0 in Table 1 below.

v	a	x_0	k
1	1	0.65 ± 0.02	0.76 ± 0.04
1	0.65 ± 0.03	1	0.75 ± 0.05
1.23 ± 0.02	1	1	0.76 ± 0.03

Table 1

According to Table 1 k is 0.75 ± 0.05, which agrees with the bounds we establish in section 3. Since $k = 0.75$, the critical line is $y = 0.75x$.

5. Conclusion

The Pursuit Curve model developed in this paper leads to a second-order nonlinear differential equation that we are unable to solve analytically. Despite this, we have successfully proven the existence and uniqueness of a critical constant that relates the three parameters x_0, v, and a together in a single equation in the case of a critical capture. As well, we have established the critical line theoretically and numerically, which forms the boundary of the region of capture. However, there are still open questions about this model. First of all, we could not figure out an analytical solution for the pursuit curve model for accelerating prey. Second, it would be very interesting to know the exact value of the constant k, which we estimate to be around .75. We have only been able to estimate k to the nearest hundredth. Overall, we believe our work provides a novel contribution to the research of radiodromes.

Appendix: Pursuit Curve with constant accelerations

In this section, we explore another variation of the Bouguer model where not only the prey are is accelerating, but the predator is also moving with a constant magnitude acceleration.

This model has the similar initial set up as the other one. We start by placing the pursuer at the origin and the target at $(x_0, 0)$ on the x-axis. While the target is still moving in the vertical direction at a constant acceleration a without initial speed, the pursuer in this case has a constant magnitude acceleration b and is always heading toward the prey.

Let $(x(t), y(t))$ be the position of the predator at time t. Since the prey in this model behaves the same as in the other model, from (2.1) we have the position of the prey at time t: $(x_0, \frac{1}{2}at^2)$, and the tangent vector to the predator's path is:

$$\frac{dy}{dx} = \frac{y - \frac{1}{2}at^2}{x - x_0} = y'$$

And

$$t = \sqrt{\frac{2y - 2y'(x - x_0)}{a}}$$

Again, the predator path can be modeled using the arc-length formula. However, this time the predator is moving with a constant magnitude acceleration b, We have:

$$\frac{1}{2}bt^2 = \int_0^x \sqrt{1 + (\frac{dy}{dx})^2}\, dx$$

By the Fundamental Theorem of Calculus,

$$bt\frac{dt}{dx} = \sqrt{1 + (\frac{dy}{dx})^2} = \sqrt{1 + y'^2}$$

Which means:

$$t\frac{dt}{dx} = \frac{1}{b}\sqrt{1 + y'^2}$$

We have the derivative of t:

$$\frac{dt}{dx} = -\frac{y''(x - x_0)}{a}\sqrt{\frac{a}{2y - 2y'(x - x_0)}}$$

We then have:

$$\sqrt{\frac{2y-2y'(x-x_0)}{a}}\left(-\frac{y''(x-x_0)}{a}\right)\sqrt{\frac{a}{2y-2y'(x-x_0)}}=\frac{1}{b}\sqrt{1+y'^2}$$

Which is:

$$-\frac{y''(x-x_0)}{a}=\frac{1}{b}\sqrt{1+y'^2}$$

So with $r=\frac{a}{b}$:

$$-y''(x-x_0)=r\sqrt{1+y'^2}$$

The constant accelerations model reduces to the classic Bouguer model. Unlike the accelerating prey model, the constant accelerations model and the Bouguer model is separable:

$$\frac{-y''}{\sqrt{1+y'^2}}=\frac{r}{x-x_0}$$

Which means:

$$\operatorname{arcsinh} y'=-r\ln(x-x_0)+C$$

But we know that $y'(0)=0$, so: $\operatorname{arcsinh} 0=-r\ln(x_0)+C$, and $C=r\ln(x_0)$
Substitute C back to (3.5)

$$y'=\sinh\big(-r\ln(x-x_0)+r\ln(x_0)\big)$$

Using the exponential form of *sinh*:

$$y'=\frac{1}{2}\left(\left(1-\frac{x}{x_0}\right)^{-r}-\left(1-\frac{x}{x_0}\right)^{r}\right)$$

So the solution for the constant accelerations model is:

$$y=\frac{1}{2}\left(-\frac{x_0(1-\frac{x}{x_0})^{1-r}}{1-r}+\frac{x_0(1-\frac{x}{x_0})^{1+r}}{1+r}\right)+C$$

with $C=\frac{x_0 r}{1-r^2}$ since $y(0)=0$.

Acknowledgement

This research is conducted as a part of the 2018-2019 Beling Program at Augustana College, where Dr. Sward is the chair. We would very much like to thank Dr. Forrest Stonedahl, Dr. Susa Stonedahl, Jonathan Reaban and Ali Rabeh for their contribution to the model of pursuit for accelerating preys.

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The Problem Corner

Edited by Pat Costello

The Problem Corner invites questions of interest to undergraduate students. As a rule, the solution should not demand any tools beyond calculus and linear algebra. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following new problems should be submitted on separate sheets before April 15, 2022. Solutions received after this will be considered up to the time when copy is prepared for publication. The solutions received will be published in the Spring 2022 issue of *The Pentagon*. Preference will be given to correct student solutions. Affirmation of student status and school should be included with solutions. New problems and solutions to problems in this issue should be sent to Pat Costello, Department of Mathematics and Statistics, Eastern Kentucky University, 521 Lancaster Avenue, Richmond, KY 40475-3102 (e-mail: pat.costello@eku.edu, fax: (859) 622-3051)

NEW PROBLEMS 881 - 890

Problem 881. *Proposed by Mathew Cropper, Eastern Kentucky University, Richmond, KY.*
Find a formula (possibly recursive) for the number of integers with n digits that contain exactly one 47 in the integer.

Problem 882. *Proposed by José Luis Díaz-Barrero, School of Civil Engineering, Barcelona Tech - UPC, Barcelona, Spain.*
Find all functions $f : \mathbb{R} \to \mathbb{R}$ that satisfy $f(x^4 + y) = f(x) + f(y^4)$ for all $x, y \in \mathbb{R}$.

Problem 883. *Proposed by Daniel Sitaru, "Theodor Costescu" National Economic College, Drobeta Turnu – Severin, Romania.*
If $a, b, c \in \mathbb{C}$ are such that $\left|a^8 + 1\right| \leq 1, \left|b^{10} + 1\right| \leq 1, \left|c^{12} + 1\right| \leq 1,$ $\left|a^4 + 1\right| \leq 1, \left|b^5 + 1\right| \leq 1,$ and $\left|c^{16} + 1\right| \leq 1$, then

$$|a + b + c| + 3 \geq |a + b| + |b + c| + |c + a|.$$

Problem 884. *Proposed by Daniel Sitaru, "Theodor Costescu" National Economic College, Drobeta Turnu – Severin, Romania.*
If $a,b,c>0$ and $a^4+b^4+c^4=3$, then

$$\frac{(a^2+b^2)^6}{(3a^8+10a^4b^4+3b^8)}+\frac{(b^2+c^2)^6}{(3b^8+10b^4c^4+3c^8)}+\frac{(c^2+a^2)^6}{(3c^8+10c^4a^4+3a^8}) \leq 12.$$

Problem 885. *Proposed by Dorin Marghidanu, Colegiul National 'A. I. Cuze', Corabia, Romania.*
If $a,b,x,y>0$ and $n\in\mathbb{N}^*$ prove that

$$\frac{(x+y)^n}{2^{(n-1)}} \leq \frac{(ax+by)^n+(bx+ay)^n}{(a+b)^n} \leq x^n+y^n.$$

Problem 886. *Proposed by George Stoica, Saint John, New Brunswick, Canada.*
Prove that for any $a\in(1,2)$ and any integer $n\geq 1$, there exist $\varepsilon_0,\varepsilon_1,\varepsilon_2,\ldots,\varepsilon_n\in\{-1,1\}$ such that

$$(a-1)\left|\varepsilon_0+\varepsilon_1 a+\varepsilon_2 a^2+\ldots+\varepsilon_n a^n\right|<1.$$

Problem 887. *Proposed by D.M. Bătinetu-Giurgiu, "Matei Basarab" National College, Bucharest, Romania and Neculai Stanciu, "George Emil Palade" School, Buzău, Romania.*
Prove that in any triangle ABC with semiperimeter s, inradius r and usual notations, the following is true

$$\frac{a^{(m+1)}}{(s-b)^m}+\frac{b^{(m+1)}}{(s-c)^m}+\frac{c^{(m+1)}}{(s-a)^m} \geq 3*2^{(m+1)}*\sqrt{3}*r.$$

Problem 888. *Proposed by D.M. Bătinetu-Giurgiu, "Matei Basarab" National College, Bucharest, Romania and Neculai Stanciu, "George Emil Palade" School, Buzău, Romania.*
Let the positive real sequence $(a_n)_{n\geq 1}$ be such that $\lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n\sqrt[n]{(n!)^2}}=a\in\mathbb{R}_+^*$.
Compute

$$\lim_{n\to\infty}\frac{1}{\sqrt[n]{(2n-1)!!}}\left(\sqrt[n+1]{a_{n+1}}-\sqrt[n]{a_n}\right).$$

Problem 889. *Proposed by Seán Stewart, Bomaderry, NSW, Australia.*
If $h_n = \sum_{k=1}^{2n} \frac{(-1)^{k+1}}{k}$, then evaluate the following two limits:
(i) item $\lim_{n\to\infty} (\log(2) - h_n)\, n$,
(ii) $\lim_{n\to\infty} \left(h_n h_{n+1} - \log^2(n)\right) n$.

Problem 890. *Proposed by Robert Stanton, St. Johns University, Jamaica, NY.*
For digits a, b, c, d, let $abcd$ represent the ordinary decimal representation $10^3 a + 10^2 b + 10c + d$. Prove that there is a unique positive integer $n = aabb$ that is a perfect square.

SOLUTIONS TO PROBLEMS 859-869

Problem 859. *Proposed by the editor.*
A regular n-gon is inscribed in a circle which is inscribed in a square with side length x. Find the length of one side of the n-gon in terms of x. [The particular case where $n = 6$ and $x = 25$ was a theMathContest.com problem.]

Solution *by Brian Beasley, Presbyterian College, Clinton, SC.*

We show that the side length s of the n-gon is $s = x\sin(\frac{\pi}{n})$. Given a regular n-gon as described, let r denote the radius of its circumcircle. Then drawing a radius from each vertex to the center divides the n-gon into n congruent triangles, each of which has side lengths r, r, and s. By the Law of Cosines, we obtain $s^2 = 2r^2 - 2r^2\cos\left(\frac{2\pi}{n}\right)$ and hence $s = cr$ where

$$c = \sqrt{(2 - 2\cos(2\pi/n))}.$$

Using the identity $\cos\left(\frac{2\pi}{n}\right) = 2cos^2\left(\frac{\pi}{n}\right) - 1$, we have

$$c = \sqrt{4 - 4cos^2\left(\frac{\pi}{n}\right)} = 2\sin\left(\frac{\pi}{n}\right).$$

Since $x = 2r$, we have that $s = x\sin\left(\frac{\pi}{n}\right)$.

Also solved by Brian Bradie, Christopher Newport University, Newport

News, VA; Albert Stadler, Herrliberg, Switzerland; Daniel Vacaru, Economic College "Maria Teiuleanu", Piteşti, Romania; and the proposer.

Problem 860. *Proposed by José Luis Díaz-Barrero, School of Civil Engineering, Barcelona Tech - UPC, Barcelona, Spain.*
Find out whether or not there is a prime p such that $p!$ ends in exactly 2020 zeroes. Is there a corresponding prime q such that $q!$ ends in exactly 2019 zeroes?

Solution *by the Missouri State University Problem Solving Group, Springfield, MO.*

It is well known that the number of zeroes that $n!$ ends in is $\sum\limits_{k=1}^{\infty} \frac{n}{5^k}$. Therefore, if $2020 = \sum\limits_{k=1}^{\infty} \left\lfloor \frac{n}{5^k} \right\rfloor \leq \sum\limits_{k=1}^{\infty} \frac{n}{5^k} = \frac{n}{4}$, we would have $n > 8080$. Taking $n = 8081$, we find that $\sum_{k=1}^{\infty} \left\lfloor \frac{8081}{5^k} \right\rfloor = 1616+323+64+12+2 = 2017$, so $8081!$ ends in 2017 zeroes. Therefore $n!$ ends in 2018 zeroes for $8085 \leq n \leq 8089$, 2019 zeroes for $8090 \leq n \leq 8094$, 2020 zeroes for $8095 \leq n \leq 8099$, and 2022 zeroes for $8100 \leq n \leq 8104$ (since 2100 contains 52 as a factor). One readily checks that 8093 is prime, so this gives us the desired q. There are no primes in the interval $8095 \leq n \leq 8099$, so no p exists. Note that there is no n, prime or not, such that $n!$ ends in exactly 2021 zeroes.

Also solved by Brian Beasley, Presbyterian College, Clinton, SC; Brian Bradie, Christopher Newport University, Newport News, VA; Corneliu Mănescu-Avram, Ploieşti, Romania; Dipyaman Mukherjee, Sodepur High School, Sodepur, Panihati, India; Henry Ricardo, Westchester Area Math Circle, Purchase, NY; Ioannis Sfikas, Athens, Greece; Albert Stadler, Herrliberg, Switzerland; Team Just Do It, Newark Academy, Livingston, NJ; Titu Zvonaru, Comăneşti, Romania; and the proposer.

Problem 861. *Proposed by José Luis Díaz-Barrero, School of Civil Engineering, Barcelona Tech - UPC, Barcelona, Spain.*
Let a_0, a_1, a_2, a_3 be any distinct nonzero real numbers. For any integer $n \geq 3$, prove that the zeroes of the polynomial

$$A(x) = x^{2n} + a_3x^3 + a_2x^2 + a_1x + a_0$$

cannot all be real.

Solution *by Brian Bradie, Christopher Newport University, Newport News, VA.*

Let a_0, a_1, a_2, a_3 be any distinct nonzero real numbers and let

$$A(x) = x^{2n} + a_3 x^3 + a_2 x^2 + a_1 x + a_0$$

where $n \geq 3$ is an integer. Because a_0 is nonzero, $x = 0$ is not a zero of the polynomial A. The table below summarizes the number of possible positive and negative real zeroes of A based on Descartes Rule of Signs. In all cases, the number of real zeroes is at most 4; however, the degree of A is $2n \geq 6$. Thus, the zeroes of polynomial A cannot all be real.

signs of coefficients	# positive real zeroes	# negative real zeroes
+,+,+,+	0	0 or 2 or 4
+,+,+,–	1	1 or 3
+,+,–,+	0 or 2	0 or 2
+,+,–,–	1	1 or 3
+,–,+,+	0 or 2	0 or 2
+,–,+,–	1 or 3	1
+,–,–,+	0 or 2	0 or 2
+,–,–,–	1	1 or 3
–,+,+,+	0 or 2	0 or 2
–,+,+,–	1 or 3	1
–,+,–,+	0 or 2 or 4	0
–,+,–,–	1 or 3	1
–,–,+,+	0 or 2	0 or 2
–,–,+,–	1 or 3	1
–,–,–,+	0 or 2	0 or 2
–,–,–,–	1	1 or 3

Also solved by Corneliu Mănescu-Avram, Ploieşti, Romania; Henry Ricardo, Westchester Area Math Circle, Purchase, NY; Albert Stadler, Herrliberg, Switzerland; Daniel Vacaru, Economic College "Maria Teiuleanu", Piteşti, Romania; and the proposer.

Problem 862. *Proposed by Daniel Sitaru, "Theodor Costescu" National Economic College, Drobeta Turnu – Severin, Romania.*
If $a, b \in \mathbb{C}$ are such that

$$\left|a^2 + 25\right| \leq 5, \left|b^2 + 36\right| \leq 6, |a+5| \leq \sqrt{5}, \text{and} \, |b+6| \leq \sqrt{6}$$

then

$$|a+b|^2 + |a-b|^2 \leq 4.$$

Solution *by the proposer.*

$$\begin{aligned}10|a| = |10a| = |(a+5)^2 - (a^2+25)| &\leq |(a+5)^2| + |a^2+25| \\ &= |a+5|^2 + |a^2+25| \\ &\leq \sqrt{5}^2 + 5 = 10.\end{aligned}$$

This implies that $|a| \leq 1$.

$$\begin{aligned}12|b| = |12b| = |(b+6)^2 - (b^2+25)| &\leq |(b+6)^2| + |b^2+36| \\ &= |b+|6^2 + |b^2+36| \\ &\leq \sqrt{6}^2 + 6 = 12.\end{aligned}$$

This implies that $|b| \leq 1$. By the parallelogram identity:

$$|a+b|^2 + |a-b|^2 = 2\left(|a|^2 + |b|^2\right) \leq 2\left(1^2 + 1^2\right) = 4.$$

Also solved by Albert Stadler, Herrliberg, Switzerland.

Problem 863. *Proposed by Daniel Sitaru, "Theodor Costescu" National Economic College, Drobeta Turnu – Severin, Romania.*
If $a,b,c > 0$ and $a+b+c = 6$, then

$$\frac{a^2}{\sqrt{b^2+6bc+5c^2}} + \frac{b^2}{\sqrt{c^2+6ca+5a^2}} + \frac{c^2}{\sqrt{a^2+6ab+5b^2}} \geq \sqrt{3}.$$

Solution *by Titu Zvonaru, Comăneşti, Romania.*

Using the AM-GM inequality and Cauchy-Schwarz inequality (Engel or Titu form), we obtain

$$\begin{aligned}\sum \frac{a^2}{\sqrt{b^2+6bc+5c^2}} &= \sum \frac{a^2\sqrt{3}}{\sqrt{(6+5c)(3b+3c)}} \geq \sum \frac{a^2\sqrt{3}}{\frac{b+5c+3b+3c}{2}} \\ &= \frac{\sqrt{3}}{2}\sum \frac{a^2}{b+2c} \geq \frac{\sqrt{3}}{2}\frac{(a+b+c)^2}{b+2c+c+2a+a+2b} \\ &= \frac{\sqrt{3}}{2} * \frac{(a+b+c)^2}{3(a+b+c)} = \frac{\sqrt{3}}{2} * \frac{6}{3} = \sqrt{3}.\end{aligned}$$

Also solved by Ioan Viorel Codreanu, Satulung, Maramures, Romania; Kee-Wai Lau, Hong Kong, China; Angel Plaza, Universidad de Las Palmas de Gran Canaria, Spain; Henry Ricardo, Westchester Area Math Circle, Purchase, NY; Albert Stadler, Herrliberg, Switzerland; Daniel Vacaru, Economic College "Maria Teiuleanu", Piteşti, Romania; and the proposer.

Problem 864. *Proposed by Toyesh Prakash Sharma (student) St. C.F Andrews School, Agra, India.*
Evaluate the following sum

$$\sum_{z=0}^{\infty}\sum_{y=0}^{\infty}\sum_{x=0}^{\infty}\frac{x+y+z}{2^{(x+y+z)}}.$$

Solution *by the Cal Poly Pomona Problem Solving Group, Pomona, CA.*

Note that the triple sum is obtained by summing across every lattice point of $\mathbb{N}^3$. Let $n = x+y+z$. For each lattice point $(x,y,z), n$ is its taxicab distance from the origin. If we can sum up $\frac{n}{2^n}$ across all lattice points, we have the answer. How many lattice points exist at a given taxicab distance n from the origin? There are $\binom{n+2}{2}$ ways to partition into 3 integers. Therefore, we can rewrite the original triple sum as follows:

$$\sum_{z=0}^{\infty}\sum_{y=0}^{\infty}\sum_{x=0}^{\infty}\frac{x+y+z}{2^{(x+y+z)}} = \sum_{n=0}^{\infty}\binom{n+2}{2}\frac{n}{2^n} = \sum_{n=0}^{\infty}\frac{(n+2)(n+1)n}{2^{n+1}}.$$

Note that $\sum_{m=0}^{\infty} x^m$is the Taylor series for $f(x) = (1-x)-1$. Taking the third derivative of $f(x)$ and its Taylor series, we obtain

$$f'''(x) = \sum_{m=0}^{\infty} m(m-1)(m-2)x^{m-3} = 6(1-x)^{-4}.$$

Shifting the index of summation to $m = n+2$, we get

$$\begin{aligned}\sum_{n=0}^{\infty}\frac{(n+2)(n+1)n}{2^{n+1}} &= \frac{2*1*0}{2^1}+\frac{3*2*1}{2^2}+\sum_{n=2}^{\infty}\frac{(n+2)(n+1)n}{2^{n+1}}\\ &= 0+\frac{3}{2}+\sum_{m=4}^{\infty}\frac{m(m-1)(m-2)}{2^{m-1}}\\ &= \frac{3}{2}+\frac{1}{2^2}\sum_{m=0}^{\infty}\frac{m(m-1)(m-2)}{2^{m-3}}-\left(0+0+0+\frac{3}{2}\right)\\ &= \frac{1}{4}f'''\left(\frac{1}{2}\right) = \frac{1}{4}\left(6(1-\frac{1}{2})^{-4}\right) = 24.\end{aligned}$$

Also solved by Brian Bradie, Christopher Newport University, Newport News, VA; Missouri State University Problem Solving Group, Springfield, MO; Dipyaman Mukherjee, Sodepur High School, Sodepur, Panihati, India; Angel Plaza, Universidad de Las Palmas de Gran Canaria, Spain; Henry Ricardo, Westchester Area Math Circle, Purchase, NY; Albert Stadler, Herrliberg, Switzerland; Seán Stewart, Bomaderry, NSW, Australia; and the proposer.

Problem 865. *Proposed by Pedro H.O. Pantoja, Natal/RN, Brazil.*
Let a,b,c be positive real numbers such that $a+b+c=1$. Prove that

$$27\left(a^5+b^5+c^5\right)+13 \leq 40\left(a^2+b^2+c^2\right).$$

Solution *Sby Ioan Viorel Codreanu, Satulung, Maramures, Romania.*

Let $\sum ab = s$ and $\prod a = p$. Using the identities
$(\sum a)^5 - \sum a^5 = 5(\prod(a+b))(\sum a^2 + \sum ab)$,
$\sum a^2 = (\Sigma a)^2 - 2\Sigma ab$,
$\prod(a+b) = (\Sigma a)(\Sigma ab) - \prod a$
and the given equality $\Sigma a = 1$, we get $\sum a^5 = 1+5s^2+5p-5ps-5s$ and $\sum a^2 = 1-2s$. The needed inequality becomes

$$27\left(1+5s^2+5p-5ps-5s\right)+13 \leq 40(1-2s)$$

which is equivalent to

$$27s^2+27p-27ps-11s \leq 0.$$

By the AM-GM inequality we have $p = abc \leq \left(\frac{a+b+c}{3}\right)^3 = 1/27$. Using the Schur inequality $s \leq \frac{1+9p}{4}$, we have

$$\begin{aligned}
27s^2+27p-27ps-11s &= s\left(27s-27p-11\right)+27p \\
&\leq \frac{1+9p}{4}\left(27*\frac{1+9p}{4}-27p-11\right)+27p \\
&= \frac{1+9p}{4}\left(\frac{27+5*27p}{4}-11\right)+27p \\
&\leq \frac{1+9*{}^{1}\!/_{27}}{4}\left(\frac{27+5*27*\frac{1}{27}}{4}-11\right)+27*\frac{1}{27} \\
&= 0.
\end{aligned}$$

Also solved by Brian Bradie, Christopher Newport University, Newport News, VA; Kee-Wai Lau, Hong Kong, China; Angel Plaza, Universidad de Las Palmas de Gran Canaria, Spain; Henry Ricardo, Westchester Area Math Circle, Purchase, NY; Albert Stadler, Herrliberg, Switzerland; Titu Zvonaru, Comăneşti, Romania; and the proposer.

Problem 866. *Proposed by Dorin Marghidanu, Colegiul National À. I. Cuze', Corabia, Romania.*
If a,b,c are the lengths of the sides of a triangle, prove the following

$$\sqrt{\frac{-a+b+c}{a}}+\sqrt{\frac{a-b+c}{b}}+\sqrt{\frac{a+b-c}{c}}>2\sqrt{2}.$$

Solution *by Kee-Wai Lau, Hong Kong, China.*

Suppose that $A \geq B \geq C$ and that the triangle is not equilateral. By AM-GM, we have

$$\begin{aligned}
&\sqrt{\frac{-a+b+c}{a}}+\sqrt{\frac{a-b+c}{b}}+\sqrt{\frac{a+b-c}{c}} \\
&\qquad \geq 2\sqrt{\left(\sqrt{\frac{a-b+c}{b}}+\sqrt{\frac{-a+b+c}{a}}\right)\left(\sqrt{\frac{a+b-c}{c}}\right)} \\
&\qquad = 2\sqrt{\sqrt{\frac{(a-b+c)(a+b-c)}{bc}}+\sqrt{\frac{(-a+b+c)(a+b-c)}{ac}}} \\
&\qquad = 2\sqrt{2}\sqrt{sin\frac{A}{2}+sin\frac{B}{2}}.
\end{aligned}$$

If $B > \frac{\pi}{/3}$, then $\sin\frac{A}{2}+\sin\frac{B}{2} \geq 2\sin\frac{B}{2} > 1$. If $\frac{\pi}{/3} > B > 0$, then

$$\begin{aligned}
\sin\frac{A}{2}+\sin\frac{B}{2} &= \cos\frac{B+C}{2}+\sin\frac{B}{2} \\
&\geq \cos B+\sin\frac{B}{2} \\
&= \left(\sin\frac{B}{2}\right)\left(1-2\sin\frac{B}{2}\right) > 1.
\end{aligned}$$

If $B = \frac{\pi}{/3}$, then since the triangle is not equilateral, we have
$\sin\frac{A}{2}+\sin\frac{B}{2} > 2\sin\frac{B}{2} = 1$.

Also solved by Brian Bradie, Christopher Newport University, Newport News, VA; Henry Ricardo, Westchester Area Math Circle, Purchase, NY; Ioannis Sfikas, Athens, Greece; Albert Stadler, Herrliberg, Switzerland; Daniel Vacaru, Economic College "Maria Teiuleanu", Piteşti, Romania; Titu Zvonaru, Comăneşti, Romania; and the proposer.

Problem 867. *Proposed by George Stoica, Saint John, New Brunswick, Canada.*
Let a,b,c be positive real numbers and $x,y,z \geq 0$ such that

$$x+aby \leq a(y+z), y+bcz \leq b(z+x), \text{ and } z+cax \leq c(x+y).$$

Prove that $x=y=z=0$. Is the conclusion still true if one assumes that $a,b,c \geq 0$?

Solution *by Albert Stadler, Herrliberg, Switzerland.*

The conclusion is not correct. For instance, take $a=b=c=x=y=z=1$. The three inequalities are true $(2 \leq 2)$, but x,y,z are not zero.

Editor's Note: Is the conclusion true if the condition $a,b,c>1$ is imposed?

Problem 868. *Proposed by D.M. Bătinetu-Giurgiu, "Matei Basarab" National College, Bucharest, Romania and Neculai Stanciu, "George Emil Palade" School, Buzău, Romania.*
Let the sequence (a_n) be defined by $a_n = \sum\limits_{k=1}^{n} \arctan\left(\frac{1}{k^2-k+1}\right)$. Compute the following:
(i) $\lim\limits_{n\to\infty} a_n = a$;
(ii) $\lim\limits_{n\to\infty} (a_n - a)n$.

Solution *by Angel Plaza, Universidad de Las Palmas de Gran Canaria, Spain.*

(i) By induction it may be proved that $a_n = \arctan(n)$, so $\lim\limits_{n\to\infty} a_n = \frac{\pi}{2}$. For $n=1$ the identity holds. Let us assume it is also true for $n-1$, that is,

$$\sum_{k=1}^{n-1} \arctan \frac{1}{k^2-k+1} = \arctan(n-1).$$

Then proving that

$$\sum_{k=1}^{n} \arctan \frac{1}{k^2-k+1} = \arctan(n)$$

becomes equivalent to proving that

$$\arctan(n-1)+\arctan\frac{1}{n^2-n+1}=\arctan(n),$$

which follows because

$$\tan\left(\arctan(n-1)+\arctan\frac{1}{n^2-n+1}\right)=\frac{n-1+\frac{1}{n^2-n+1}}{1-\frac{1}{n^2-n+1}}=n.$$

(ii) By using the Laurent series expansion of $\arctan(n)$ at $n=\infty$, it follows that

$$\lim_{n\to\infty}(\frac{\pi}{2}-\arctan(n))n=\lim_{n\to\infty}\left((\frac{\pi}{2}-\left(\frac{\pi}{2}-\frac{1}{n}+\frac{1}{3n^3}+\dots\right)\right)n=1.$$

Also solved by Florică Anastase, "Alexandru Odobescu High School, Lehliu-Gară, Călăraşi, Romania; Brian Bradie, Christopher Newport University, Newport News, VA; Missouri State University Problem Solving Group, Springfield, MO; Henry Ricardo, Westchester Area Math Circle, Purchase, NY; Albert Stadler, Herrliberg, Switzerland; Seán Stewart, Bomaderry, NSW, Australia; Daniel Vacaru, Economic College "Maria Teiuleanu", Piteşti, Romania; Titu Zvonaru, Comăneşti, Romania; and the proposers.

Problem 869. *Proposed by D.M. Bătinetu-Giurgiu, "Matei Basarab" National College, Bucharest, Romania and Neculai Stanciu, "George Emil Palade" School, Buzău, Romania.*

If $\gamma_n=-\ln n+\sum\limits_{k=1}^{n}\frac{1}{k}$ with $\lim\gamma_n=\gamma$, the Euler-Mascheroni constant, compute the following:

(i) $\lim\limits_{n\to\infty}(\gamma_n-\gamma)n$;

(ii) $\lim\limits_{n\to\infty}\left(\gamma_n\gamma_{n+1}-\gamma^2\right)n$.

Solution *by Henry Ricardo, Westchester Area Math Circle, Purchase, NY.*

(i) It is known that $\frac{1}{2(n+1)}<\gamma_n-\gamma<\frac{1}{2n}$ for $n\geq 1$. Therefore, $\frac{n}{2(n+1)}<(\gamma_n-\gamma)n<\frac{n}{2n}$, and $\lim\limits_{n\to\infty}(\gamma_n-\gamma)n=\frac{1}{2}$ by the Squeeze Principle.

(ii) We see that

$$\gamma_{n+1}-\gamma_n=\left(\sum_{k=1}^{n+1}\frac{1}{k}-\ln(n+1)\right)-\left(\sum_{k=1}^{n}\frac{1}{k}-\ln n\right)=\frac{1}{n+1}+\ln\left(\frac{n}{n+1}\right).$$

Using this, we see that

$$\begin{aligned}(\gamma_n\gamma_{n+1}-\gamma^2)n &= (\gamma_n(\gamma_{n+1}-\gamma_n)+(\gamma_n+\gamma)(\gamma_n-\gamma))n \\ &= \frac{n}{n+1}*\gamma_n+\gamma_n*\ln\left(\frac{n}{n+1}\right)+(\gamma_n+\gamma)(\gamma_n-\gamma)n\end{aligned}$$

As $n\to\infty$, the first term tends to γ, the second term tends to $-\gamma$ (using L'Hopitals rule) and (using the result of part (i)) the last term approaches γ. Therefore,

$$\lim_{n\to\infty}(\gamma_n\gamma_{n+1}-\gamma^2)n = \gamma - \gamma+\gamma=\gamma.$$

Also solved by Florică Anastase, "Alexandru Odobescu High School, Lehliu-Gară, Călăraşi, Romania; Brian Bradie, Christopher Newport University, Newport News, VA; Toyesh Prakash Sharma (student) St. C.F Andrews School, Agra, India; Albert Stadler, Herrliberg, Switzerland; Seán Stewart, Bomaderry, NSW, Australia; and the proposers.

Kappa Mu Epsilon News

Edited by Cynthia Huffman, Past Historian, and Mark Hughes, Historian
Updated information as of January 2021

News of chapter activities and other noteworthy KME events should be sent to

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KAPPA MU EPSILON
Installation Report
Arkansas Gamma, Harding University
Searcy, Arkansas

The Arkansas Gamma Chapter of Kappa Mu Epsilon was installed at Harding University in Searcy, Arkansas. The ceremony was held via Zoom at 4 p.m. on April 27, 2021. The meeting was hosted by Professor Ronald Smith, who was installed as the Corresponding Secretary of the chapter. KME national president, Dr. Don Tosh, served as the installing officer.
Ten students and four faculty members were initiated as the charter members of the Arkansas Gamma Chapter. The four faculty are Professor of Mathematics, Dr. Deborah Duke, Associate Professors of Mathematics, Dr. Ronald Smith and Dr. James Burke, and Assistant Professor of Mathematics, Ms. Laurie Walker. The ten students are Camille Starck, Jason Hendrix, Mason LaFerney, Eric Traughber, Dylan Hendricks, Kyler Hensley, Hallie Martin, Anna Beth Neely, Alina Westbrook, and Laura Grace Ashburn.

Arkansas Gamma

Chapter News

AL Zeta – Birmingham-Southern College
Corresponding Secretary – Dr. Allie Ray; 619 Total Members; 8 New Members
New Initiates – Alexis Chambers, Caroline Creager, Claude Hall, Brianne Kendall, Hyun Lee, Christopher McClintock, Caroline Meffert, and David Minton.

AL Theta – Jacksonville State University
Chapter President – Hannah Davis; 303 Total Members; 14 New Members
Other Spring 2021 Officers: Hannah Boozer, Vice President; Marcus Shell, Secretary; Bronte Ray, Treasurer; Dr. David Dempsey, Corresponding Secretary; and Dr. Jason Cleveland, Faculty Sponsor.
The Alabama Theta chapter met monthly in online meetings (MS Teams) due to Covid-19 restrictions, but we made the best of it, finding new ways to play online games while seeing each other virtually. We held a virtual initiation ceremony on March 19, including the 14 new initiates, as well as the 13 who were to be initiated in Spring 2020, just as our campus closed. Several of us attended the KME National Convention, meeting together in a large classroom to watch the proceedings together, while interacting individually on laptops. We are proud of Dakota Heathcock, a Spring 2021 initiate, who was a student speaker for the convention. All in all, I am proud of the way our students found ways to stay engaged with each other,

and we look forward to a (hopefully) more normal Fall semester when we can meet in person, elect new officers, and plan new activities.

New Initiates – Emily Barfield, Rachel Bonner, Harley Abigail Carson, Pacey Mikaela Carson, Bailey Kyle Cochran, Kevin Michael Grubbs, Dakota Lee Heathcock, James Mitchell Jensen II, Jase Cameron Jones, Tajuddin Mwijage, Tyler Jase Nichols, Andrew Evan Parton, Stephen Christopher Thompson, and Christopher James White.

AR Beta – Henderson State University

Chapter President – Rachel Pepper; 56 Total Members; 8 New Members
Other Spring 2021 Officers: Sawyer Hardage, Vice President; Greg Jordan, Secretary; Madison Rushing, Treasurer; Fred Worth, Corresponding Secretary; and Carolyn Eoff, Faculty Sponsor.

Due to the pandemic, we did not have any meetings or new members in 2020. We initiated eight new members on April 27, via Zoom. We hope to get back to a more regular routine in the fall 2021 semester.

AR Gamma – Harding University

Corresponding Secretary – Ronald Smith; 15 Total Members; 14 New Members

Harding University started a new chapter off Kappa Mu Epsilon on April 27, 2021 in Searcy, Arkansas. Ten students and four faculty were installed at that time. We have one faculty member who is already a member of KME. Dr. Don Tosh conducted the ceremony via Zoom. All but 2 of those being initiated were present in person at Harding University. Several parents and guests were able to attend the ceremony.

New Initiates – Camille Starck, Jason Hendrix, Mason LaFerney, Eric Traughber, Dylan Hendricks, Kyler Hensley, Hallie Martin, Anna Beth Neely, Alina Westbrook, Laura Grace Ashburn, Ronald Smith, Deborah Duke, Laurie Walker, and James Burk.

CA Epsilon – California Baptist University

Corresponding Secretary – James Buchholz; 281 Total Members; 17 New Members

New Initiates – Crioni Cuenca, Chelsea Dones, Rebecca Doshier, Tracey Grisham, Victoria Grooters, Lauren Helt, Wesley Higbee, Haley Hillrich, Melissa Lebow, Keri Long, Katherine Luiten, Micah Means, Nathan Moretti, Isabella Rhodes, Hayley Thrapp, Tatiana Tim, and Sarah Valenzuela.

CT Beta – Eastern Connecticut State University

Corresponding Secretary and Faculty Sponsor – Dr. Mehdi Khorami; 534 Total Members; 19 New Members

New Initiates – Jafet Aparicio Santos, Caroline Banning, Samuel Bielaczyc, Sydney

Calvert, Micah Conway, John Fiester, Ray Flegert, James Hemeon, Bailey Holloway, Natalie Iacocca, Rossana Izaguirre, Tristan Larose, Arturo Martinez del Rello, Alexa O'Sullivan, Lauren Parks, Olivia Rovalino, Carly Sebastian, Irene Swenson, and Peyton Moore.

CT Gamma – Central Connecticut State University
Chapter President – William Caron; 78 Total Members; 8 New Members
Other Spring 2021 Officers: Bradley Doolgar, Vice President; Emma Johnson, Secretary; Micalyia Douglas, Treasurer; Leah Frazee, Corresponding Secretary; and Marian Anton, Faculty Sponsor.
We hosted a virtual initiation ceremony on Spring 2021 in light of the pandemic.

FL Gamma – Southeastern University
Corresponding Secretary – Dr. Berhane Ghaim; 67 Total Members; 4 New Members
New Initiates – Zhenlin Chen, Elizabeth Davison, Gabriella Gonzalez, and Kenneth Winters.

FL Delta – Embry-Riddle Aeronautical University
Corresponding Secretary – Dr. Sirani Mututhanthrige Perera; 84 Total Members; 19 New Members
New InitiatesNew Initiates – Justin Badalamento, Benjamin Banner, Gabrielle Bonowski, Matthew Caccomo, Lance Cross, Harrison Dinius, Samuel Feldman, Jorge Forgues, Bianca Ito, Cole Lanning, Haley Lowe, Mariah Marin, Alexandra Reese, Nikolaus Rentzke, Justin Scott, Sumit Shibib, Bradley Sweet, Christopher Swinford, and Harrison Turnage.

GA Beta – Georgia College & State University
Corresponding Secretary – Rodica Cazacu; 255 Total Members; 3 New Members
New Initiates – Samuel Eichel, Stephen Mosley, and James Pangia.

HI Alpha – Hawaii Pacific University
Corresponding Secretary – Tara Davis; 116 Total Members; 3 New Members
We initiated 3 new members this year. We also had one student and one faculty participant in the annual meeting. This is the first time to my knowledge that we have been able to participate in the meeting, so we appreciated the virtual format. We hope that with things opening up in fall we may resume more of our regular activities.
New Initiates - Rebekah Lee Cornish, Kristina Lara Kelly Bechthold, and Ryan Jake S.

Constantino.

IA Alpha – University of Northern Iowa
Chapter President – Matthew Adams; 1100 Total Members; 5 New Members
Other Spring 2021 Officers: Lauren Dierks, Vice President; Ashley DeWispelaere, Secretary; and Dr. Mark D. Ecker, Corresponding Secretary and Faculty Sponsor.
Due to the coronavirus pandemic, no KME meetings were held during the Spring 2021 semester at the University of Northern Iowa. However, five new members were initiated into KME this semester.

IA Epsilon – Central College
Chapter President – Thomas Spoehr; 47 Total Members; 6 New Members
Other Spring 2021 Officer: Dr. Russell E. Goodman, Corresponding Secretary and Faculty Sponsor.
New Initiates – Nolan Freymark, Kailee Meyer, Liam Mock, Katherine Nop, Thomas Spoehr, and Ryan Stallman.

IL Zeta – Dominican University
Corresponding Secretary – Mihaela Blanariu; 450 Total Members; 12 New Members
We had our KME Initiation Ceremony on April 15, 2021 where we initiated 12 new members.
New Initiates – Samantha Armijo, Isabel Batres, Madison Fette, Michael Guadarrama, Iyleah Hernandez, Isai Hipolito, Abigail Hosek, Brian Manuel, Alondra Montenegro, Saul Quintero, Daniela Salgado, and Henry Smith.

IN Beta – Butler University
Corresponding Secretary – Chris Wilson; 440 Total Members; 8 New Members
New Initiates – Kailee Callaghan, Katie Crowe, David Gregory, Olivia Herrmann, Lindsey Koiro, Dylan Laudenschlager, Katherine Olsen, and Kelly Ryan.

KS Beta – Emporia State University
Chapter President – Shelby Hettenbach; 1528 Total Members; 2 New Members
Other Spring 2021 Officers: Austin Crabtree, Vice President; Mackenzee Olson, Secretary; Ian Hull, Treasurer; Tom Mahoney, Corresponding Secretary; and Brian Hollenbeck, Faculty Sponsor.
New Initiates – Tommy Nguyen, Kyle Brinker.

KS Delta – Washburn University

Chapter President – Abigail Beliel; 822 Total Members; 12 New Members
Other Spring 2021 Officers: Kael Ecord, Vice President; Madison Henley, Secretary; Clare Bindley, Treasurer; and Sarah Cook, Corresponding Secretary and Faculty Sponsor.

On Tuesday, April 6, the Kansas Delta Chapter of Kappa Mu Epsilon initiated 12 new members. Due to the pandemic, the initiation was conducted virtually, which gave more parents and grandparents the opportunity to attend the ceremony. Although there were no scheduled meetings in the Spring term, three faculty and three students attended the National KME Convention.

New Initiates – Ajar Basnet, Clare Bindley, Katherine G. Cook, Viktoria Kassidy Donetz, Kael Ecord, Hailey S. Franson, Sara Johnson, Jarrod A. Saathoff, Micah Alan Skebo, Crystal M. Stohs, Dane Andrew Vanderbilt, and Alexander M. Yelland.

KS Eta – Sterling College

Corresponding Secretary – Amy Kosek; 19 Total Members; 9 New Members
Other Spring 2021 Officer: Pete Kosek, Faculty Sponsor.

New Initiates – Daylan Faltysek, Timothy Ferri, Blake Gladson, Jackson Lewis, Sydney Schmidt, Shara Shepard, Robert Stansbury, Nathan Wells, and Sara Welsch.

KY Gamma – Bellarmine University

Corresponding Secretary – Jen Miller; 24 Total Members; 7 New Members

New Initiates – Jack Clines, Megan Justice, Haley Mussler, Sheridan Payne, Rachael Rahe, Evan Seely, and Jeremiah Zonio.

MD Beta – McDaniel College

Corresponding Secretary – Spencer Hamblen; 441 Total Members; 5 New Members

New Initiates – Nicole Averinos, Stanley Gao, Clayton Herbst, Lan Mai, and Matthew O'Neill.

MD Delta – Frostburg State University

Chapter President – Bailey Brewer; 540 Total Members; 3 New Members
Other Spring 2021 Officers: Madison Green, Vice President; Ashley Armbruster, Secretary; Jay Collins, Treasurer; Mark Hughes, Corresponding Secretary and Faculty Sponsor; and Frank Barnet, Faculty Sponsor.

Maryland Delta Chapter held an initiation ceremony in March where we

welcomed three new members to the chapter. We met in person while observing Covid protocols. Faculty sponsor Mark Hughes gave a presentation entitled "Huygens, Curvature, and the Pendulum Clock". The chapter also participated in the National Convention which was held online in April. Six students and two faculty attended. We met together in a large conference room, brought in food, and enjoyed the convention! We offer our best wishes to Chapter President Bailey Brewer who will be attending graduate school at Duquesne University in the fall as well as to chapter member Andrew Kastner who will be attending graduate school at George Mason University.

New Initiates – Jessica Farrell, Jared Hose, and Brynn Lewis.

MI Beta – Central Michigan University

Chapter President – Austin Konkel; 1757 Total Members; 3 New Members Other Spring 2021 Officers: Emily Naegelin, Vice President; Kelsey Knoblock, Secretary; Robert Mason, Treasurer; Camilla Madacki, Public Relations; and Ben Salisbury, Corresponding Secretary and Faculty Sponsor.

1. On January 19th, KME held a Math Pictionary game night where members took turns drawing pictures of mathematical ideas while others guess.
2. On February 2nd, KME President Austin Konkel gave a virtual presentation titled the "Foundations of Math," in which he discussed the foundations of propositional logic and the axioms of Zermelo–Fraenkel set theory.
3. On February 16th, KME President Austin Konkel alongside KME Treasurer Robert Mason virtually presented on the "Map of Mathematics," in which several branches of pure and applied math were discussed along with some connections between these subfields of mathematics.
4. On March 2nd, KME Faculty Advisor Ben Salisbury gave an introduction to LaTeX. The talk was presented virtually through the group's Discord channel, and participants were encouraged to follow along using Overleaf.
5. On Marth 16th, KME held a Math Wikipedia Game which involves selecting two Wikipedia pages (relating to mathematics), then either working in teams to find an optimal route of clicking Wikipedia links between them or racing as individuals.
6. On April 6th, KME hosted a Math Kahoot which included math trivia questions.
7. On April 11th, KME held its initiation ceremony, where KME mem-

bers were invited to visit in person while social distancing, and guests could visit online. Professor Lisa DeMeyer gave a presentation regarding zero divisors and graphs.

8. Near the end of the semester, KME members helped to raise awareness to gather donations for the Special Olympics.

New Initiates – Kelsey Knoblock, Jeremy Proksch, and Jena Wazny.

MI Delta – Hillsdale College

Chapter President – Nicholas West; 389 Total Members; 26 New Members
Other Spring 2021 Officers: Benjamin Hufford, Vice President; Aaron Jacobson, Secretary; Abigail Price, Treasurer; and Kevin Gerstle, Corresponding Secretary and Faculty Sponsor.

We initiated 26 new members in April 2021. During the semester, we had two other events: a faculty panel on applying to graduate school and an event where we learned two different "math dances".

MI Epsilon – Kettering College A

Chapter President – Danny Boyle; 1070 Total Members; 102 New Members
Other Fall 2020 Officers: Makayla Carpenter (A) and Alley Broom (B), Vice President; Lindsey Malston (A) and Mary Allen (B), Secretary; Rebecca Abbott-Mccune (A) and Charles Cook (B), Treasurer; Dr. Boyan Dimitrov, Corresponding Secretary; and Matthew Causley, Faculty Sponsor.

We have had a winter term initiation for students in our Section A, which brought in over 30 new members. We also had a T-shirt design competition that semester, but final results were not made due to the changes in Institutional policy, such as I suppose were introduced elsewhere in the US. Here is the group picture of our Section A new initiated students and some faculty KME involved. Picture was taken by Hee Seok Nam, our newest Math Faculty colleague, and one of the most enthusiastic workers in the field of Actuarial teaching and research.

The Summer and Fall Terms started completely under the new conditions of partially f2f classes and most online teaching. Our faculty have had experienced this new technology of teaching and some intermission weeks to prepare it better. It seems that our Terms were gone partially and completely online. However, we have had some achievements, despite the challenges. We set this event up: The Friday night "Math Games and Olympiad Preview." Cherie Taylor organized a Zoom account for Friday, 11/13, 2020 at 6:00 - 8:00pm. Program agenda provided:

- 6:00-6:05 Introductions
- 6:05-6:10 Mathematical Magic
- 6:10-6:30 Proofs without words (includes breakout rooms with faculty supervision)
- 6:30 - 7:15 The game of Set (breakout rooms with faculty supervision)
- 7:15 - 7:55 What is an Olympiad problem? (Ruben)
- 7:55 - 8:00 Invitation to the Olympiad on Saturday and Q&A's

Everything was online. Here is the list of faculty who agreed to participate in the Math Olympiad week-end, specifically, who will attend the Friday session (Friday, November 13, 6:00-8:00pm, Hee-Seok Nam; David Henicken; Matthew O'Toole; Ruben Hayrapetyan; Ed Masha; Leszek Gawarecki. This event went with a great success. We have had the winners, thanks to the work of Professor Ruben Hayrapetyan: Olympiad 2020 Ranking

1. Gabriel Howald
2. Trevor Arcieri
3. Jack Taylor

Unfortunately, due to the absence of students on campus, we could not make the new initiation action at Kettering. Very sad news: We lost our longest working at the university colleague, Mathematics Professor Duane McKeachie. Here are some of the words of our president Dr. Robert K. McMahan about him: Professor McKeachie joined the university in 1949 when he was hired to teach Mathematics and Mechanical Engineering at General Motors Institute. Shortly after his 80th birthday, he retired after 57 active years of instilling a passion for mathematics to generations of students. He was a distinguished educator, loved by students and his colleagues. After he retired, he retained a strong connection to Kettering throughout his remaining years. In 2017, he again returned to campus for the dedication of the McKeachie-Brown Founder's Room, located in the Mathematics Department of the AB. The lounge, which has since become a dynamic gathering space for students, faculty and alumni was created to inspire collaboration and creativity. It is named to honor the legacy of two long-time faculty members who made an impact on the University – Pro-

fessor McKeachie and his longtime friend and colleague, Professor Robert Brown, another longtime member of Kettering's faculty from 1955 to 1995 who himself passed away in 2013. Words cannot express how much Duane has meant to this University and how much he will be missed.

MI Epsilon – Kettering College B

Chapter President – Robin Pelayo; 1100 Total Members; 40 New Members Other Winter 2021 Officers: Lindsey Malson; Rufus Kurapati, Treasurer; Dr. Boyan Dimitrov, Corresponding Secretary; and Matthew Causley, Faculty Sponsor

Note from Makayla Carpenter as President:

Being a member of KME has been an amazing opportunity as I have passed through the positions of Secretary and President for the past 2 years. I believe that those who would like to follow in these positions should have a love of mathematics, as well as have the drive to deal with the current circumstances and create new traditions here at Kettering. The new convention for the new initiated KME members was organized on March 17, 2021 as an ONLINE event by the Faculty Sponsor Matthew Causley, Assistant Professor at the Mathematics Department, It was kind of challenging new experience here and successfully implemented, gathering both sections A and B of Kettering University. Great work Matt, congratulations.

Last news is that your Kettering KME Corresponding Secretary, Dr. Boyan Dimitrov will retire after the end of this Academic year on June 30. 2021, after 23 years of service for KME. He got the Kettering Distinguished Research Award for the numerous articles published in 2020, and all together 193 in his about 52 years of research activity. Since this is his 4th Kettering major award (3 for research and 1 for teaching), by the rules at Kettering his portrait will be posted on the Kettering Wall of Fame.

MO Beta – University of Central Missouri

Corresponding Secretary – Dr. Rhonda McKee; 1541 Total Members; 4 New Members

New Initiates – Faith Franz, Kaitlynn Heussner, Annabelle Mandina, and Connor Stoehr.

MO Gamma – William Jewell College

Corresponding Secretary – Erin Martin; 763 Total Members; 3 New Members

New Initiates – Katie Bird, Abby Christensen, and Matthew Martel.

MO Theta – Evangel University

Chapter President – Peter Russell; 294 Total Members; 4 New Members

Other Spring 2021 Officers: Hannah Tower, Vice President; and Dianne Twigger, Corresponding Secretary and Faculty Sponsor.
We were able to hold in person initiations this spring, as well as three other socially distanced meetings.
New Initiates – Savannah Cosme, Annaliese Marsiglio, Ericsson McDermot, and Samantha Nicholls.

MO Nu – Columbia College
Corresponding Secretary and Faculty Sponsor – Kenny Felts; 89 Total Members
We have not had any activities or initiations since the pandemic began.

MO Omicron – Rockhurst University
Chapter President – Rachel Torre; 13 Total Members
Other Spring 2021 Officers: John Miller, Corresponding Secretary; and Zdenka Guadarrama, Faculty Sponsor.
We've struggled to recruit for the new chapter due to covid and not having any F2F meetings, we're hopeful that this will change in the fall.

MS Alpha – Mississippi University for Women
Corresponding Secretary – Dr. Joshua Hanes; 837 Total Members; 2 New Members

MS Gamma – University of Southern Mississippi
Corresponding Secretary – Zhifu Xie; 1132 Total Members; 7 New Members
New Initiates – Barbara Jones, Shailee Manandhar, Emma Mancevski, Minh Nguyen, Mark Parish, Megan Elizabeth Sickinger, and Jordan Matthew Wilson.

NC Epsilon – North Carolina Wesleyan College
Corresponding Secretary – Gail Stafford; 104 Total Members; 3 New Members
New Initiates – Latrell McDougald, Eugene Jeantou Okoko, and Summer Gabrielle Phillips.

NC Zeta – Catawba College
Chapter President – Amber White; 88 Total Members; 12 New Members
Other Spring 2021 Officers: Kaitlin Koons, Vice President; Allison Baker, Secretary; Dr. Doug Brown, Corresponding Secretary; and Dr. Katherine Baker, Faculty Sponsor.

NE Alpha – Wayne State College

Corresponding Secretary and Faculty Sponsor – Dr. Jennifer Langdon; 1047 Total Members; 8 New Members

New Initiates: Sam Anzalone, Trystan Bennett, Delaney Craig, Maddie Duffy, Hannah McGill, Samuel Morrill, Paytra TeBrink, and Alyssa Verros.

NE Beta – University of Nebraska Kearney

Corresponding Secretary and Faculty Sponsor – Dr. Katherine Kime; 929 Total Members; 7 New Members

In spring 2020, initiation was cancelled. About April this spring (2021), Trenton Chramosta, who had been accepted in spring 2020, suggested a virtual initiation. Trenton and possibly others visited classrooms to speak and give out initiation forms, as is our standard practice. Three initiates from spring 2020 (including Trenton) and four from spring 2021 were initiated during the last week of the spring semester. We had not had officer elections or meetings since spring 2020. In the reading of the initiation ceremony, existing member Amanda Jensen played the role of President, member Lena Janssen played the role of Vice-President and member Brooke Carlson played the role of Secretary. The virtual initiation was successful and it was extremely helpful to have electronic copies of the ceremony, and the Power Point slides, available from the KME website. Also, Dr. Kime and Amanda Jensen attended the virtual National Convention. Dr. Kime served as chair of the Nominating Committee. Finally, several KME members who graduated purchased our pink and grey honor cords, to wear over their robes.

NE Gamma – Chadron State College

Chapter President – Dylan Koretko; 534 Total Members; 7 New Members
Other Spring 2021 Officers: Kyeisha Garza, Vice President; Manou Mbombo, Secretary; Louis Christopher, Treasurer; and Gregory Moses, Corresponding Secretary and Faculty Sponsor.

New Initiates – Jordyn Beebe, Louis Christopher, Jung Colen, Katelyn Eldredge, Kyeisha Garza, Dylan Koretko, and Manou Mbombo.

NE Delta – Nebraska Wesleyan University

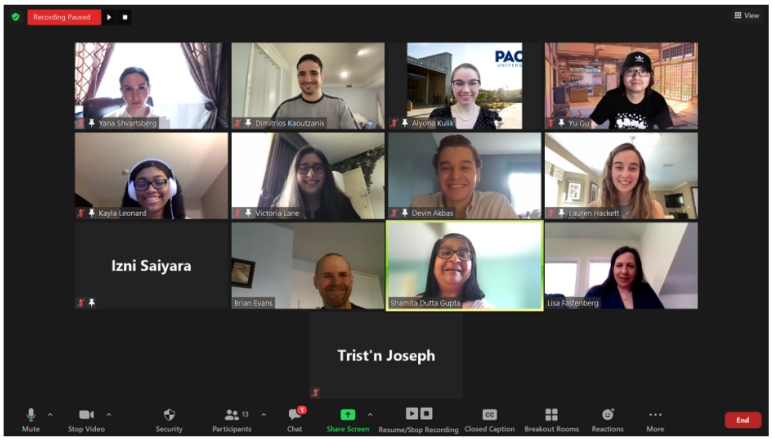
Chapter President – Enver Stading; 304 Total Members; 11 New Members
Other Spring 2021 Officers: Max Rademacher, Vice President; Amanda Hultgren, Secretary; Xander Schmit, Treasurer; and Melissa Erdmann, Corresponding Secretary and Faculty Sponsor.

We had a nice initiation followed by an outdoor picnic on April 29th, 2021. Also, this spring we had a fun virtual problem solving event

New Initiates – Kobe Hansen, Margaret Harris, Amanda Hultgren, Grace Moravec, Max Rademacher, Xander Schmit, Sydni Springer, Enver Stading, Benjamin Vyzourek, Ryan Wall, and Norbert Weijenberg.

NJ Epsilon – New Jersey City University
Corresponding Secretary and Faculty Sponsor – Dr. Alemtsehai Turasie; 151 Total Members
Other Spring 2021 Officer: Dr. Debananda Chakraborty, Faculty Sponsor.
Activities were suspended due to the pandemic.

NY Kappa – Pace University
Corresponding Secretary and Faculty Sponsor – Shamita Dutta Gupta; 408 Total Members; 11 New Members
NY Kappa held its Initiation Ceremony via Zoom on April 30, 2021. Guest speaker, Dr. Yana Shvartsberg, (Clinical Assistant Professor, Mathematics Department, NYC) gave a presentation entitled, "Towards the History of Differentiated Mathematics Education."



New Initiates – Devin Akbas, Brian Evans, Yu Gu, Lauren Hackett, Trist'n Joseph, Dimitrios Kaoutzania, Alyona Kulik, Victoria Lane, Kayla Leonard, Izni Saiyara, and Yana Shvartsberg.

NY Lambda – LIU Post, Long Island University
Corresponding Secretary and Faculty Sponsor – Dr. Corbett Redden; 463 Total Members; 19 New Members
On April 25, 2021, the NY Lambda Chapter at LIU Post held a virtual KME Initiation and Departmental Awards Ceremony on Zoom that was well attended by students, parents, faculty, and alumni.

NY Nu – Hartwick College
Chapter President – Dell Potts; 349 Total Members; 10 New Members
Other Spring 2021 Officers: Shane Lamparter, Vice President; Hannah Bochniak, Secretary; James Lukasik, Treasurer; and Gerald Hunsberger, Corresponding Secretary and Faculty Sponsor.
New Initiates – Hannah Bochniak, Justin Curreri, Ciara Davis, Neiva Fortes, Shane Lamparter, James Lukasik, James Macak, Lucus Mussi, Dell Potts, and Raymond Mess.

NY Pi – Mount Saint Mary College
Corresponding Secretary – Dr. Lee Fothergill; 135 Total Members; 6 New Members
New Initiates – Alyssa Marie Barbara, Lindsay E. Byer, Renée Michelle Hydo, Nasayah Israel, Michael Joseph Marchitelli, and Margaret F. Nuss.

OH Gamma – Baldwin Wallace University
Chapter President – Sydney Leither; 1027 Total Members; 15 New Members
Other Spring 2021 Officers: Jessica Blakley, Vice President; Harry Rouse, Secretary; and David Calvis, Corresponding Secretary and Faculty Sponsor.
The Ohio Gamma chapter conducted its Spring 2021 initiation ceremony online on April 18, 2021. We had not been able to hold an initiation the previous year.
New Initiates – Izzy Andrews, Nathan Bianco, Paul Blazek, Jeffery Borovac Jr., Kailin Breedlove, Erin Carr, Christina Chukri, Brianna Delewski, Carrie Eierman, Yana Kryvyak, Noa Mayer, Jared Rudge, Gregory Steinberger, Maddie Syrowski, and Lindsey Wodzisz.

OH Zeta – Muskingum University
Corresponding Secretary – Richard Daquila; 613 Total Members; 4 New Members
New Initiates – Madison Dodd, Carter Tram, Camryn Woodley, and John Yoder.

OH Eta – Ohio Northern University
Corresponding Secretary – Ryan Rahrig; 484 Total Members; 8 New Members
New Initiates – Aiwyn Brock, Timothy F. Dunn, Brady Michael Harmon, Greg Hassenpflug, Andrew LaSorsa, Holly Poremba, Logan Reichling, and Adia Welch.

OK Gamma – Southwestern Oklahoma State University
Corresponding Secretary and Faculty Sponsor – Dr. Tom McNamara; 616 Total Members; 4 New Members

PA Alpha – Westminster College

Corresponding Secretary – Natacha Fontes-Merz; 976 Total Members; 13 New Members

New Initiates – Michael Ace, Matthew Bollinger, Erynn Daubenmire, Montana Ferita, Alex Georgescu., Anna Grimenstein, Carinna Lapson, Owen Meilander, Jessica Nelson, Tsubomi Poley, Seth Schrader, Tanner Smith, and Emilee Spozarski.

PA Theta – Susquehanna University

Corresponding Secretary – Kenneth Brakke; 603 Total Members; 16 New Members

New Initiates – Aychin Aliyeva, Elizabeth A. Balas, Derek Borza, Ashton B. Buchanan, Arata Fujikawa, Nicolaus B. Gagliano, Samuel D. Hebert, Lindsey M. Hunter, Brad S. Knepp, Katherine A. Koch, Samantha Koitz, Jonathan Lewis, John Morris, Matthew J. Toven, Connor P. Van Zijl, and Tersis Wolle.

PA Mu – Saint Francis University

Chapter President – Teresa Reid; 494 Total Members; 11 New Members
Other Spring 2021 Officers: Kari Lagan, Vice President; Michael Gallagher, Secretary; Nathan Moore, Treasurer; and Dr. Brendon LaBuz, Corresponding Secretary and Faculty Sponsor.

The Pennsylvania Mu Chapter held their annual initiation ceremony in person (with virtual option) on Monday, March 8, 2021. In lieu of serving pies for pi day, we created a small booklet of pie recipes to distribute to the campus community. Two faculty members and two students attended the virtual national convention.

New Initiates – Jenna Beitel, Cara Bintrim, Regina Edgington, Morgan Kiesewetter, Tate Myers, Jared Ohler, Charles Pelsang, Isaac Repko, Kevin Rowland, Nathan Wolfe, and Madison Wright.

PA Xi – Cedar Crest College

Corresponding Secretary – Dr. Joshua Harrington; 124 Total Members; 5 New Members
Other Fall 2017 Officers: ETC

NEWS ITEMS

New Initiates – Christina Alberici, Khoula Jaber, Sydney Jones, Kelly Reid, and Solomaya Schwab.

PA Pi –Slippery Rock University

Chapter President – Spencer Kahley; 140 Total Members; 5 New Members
Other Spring 2021 Officers: Elise Grabner, Corresponding Secretary; and

Amanda Goodrick, Faculty Sponsor.
New Initiates – Dr. Jana Asher, Dr. Boris Brimkov, Dr. Jeffrey Musyt, Spencer Kahley, and Michael Zirpoli.

PA Rho – Thiel College
Chapter President – Macy Siefert; 139 Total Members; 3 New Members
Other Spring 2021 Officers: Emily Groves, Vice President; Cassie Brown, Secretary; Kara Baumgardner, Treasurer; Dr. Russell Richins, Corresponding Secretary; and Dr. Jie Wu, Faculty Sponsor.
Unfortunately, due to COVID restrictions and other complications, we were limited to only a virtual initiation ceremony this spring.
New Initiates – Madison Hawthorne, Jeremy Groves, and Camryn Sankey.

PA Sigma – Lycoming College
Corresponding Secretary – Andrew Brandon; 142 Total Members; 6 New Members
Other Spring 2021 Officer: Christopher Reed, Faculty Sponsor.
We hope to hold elections in fall 2021. Due to covid-19, in-person meetings were severely limited/restricted on our campus during the 2020/21 academic year.

RI Beta – Bryant University
Chapter President – Christopher Ethier; 188 Total Members; 17 New Members
Other Spring 2021 Officers: Constance Tang, Vice President; Alexandra Sherman, Secretary; Liam Mahler, Treasurer; Prof. John Quinn, Corresponding Secretary; and Prof. Alan Olinsky, Faculty Sponsor.
For the 43rd Biennial Convention in April 2021:

- Professor Son Nguyen, presented a KME Workshop entitled: "Building an image recognition model from scratch." There were also two student presentations:
- Luis Sanchez Mercedes, Title: "Cryptonomics: An Empirical Analysis of the Internal and External Performance of Blockchain Networks to the Determine Their Changes in Price."
- Christopher McHeffey, Title: "Improve the accuracy of Tuberculosis Detection from Chest X-ray using Transfer Learning."

TN Gamma – Union University
Chapter President – Emory Craft; 517 Total Members; 12 New Members
Other Spring 2021 Officers: John Mayer, Vice President; Davina Norris, Secretary and Treasurer; Michael Drury, Webmaster and Historian; Bryan

Dawson, Corresponding Secretary; and Matt Lunsford, Faculty Sponsor.
TN Gamma held its first initiation since the beginning of the COVID-19 pandemic on April 12, 2021. Instead of our usual banquet at a local restaurant, we used a meeting room on campus (Coburn) and the only food was homemade cupcakes distributed at the end of the meeting for consumption elsewhere. In addition to the initiation ceremony, we held a socially distanced group activity where teams competed to be the first to solve a cryptographic murder mystery that our faculty sponsor found on the internet. A good time was had by all. Officers for the following year were elected.
New Initiates – Caleb Atkins, Abigal Branson, Rylee Iorio, Taylor Overcast, Lisa Reed, Jacob Roessler, Shawn Ross, Joya Schrock, Gavyn Thorne, Benjamin Trainor, Braden Watkins, and Whitney Wheeler.

TN Zeta – Lee University
Corresponding Secretary – Caroline Maher-Boulis; 77 Total Members; 5 New Members
New Initiates – Emma Chase, Lauren Lester, Jon Rowland, Audrey Royer, and Joshua Schlabach.

TX Lambda – Trinity University
Corresponding Secretary – Dr. Hoa Nguyen; 307 Total Members; 17 New Members

VA Gamma – Liberty University
Corresponding Secretary – Timothy Van Voorhis; 434 Total Members; 17 New Members
New Initiates – Jerome C. Arrington, Liam Barham, Eli R. Best, Christopher C. Cockes, Rebecca DeLee, Simeon Eberz, Kayla Gentile, Nathanael Gentry, Bryce Jones, Jacob Litzau, Noah Midkiff, Hallie Morris, Paul Savas, Arthur N. Tanyel, Lydia J. Wu, Yucheng Zhong, and Caroline Zielke.

WI Alpha – Mount Mary University
Chapter President – Melissa Golo; 306 Total Members; 2 New Members
Other Spring 2021 Officers: Hannah Ashbach, Vice President and Secretary; Melissa Golo, Treasurer; Sherrie Serros, Corresponding Sectretary; and Jeremy Edison, Faculty Sponsor.
At the virtual spring initiation ceremony, alumna Jeannette Ingabire presented *Validating Fast Magnetogenetics Technology in the Brain* and discussed graduate school applications.
New Initiates – Marissa Heraly and Mary Parlier.

WV Alpha – Bethany College

Chapter President – Joseph A. Makowski; 194 Total Members; 10 New Members

Other Spring 2021 Officers: Ethan J. Young, Vice President; Lena A. Grogan, Secretary and Treasurer; and Dr. Adam C. Fletcher, Corresponding Secretary and Faculty Sponsor.

West Virginia Alpha chapter, like so many other chapters across the country, has had an interesting semester. The College remained predominately on-campus throughout the spring semester, with strict COVID protocols in place throughout. The local and national restrictions canceled or postponed a number of the chapter's usual activities, like the annual Math/Science Day competition for local high school students and a number of professional meetings. West Virginia Alpha chapter and our local Mathematics and Computer Science Club did, however, have a handful of hybrid game nights and computer gaming competitions. The chapter had a banner year for the (hybrid) initiation ceremony, initiating ten new members, and had four students and a faculty member attend the hybrid Biennial Convention. Additionally, the chapter assisted with the Upsilon Pi Epsilon (international computing sciences honor society) initiation ceremony (also hybrid) for four new members. The chapter looks forward to a fall semester when travel and competition will again be possible, and new opportunities to "appreciate the beauty of mathematics" will be available!

New Initiates – Haley Shaw Bensinger, Antonio G. Caputo, Benjamin James Fedoush, Patrick M. Gleason, Marcus Alexander Kozloff, Amanda Marie Reynolds, Lauren E. Starr, Jacob Calvin Thornburg, Courtney E. Walker, and Cullen Jeffrey Wise.

*A*ctive Chapters of Kappa Mu Epsilon

Listed by date of installation

Chapter	Location	Installation Date
OK Alpha	Northeastern State University, Tahlequah	18 Apr 1931
IA Alpha	University of Northern Iowa, Cedar Falls	27 May 1931
KS Alpha	Pittsburg State University, Pittsburg	30 Jan 1932
MO Alpha	Missouri State University, Springfield	20 May 1932
MS Alpha	Mississippi University for Women, Columbus	30 May 1932
NE Alpha	Wayne State College, Wayne	17 Jan 1933
KS Beta	Emporia State University, Emporia	12 May 1934
AL Alpha	Athens State University, Athens	5 Mar 1935
NM Alpha	University of New Mexico, Albuquerque	28 Mar 1935
IL Beta	Eastern Illinois University, Charleston	11 Apr 1935
AL Beta	University of North Alabama, Florence	20 May 1935
AL Gamma	University of Montevallo, Montevallo	24 Apr 1937
OH Alpha	Bowling Green State University, Bowling Green	24 Apr 1937
MI Alpha	Albion College, Albion	29 May 1937
MO Beta	University of Central Missouri, Warrensburg	10 Jun 1938
TX Alpha	Texas Tech University, Lubbock	10 May 1940
KS Gamma	Benedictine College, Atchison	26 May 1940
IA Beta	Drake University, Des Moines	27 May 1940
TN Alpha	Tennessee Technological University, Cookeville	5 Jun 1941
MI Beta	Central Michigan University, Mount Pleasant	25 Apr 1942
NJ Beta	Montclair State University, Upper Montclair	21 Apr 1944
IL Delta	University of St. Francis, Joliet	21 May 1945
KS Delta	Washburn University, Topeka	29 Mar 1947
MO Gamma	William Jewell College, Liberty	7 May 1947
TX Gamma	Texas Woman's University, Denton	7 May 1947
WI Alpha	Mount Mary College, Milwaukee	11 May 1947
OH Gamma	Baldwin-Wallace College, Berea	6 Jun 1947
MO Epsilon	Central Methodist College, Fayette	18 May 1949
MS Gamma	University of Southern Mississippi, Hattiesburg	21 May 1949
IN Alpha	Manchester College, North Manchester	16 May 1950
PA Alpha	Westminster College, New Wilmington	17 May 1950
IN Beta	Butler University, Indianapolis	16 May 1952
KS Epsilon	Fort Hays State University, Hays	6 Dec 1952
PA Beta	LaSalle University, Philadelphia	19 May 1953
VA Alpha	Virginia State University, Petersburg	29 Jan 1955
IN Gamma	Anderson University, Anderson	5 Apr 1957
CA Gamma	California Polytechnic State University, San Luis Obispo	23 May 1958
TN Beta	East Tennessee State University, Johnson City	22 May 1959
PA Gamma	Waynesburg College, Waynesburg	23 May 1959
VA Beta	Radford University, Radford	12 Nov 1959
NE Beta	University of Nebraska—Kearney, Kearney	11 Dec 1959
IN Delta	University of Evansville, Evansville	27 May 1960
OH Epsilon	Marietta College, Marietta	29 Oct 1960
MO Zeta	University of Missouri—Rolla, Rolla	19 May 1961
NE Gamma	Chadron State College, Chadron	19 May 1962
MD Alpha	College of Notre Dame of Maryland, Baltimore	22 May 1963
CA Delta	California State Polytechnic University, Pomona	5 Nov 1964
PA Delta	Marywood University, Scranton	8 Nov 1964
PA Epsilon	Kutztown University of Pennsylvania, Kutztown	3 Apr 1965
AL Epsilon	Huntingdon College, Montgomery	15 Apr 1965
PA Zeta	Indiana University of Pennsylvania, Indiana	6 May 1965
TN Gamma	Union University, Jackson	24 May 1965
IA Gamma	Morningside College, Sioux City	25 May 1965
MD Beta	McDaniel College, Westminster	30 May 1965
IL Zeta	Dominican University, River Forest	26 Feb 1967
SC Beta	South Carolina State College, Orangeburg	6 May 1967
PA Eta	Grove City College, Grove City	13 May 1967
NY Eta	Niagara University, Niagara University	18 May 1968
MA Alpha	Assumption College, Worcester	19 Nov 1968
MO Eta	Truman State University, Kirksville	7 Dec 1968
IL Eta	Western Illinois University, Macomb	9 May 1969
OH Zeta	Muskingum College, New Concord	17 May 1969
PA Theta	Susquehanna University, Selinsgrove	26 May 1969
PA Iota	Shippensburg University of Pennsylvania, Shippensburg	1 Nov 1969
MS Delta	William Carey College, Hattiesburg	17 Dec 1970
MO Theta	Evangel University, Springfield	12 Jan 1971
PA Kappa	Holy Family College, Philadelphia	23 Jan 1971
CO Beta	Colorado School of Mines, Golden	4 Mar 1971
KY Alpha	Eastern Kentucky University, Richmond	27 Mar 1971
TN Delta	Carson-Newman College, Jefferson City	15 May 1971
NY Iota	Wagner College, Staten Island	19 May 1971
SC Gamma	Winthrop University, Rock Hill	3 Nov 1972
IA Delta	Wartburg College, Waverly	6 Apr 1973
PA Lambda	Bloomsburg University of Pennsylvania, Bloomsburg	17 Oct 1973
OK Gamma	Southwestern Oklahoma State University, Weatherford	1 May 1973

NY Kappa	Pace University, New York	24 Apr 1974
TX Eta	Hardin-Simmons University, Abilene	3 May 1975
MO Iota	Missouri Southern State University, Joplin	8 May 1975
GA Alpha	State University of West Georgia, Carrollton	21 May 1975
WV Alpha	Bethany College, Bethany	21 May 1975
FL Beta	Florida Southern College, Lakeland	31 Oct 1976
WI Gamma	University of Wisconsin—Eau Claire, Eau Claire	4 Feb 1978
MD Delta	Frostburg State University, Frostburg	17 Sep 1978
IL Theta	Benedictine University, Lisle	18 May 1979
PA Mu	St. Francis University, Loretto	14 Sep 1979
AL Zeta	Birmingham-Southern College, Birmingham	18 Feb 1981
CT Beta	Eastern Connecticut State University, Willimantic	2 May 1981
NY Lambda	C.W. Post Campus of Long Island University, Brookville	2 May 1983
MO Kappa	Drury University, Springfield	30 Nov 1984
CO Gamma	Fort Lewis College, Durango	29 Mar 1985
NE Delta	Nebraska Wesleyan University, Lincoln	18 Apr 1986
TX Iota	McMurry University, Abilene	25 Apr 1987
PA Nu	Ursinus College, Collegeville	28 Apr 1987
VA Gamma	Liberty University, Lynchburg	30 Apr 1987
NY Mu	St. Thomas Aquinas College, Sparkill	14 May 1987
OH Eta	Ohio Northern University, Ada	15 Dec 1987
OK Delta	Oral Roberts University, Tulsa	10 Apr 1990
CO Delta	Mesa State College, Grand Junction	27 Apr 1990
PA Xi	Cedar Crest College, Allentown	30 Oct 1990
MO Lambda	Missouri Western State College, St. Joseph	10 Feb 1991
TX Kappa	University of Mary Hardin-Baylor, Belton	21 Feb 1991
SC Delta	Erskine College, Due West	28 Apr 1991
NY Nu	Hartwick College, Oneonta	14 May 1992
NH Alpha	Keene State College, Keene	16 Feb 1993
LA Gamma	Northwestern State University, Natchitoches	24 Mar 1993
KY Beta	Cumberland College, Williamsburg	3 May 1993
MS Epsilon	Delta State University, Cleveland	19 Nov 1994
PA Omicron	University of Pittsburgh at Johnstown, Johnstown	10 Apr 1997
MI Delta	Hillsdale College, Hillsdale	30 Apr 1997
MI Epsilon	Kettering University, Flint	28 Mar 1998
MO Mu	Harris-Stowe College, St. Louis	25 Apr 1998
GA Beta	Georgia College and State University, Milledgeville	25 Apr 1998
AL Eta	University of West Alabama, Livingston	4 May 1998
PA Pi	Slippery Rock University, Slippery Rock	19 Apr 1999
TX Lambda	Trinity University, San Antonio	22 Nov 1999
GA Gamma	Piedmont College, Demorest	7 Apr 2000
LA Delta	University of Louisiana, Monroe	11 Feb 2001
GA Delta	Berry College, Mount Berry	21 Apr 2001
TX Mu	Schreiner University, Kerrville	28 Apr 2001
CA Epsilon	California Baptist University, Riverside	21 Apr 2003
PA Rho	Thiel College, Greenville	13 Feb 2004
VA Delta	Marymount University, Arlington	26 Mar 2004
NY Omicron	St. Joseph's College, Patchogue	1 May 2004
IL Iota	Lewis University, Romeoville	26 Feb 2005
WV Beta	Wheeling Jesuit University, Wheeling	11 Mar 2005
SC Epsilon	Francis Marion University, Florence	18 Mar 2005
PA Sigma	Lycoming College, Williamsport	1 Apr 2005
MO Nu	Columbia College, Columbia	29 Apr 2005
MD Epsilon	Stevenson University, Stevenson	3 Dec 2005
NJ Delta	Centenary College, Hackettstown	1 Dec 2006
NY Pi	Mount Saint Mary College, Newburgh	20 Mar 2007
OK Epsilon	Oklahoma Christian University, Oklahoma City	20 Apr 2007
HA Alpha	Hawaii Pacific University, Waipahu	22 Oct 2007
NC Epsilon	North Carolina Wesleyan College, Rocky Mount	24 Mar 2008
NY Rho	Molloy College, Rockville Center	21 Apr 2009
NC Zeta	Catawba College, Salisbury	17 Sep 2009
RI Alpha	Roger Williams University, Bristol	13 Nov 2009
NJ Epsilon	New Jersey City University, Jersey City	22 Feb 2010
NC Eta	Johnson C. Smith University, Charlotte	18 Mar 2010
AL Theta	Jacksonville State University, Jacksonville	29 Mar 2010
GA Epsilon	Wesleyan College, Macon	30 Mar 2010
FL Gamma	Southeastern University, Lakeland	31 Mar 2010
MA Beta	Stonehill College, Easton	8 Apr 2011
AR Beta	Henderson State University, Arkadelphia	10 Oct 2011
PA Tau	DeSales University, Center Valley	29 Apr 2012
TN Zeta	Lee University, Cleveland	5 Nov 2012
RI Beta	Bryant University, Smithfield	3 Apr 2013
SD Beta	Black Hills State University, Spearfish	20 Sept 2013
FL Delta	Embry-Riddle Aeronautical University, Daytona Beach	22 Apr 2014
IA Epsilon	Central College, Pella	30 Apr 2014
CA Eta	Fresno Pacific University, Fresno	24 Mar 2015
OH Theta	Capital University, Bexley	24 Apr 2015
GA Zeta	Georgia Gwinnett College, Lawrenceville	28 Apr 2015
MO Xi	William Woods University, Fulton	17 Feb 2016
IL Kappa	Aurora University, Aurora	3 May 2016
GA Eta	Atlanta Metropolitan University, Atlanta	1 Jan 2017
CT Gamma	Central Connecticut University, New Britan	24 Mar 2017
KS Eta	Sterling College, Sterling	30 Nov 2017
NY Sigma	College of Mount Saint Vincent, The Bronx	4 Apr 2018
PA Upsilon	Seton Hill University, Greensburg	5 May 2018

KY Gamma	Bellarmine University, Louisville	23 Apr 2019
MO Omicron	Rockhurst University, Kansas City	13 Nov 2020
AK Gamma	Harding University, Searcy	27 Apr 2021