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A Generalization of the Collatz Conjecture

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Abstract

The Collatz Conjecture is one of the most famous (or perhaps infamous) unsolved problems in modern mathematics. By comparison with other major unsolved problems like the Riemann Hypothesis or the Birch and Swinnerton-Dyer Conjecture, the Collatz Conjecture is remarkably easy to state. Begin with a positive integer. If the integer is even, divide by 2. If not, multiply by 3 and add 1 (hence an alternative name, the "$3n+1$ conjecture"). Repetition of this process will generate an infinite sequence—and a different sequence for each starting point. In practice these sequences seem to always lead to an infinite repetition of the integers 4, 2, 1, and the Collatz Conjecture claims that this is in fact always the case. Despite the simplicity of the conjecture, no proof has yet been found. We unfortunately do not have a proof to offer here. However, we explore the possibility of generalizing the Collatz Conjecture, with a focus on this question: is there anything special about the numbers 3 and 1 from the original, or could we have, for example, multiplied by 5 and added 8 to get similar results? We find that some alternative versions of the conjecture lead to sequences diverging to infinity, but a subset seem to have similar behavior to the original. We offer here a set of theorems classifying these cases.

Introduction

The Collatz Conjecture, also known as the $3n+1$ problem, is a famous example of the sort of mathematical question that is easy to understand but very difficult to answer. We begin with a very simple function $f : \mathbb{Z}^+ \to \mathbb{Z}^+$ defined by

$$f(n) = \begin{cases} 3n+1 & \text{if } n \text{ is odd} \\ n/2 & \text{if } n \text{ is even} \end{cases}$$

We then apply the function f iteratively to some starting integer n to generate a sequence. For example, if we begin with the value of $n = 10$, then

$$
\begin{aligned}
f(10) &= 10/2 = 5 \\
f(5) &= 3 \cdot 5 + 1 = 16 \\
f(16) &= 16/2 = 8 \\
f(8) &= 8/2 = 4 \\
f(4) &= 4/2 = 2 \\
f(2) &= 2/2 = 1
\end{aligned}
$$

Thus, $f(f(f(f(f(f(10)))))) = f^{(6)}(10) = 1$. Then since 1 is odd, $f(1) = 3 \cdot 1 + 1 = 4$, and the cycle of values 4, 2, 1 repeats forever. (Note that the superscript (6) represents the number of applications of the function, not the 6^{th} derivative as in many calculus texts). A little numerical experimentation shows that this behavior is common—that is, that many starting values generate sequences ending in an infinite repetition of the three numbers 4, 2, 1—though sometimes it takes a very long time to reach the first such repetition. This initially seems a bit surprising: since the function more than doubles its odd inputs, and merely halves its even inputs, one might expect the resulting sequence to grow without bound in most cases. However, even and odd inputs do not occur with equal frequency: an odd entry n in the sequence will always be followed by an even entry $3n+1$, whereas an even entry may be followed by more even entries. Hence there may be on average more than one division by 2 corresponding to each (approximate) tripling. As a result, sequences generated by f frequently decrease until they end with 4, 2, 1 repeating forever, as noted above.

According to the Collatz Conjecture, this will always happen eventually—regardless of the value of the starting integer n. Stated more formally:

Collatz Conjecture - *Let $f : \mathbb{Z}^+ \to \mathbb{Z}^+$ be defined by*

$$
f(n) = \begin{cases} 3n+1 & \text{if } n \text{ is odd} \\ n/2 & \text{if } n \text{ is even} \end{cases}
$$

Then for any positive integer n there exists some nonnegative integer k such that $f^{(k)}(n) = 1$.

This is a very simple conjecture to state—with less notation, it could even be made into a sort of game for elementary school students practicing basic arithmetic operations—but it has resisted attempts by several generations of mathematicians to find a proof. "Experienced mathematicians warn up-and-comers to stay away from the Collatz conjecture. It's a siren song, they say: Fall under its trance and you may never do meaningful work again" (Harnett 2019). It is "considered a textbook example of wild deterministic chaos, so much so that Paul Erdos famously declared 'mathematics may not be ready for such problems'" (Aberkane 2017).

In the absence of an actual proof, some progress can be made by rephrasing the conjecture in alternative but equivalent forms, in the hope that tools from other branches of mathematics may eventually suggest a solution. Jeffrey Lagarias, one of the most prolific mathematicians in the study of this problem, developed several such alternative formulations using—among other things—tools from graph theory (Lagarias 1985). Jan Kleinnijenhuis and collaborators looked at the Collatz function from a view of binary trees and cycles, finding automorphisms between cotrees and showing subsets of the natural numbers are contained in these cotrees within the large binary tree of the natural numbers (Kleinnijenhuis 2022). Carnielli generalized the Collatz conjecture so that the original modulus became some natural number d, with all n congruent to zero modulo d were divided by d and the other natural numbers were mapped to a number divisible by d. From there, the goal was to show that there were finitely many finite cycles, showing no natural number expands infinitely (Carnielli 2015).

Toward a Generalization of the Collatz Conjecture

We can also consider the following generalized version of the function f from the Collatz Conjecture: since there is nothing necessarily special about the integers 3 and 1, we can replace $3n+1$ in the definition of f with $an+b$, where a and b are any two natural numbers (see also EMN 2020).

Definition. *A Collatz rule is a function $f : \mathbb{Z}^+ \to \mathbb{Z}^+$ of the form:*

$$f(n) = \begin{cases} an+b & \text{if } n \text{ is odd} \\ n/2 & \text{if } n \text{ is even} \end{cases}$$

where $a,\ b \in \mathbb{Z}^+$. The Collatz rule with $a = 3$ and $b = 1$ is the standard Collatz rule. The Collatz sequence generated by the Collatz rule f and the initial value $n \in \mathbb{Z}^+$ is the sequence $\left\{f^{(k)}(n)\right\}_{k=0}^{\infty}$.

We would like to propose a generalized version of the Collatz conjecture using these new functions, but before we can pose a plausible conjecture, we need to address some subtleties brought about by these new functions.

The first difference between our new Collatz rules and the original is that sequences generated by such rules will not necessarily end in a repetition of 4, 2, 1 as before. For example, the rule with $an+b = 5n+1$ can end in the repetition of the more complicated: 1, 6, 3, 16, 8, 4, 2, 1. Furthermore, some such sequences do end with repetition, but do not reach 1. For example, the rule with $an+b = 3n+3$ can lead to a repetition of 3, 12, 6, 3. We might still conjecture that all such sequences (or perhaps those satisfying some conditions) still always end in an infinite repetition of some finite sequence. However, we will need to define the rank of a natural number n not as the number of iterations needed to reach 1, but the number of iterations needed to reach the first integer in one of these repeating cycles (for example, the first 3 above).

Definition. *Let f be a Collatz rule and let n be a natural number. If there exists a nonnegative integer k such that* $f^{(k)}(n) = f^{(k+r)}(n)$ *for some integer* $r > 0$*, then the smallest such k is the rank of n. If no such k exists, the rank of n is infinite.*

Note that the definition does not explicitly say that the repetition continues forever, but it does not need to. A good exercise (which we do not include here) would be to show that if some integer is ever repeated after r iterations, then it is repeated again after $2r$ iterations and $3r$ iterations and so on, though the same is not necessarily true in reverse.

A second difference between the new rules and the original is that some of the new rules definitely do not result in cyclic behavior as the original is conjectured to. In order to pose a plausible conjecture, we need to begin by identifying and excluding these obvious counterexamples. We could then phrase the generalized Collatz conjecture by claiming that for any Collatz rule (with perhaps some restrictions) every natural number will have finite rank.

Before we determine what restrictions on f would be necessary, it is also worth noting that some Collatz rules are essentially alternative formulations of others. For example, the rule with $an+b = 6n+2$ is essentially the same as the standard Collatz rule; since every application of $6n+2$ is followed by a division by 2, the result is simply to compute $3n+1$ in two steps rather than one. This rule will technically generate a different sequence than the original, but the sequence generated by the original Collatz rule is a subsequence of this one. Similarly, the rule with $an+b = 15n+5$ will generate a sequence whose terms are each five times as large as the corresponding terms of the standard Collatz rule. It will therefore be useful to have a terminology for comparing similar Collatz rules.

Definition. *A reduced Collatz Rule is found by taking a Collatz Rule and removing all common divisors of a and b. A Collatz family is the set of all Collatz rules with the same reduced form. A Collatz Rule is said to have "parity" if a and b are both even or both odd. We may sometimes distinguish between even-even parity and odd-odd parity. A Collatz Rule is said to have "complete parity" if the a and b of its reduced form are both even or both odd.*

Complete parity is of course the stronger condition. (Another easy exercise would be to check that Collatz families are in fact equivalence classes and therefore form a partition of the space of all Collatz rules.)

Parity Expansion Theorem (PET)

We now need to ask what form a generalized Collatz conjecture could reasonably take now that we are using the modified forms of f described above. When we used the standard Collatz rule, it was conjectured—but not proven—that all Collatz sequences eventually lead to one—that is, all natural numbers have finite rank. As it turns out, this is not the case for all Collatz rules. For example, consider the rule:

$$f(n) = \begin{cases} 3n+2 & \text{if } n \text{ is odd} \\ n/2 & \text{if } n \text{ is even} \end{cases}$$

If n is odd, then $f(n) = 3n+2$ is also odd, but larger, so that the second part of the rule is never used. Thus, for odd n the sequence $f^{(k)}(n)$ is monotone increasing and diverges to infinity. If n is even, then eventually successive divisions by 2 will produce an odd integer (possibly as small as 1) and after that, all subsequent iterations will produce once again a monotone increasing sequence of odd integers that diverges to infinity.

From this emerges a pattern, in which if $(a+b) \pmod 2$ is an odd number, the Collatz Rule will continually expand. This can be formalized in the theorem below:

Parity Expansion Theorem. *Let f be a Collatz Rule of the form*

$$f(n) = \begin{cases} an+b & \text{if } n \text{ is odd} \\ n/2 & \text{if } n \text{ is even} \end{cases}$$

Then if $(a+b) \equiv 1 \pmod 2$, *the sequence* $f^{(k)}(n)$ *will diverge to infinity for all natural numbers n.*

Proof. Let a and b be natural numbers with This implies either a is even and b is odd, or b is even and a is odd.

Case 1: Let a be even and b be odd. Consider n in the natural numbers.

Case 1a: Let n be odd. Then by the Collatz Rule, $f(n) = an+b$. If n is odd, then an even number multiplied by an odd number is odd, plus an even number is odd. So, if n is odd, then $f(n)$ is also odd. Note: Since a and b are both part of the natural numbers, clearly $f(n)$ is greater than n. Since n was an arbitrary odd natural number, $f(n)$ is greater than n for all odd natural numbers. Thus, $f(n)$ will diverge to infinity.

Case 1b: Let n be even. By Prime Factorization, n can be rewritten as $2^k {p_1}^{a_1} {p_2}^{a_2} \cdots {p_m}^{a_m}$ for some k and m in the natural numbers, where p_i is a unique prime greater than two and a is an integer greater than or equal to zero. By the Collatz Rule, $f(n) = n/2$, which implies that

$$f\left(2^k {p_1}^{a_1} {p_2}^{a_2} \cdots {p_m}^{a_m}\right) = \frac{\left(2^k {p_1}^{a_1} {p_2}^{a_2} \cdots {p_m}^{a_m}\right)}{2} = 2^{k-1} {p_1}^{a_1} {p_2}^{a_2} \cdots {p_m}^{a_m}.$$

Since k is finite, $f^{(k)}(n) = {p_1}^{a_1} {p_2}^{a_2} \cdots {p_m}^{a_m}$. Since p_i is prime and greater than 2 for all i between 1 and m, ${p_1}^{a_1} {p_2}^{a_2} \cdots {p_m}^{a_m}$ is a product of odd numbers and thus, odd. By Case 1a, ${p_1}^{a_1} {p_2}^{a_2} \cdots {p_m}^{a_m}$ will diverge to infinity.

Therefore, by cases 1a and 1b, if a is even and b is odd, $f(n)$ will diverge to infinity.

Case 2: Let a be odd and b be even. Consider n in the natural numbers.

Case 2a: Let n be odd. Then by the Collatz Rule, $f(n) = an + b$. If n is odd, then an odd number times and odd number is odd, plus an even number is odd. So if n is odd, then $f(n)$ is odd. By similar reasoning to Case 1a, $f(n)$ will diverge to infinity.

Case 2b: Let n be even. Then by similar reasoning to Case 1b, $f^{(k)}(n) = {p_1}^{a_1}{p_2}^{a_2}\cdots{p_m}^{a_m}$ for some unique primes p_i. Then by Case 2a, ${p_1}^{a_1}{p_2}^{a_2}\cdots{p_m}^{a_m}$ will diverge to infinity.

Therefore, by cases 2a and 2b, if a is odd and b is even, $f(n)$ will diverge to infinity.

Therefore, by Case 1 and Case 2, if $(a+b) \equiv 1 \pmod 2$, then $f(n)$ will diverge to infinity. ■

Thus, if a Collatz Rule does not have parity, the resulting sequence will diverge to infinity. However, the inverse of this statement is not true. An immediate corollary is that if a Collatz Rule's reduced form does not have parity, then the sequences generated by that Collatz Rule itself and its reduced form will both diverge to infinity. Stated more formally:

Corollary 1. *Let f be a Collatz Rule of the form*

$$f(n) = \begin{cases} an+b & \text{if } n \text{ is odd} \\ n/2 & \text{if } n \text{ is even} \end{cases}$$

Let d=gcd(a,b). Let g be the reduced form of the Collatz rule f:

$$f(n) = \begin{cases} \left(\frac{a}{d}\right)n + \left(\frac{b}{d}\right) & \text{if } n \text{ is odd} \\ n/2 & \text{if } n \text{ is even} \end{cases}$$

Then if $\left(\frac{a}{d} + \frac{b}{d}\right) \equiv 1 \pmod 2$, *the sequences* $f^{(k)}(n)$ *and* $g^{(k)}(n)$ *will both diverge to infinity for all natural numbers n.*

Example.

Let

$$f(n) = \begin{cases} 6n+8 & \text{if } n \text{ is odd} \\ n/2 & \text{if } n \text{ is even} \end{cases}$$

This rule has parity since a and b are both even, but its reduced form is:

$$f(n) = \begin{cases} 3n+4 & \text{if } n \text{ is odd} \\ n/2 & \text{if } n \text{ is even} \end{cases}$$

The reduced form does not have parity. If we try a specific starting value of $n = 10$, we get the sequence 10, 5, 19, 61, 187, 565, 1699, ... The sequence diverges to infinity as expected, since after the first step, the terms are all odd and the $n/2$ calculation is never used. However, if we had used the original sequence, we would instead get the sequence 10, 5, 38, 19, 122, 61, 374, 187, 1130, 565, 3398, 1699, ... This sequence also diverges to infinity and in fact contains the previous sequence as a subsequence. More generally, the proof of the Corollary would come down to showing that if a reduced Collatz rule diverges to infinity, then the original does the same.

Also, if the greatest common divisor of a and b in a Collatz rule is even, the rule may cause $f(n)$ to collapse a step, but it will never return to a loop and will gradually increase in size and overall expand to infinity. Along this train of thought, if a Collatz Rule has both an odd a and an odd b, then the Collatz Rule will always have complete parity, since the greatest common divisor of two odd numbers must be odd and the remaining reduced form will also have an odd a and an odd b. This implies that if the complete parity of a Collatz rule implies that the rule collapses to a loop, then every Collatz Rule with odd a and odd b, including the original Collatz conjecture $3n+1$, will all collapse. This does not indicate whether every natural number in a Collatz Rule collapses to the same loop or not. We are now finally able to state a generalization of the Collatz conjecture.

Conjecture. *Let f be a Collatz rule of the form*

$$f(n) = \begin{cases} an+b & \text{if } n \text{ is odd} \\ n/2 & \text{if } n \text{ is even} \end{cases}$$

Let $d = gcd(a,b)$. If this rule has complete parity—that is, if $\frac{a}{d}$ and $\frac{b}{d}$ are either both even or both odd—then every natural number n has finite rank under this rule.

We do not offer a proof of this conjecture.

Comparison of Families

The Generalized Collatz Conjecture applies to rules with complete parity, and the Parity Expansion Theorem applies to all the others. The next question is—which case is more common? In other words, how many rules fall into the classes without parity, with only parity, and with complete parity? Figure 1 shows the parities of each rule for a and b from 2 to 50.

	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50
2	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
4	1	0	1	1	1	0	1	1	1	0	1	1	1	0	1	1	1	0	1	1	1	0	1	1	1
6	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
8	1	1	1	0	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	0	1	1	1	1	1
10	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
12	1	0	1	1	1	0	1	1	1	0	1	1	1	0	1	1	1	0	1	1	1	0	1	1	1
14	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
16	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1
18	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
20	1	0	1	1	1	0	1	1	1	0	1	1	1	0	1	1	1	0	1	1	1	0	1	1	1
22	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
24	1	1	1	0	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	0	1	1	1	1	1
26	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
28	1	0	1	1	1	0	1	1	1	0	1	1	1	0	1	1	1	0	1	1	1	0	1	1	1
30	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
32	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1
34	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
36	1	0	1	1	1	0	1	1	1	0	1	1	1	0	1	1	1	0	1	1	1	0	1	1	1
38	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
40	1	1	1	0	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	0	1	1	1	1	1
42	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
44	1	0	1	1	1	0	1	1	1	0	1	1	1	0	1	1	1	0	1	1	1	0	1	1	1
46	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
48	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1
50	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0

Figure 1: a is on the vertical axis and b is on the horizontal axis. White cells represent Collatz rules with complete parity.

A quick count shows that 50% of these rules have no parity, 16.56]% have only parity, and 33.44% have complete parity. To see why this occurs, note that, loosely speaking, 1/4 of all rules will have both a and b even, 1/4 will have both a and b odd, and 1/4will have neither of the above. (We will need to be more precise about this later since 1/4 of an infinite set is not a rigorously defined concept). The 1/4 of cases in which one of a and b is even and the other is odd represent rules without parity. The 1/4 of cases in which both a and b are odd represent cases with complete parity.

The 1/4 of cases in which a and b are both even are a more complicated set. Suppose that for these 1/4 of the rules, we divide a and b each by 2. The remaining values of $a/2$ and $b/2$ will then both be odd in 1/4 of such cases, which means those rules will have complete parity. In 1/2 of such cases, one of $a/2$ and $b/2$ will be even and the other odd, so those rules have only parity. The remaining 1/4 will have $a/2$ and $b/2$ both even, and thus we can divide by 2 yet again and break them down further. Thus, 1/4 of the original (a,b) pairs have complete parity since both numbers are odd, 1/16 have complete parity because both numbers are odd after dividing by 2 once, 1/64 have complete parity because both numbers are odd after dividing by 2 twice, and so on. Thus, the number of rules in this 1/4-size set that have even-even parity is

$$\sum_{n=1}^{\infty} \left(\frac{1}{4}\right)^n = \frac{1/4}{1-1/4} = \frac{1}{3}$$

Figure 2 shows this schematically. The set of rules with complete parity is green.

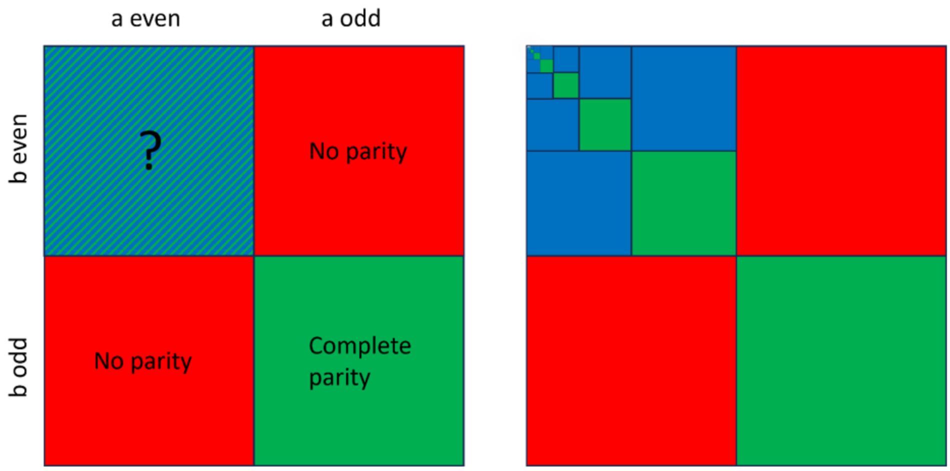

Figure 2: Green regions represent complete parity, red regions represent no parity, and blue regions represent parity only. Note that green regions use up 1/3 of the total area of the square.

The above argument is somewhat informal, but it can be formalized by considering a set of ordered pairs (and corresponding Collatz rules) (a,b) in which $1 \leq a \leq N$ and $1 \leq b \leq N$. Then the statements above should be true in the limit as $N \to \infty$.

Theorem. *Consider the set of ordered pairs (and corresponding Collatz rules) (a,b) in which* $1 \leq a \leq N$ *and* $1 \leq b \leq N$. *Let* $F_{NP}(N)$ *be the fraction of these Collatz rules that have no parity, let* $F_{OOP}(N)$ *be the fraction that have complete odd-odd parity, and let* $F_{EEP}(N)$ *be the fraction that have complete even-even parity. Then*

$$\lim_{n\to\infty} F_{NP}(N) = \frac{1}{2}$$
$$\lim_{n\to\infty} F_{OOP}(N) = \frac{1}{4}$$
$$\lim_{n\to\infty} F_{EEP}(N) = \frac{1}{12}$$

Thus, in the limit as $N \to \infty$, *1/3 of all Collatz rules have complete parity of either odd-odd form or even-even form. The remaining 1/6 have parity but not complete parity.*

Proof. Let $\kappa_m(N)$ be the number of positive integers less than or equal to N which are divisible by 2^m. (That is, such an integer has at least m factors of 2 in its prime factorization.) The number of integers from 1 to N that have at least one factor of 2 in their prime factorization is either $N/2$ (if N is even) or $(N-1)/2$ (if N is odd). Thus,

$$\frac{N-1}{2} \leqslant \kappa_2(N) \leqslant \frac{N}{2}.$$

Similarly, the number of integers from 1 to N that have at least m factors of 2 in their prime factorization is at most $N/2^m$ (if $N \equiv 0 \pmod{2^m}$) and at least $(N-2^m+1)/2^m$ (with equality in the case that $N \equiv -1 \pmod{2^m}$). Thus, we may generalize the above equation to:

$$\frac{N-2^m+1}{2^m} \leqslant \kappa_m(N) \leqslant \frac{N}{2^m}.$$

Consider integers a and b with $1 \leq a \leq N$ and $1 \leq b \leq N$. The Collatz rule corresponding to the pair (a,b) has no parity iff a is even and b is odd or vice-versa. The fraction of pairs for which this occurs-denoted $F_{NP}(N)$–is:

$$F_{NP}(N) = \frac{2\kappa_2(N)(N-\kappa_2(N))}{N^2}.$$

Thus,

$$2\left(\frac{N-1}{2}\right)\left(N-\frac{N}{2}\right)\left(\frac{1}{N^2}\right) \leqslant F_{NP}(N) \leqslant 2\left(\frac{N}{2}\right)\left(N-\frac{N-1}{2}\right)\left(\frac{1}{N^2}\right)$$

or

$$\left(\frac{N-1}{2}\right)\frac{1}{N} \leqslant F_{NP}(N) \leqslant \left(\frac{N+1}{2}\right)\frac{1}{N}.$$

As $N \to \infty$, the upper and lower bounds both approach 1/2, and therefore, by the Squeeze Theorem, so does $F_{NP}(N)$. Thus, in the limit as $N \to \infty$, half of all Collatz rules have no parity. Similarly, for a and b in the same range as above, the Collatz rules that have complete odd-odd parity will correspond to (a,b) pairs in which both numbers are odd. The calculation of the fraction of cases–denoted $F_{OOP}(N)$–in which this occurs will be nearly identical to the above, except that the factor of 2 will be absent, resulting in the conclusion that $F_{OOP}(N) \to \frac{1}{4}$ as $N \to \infty$.

Complete even-even parity is slightly more challenging. Let $F_{EEP}(N)$ denote the fraction of cases in which this occurs. $F_{EEP}(N)$ will be a sum of the form $F_{EEP}(N) = \sum_{m=1}^{\infty} \phi_m(N)$ in which $\phi_m(N)$ is the fraction of all Collatz rules in which a and b each have exactly m factors of 2 in their prime factorization, and in which a and b are bounded above by N, as before. (This is of course the same m for both a and b.) We write this as an infinite sum, but in practice all terms will be zero after some point since we have a finite upper bound N on a and b. This point will differ depending on the prime factorization of N itself but cannot exceed $\lfloor \log_2 N \rfloor$ (since a and b can have at most as many factors of 2 as N has). We can write bounds on the quantity $\phi_m(N)$ as follows. First, note that the total number $\kappa_m(N)$ of integers in $[1,N]$ with at least m factors of 2 is between $(N-2^m+1)/2^m$ and $N/2^m$, as discussed above. For example, the number of integers in $[1,N]$ divisible by 4 is at most $N/4$ and at least $(N-3)/4$. However, to find the number of integers with exactly m factors of 2, we must subtract the number $\kappa_{m+1}(N)$ of integers with at least $m+1$ factors of 2. Thus, we consider the two bounds:

$$\frac{N-2^m+1}{2^m} \leqslant \kappa_m(N) \leqslant \frac{N}{2^m}$$

and

$$\frac{N-2^{m+1}+1}{2^{m+1}} \leqslant \kappa_{m+1}(N) \leqslant \frac{N}{2^{m+1}}.$$

The second needs to be multiplied by -1, giving:

$$-\frac{N}{2^{m+1}} \leqslant -\kappa_{m+1}(N) \leqslant -\frac{N-2^{m+1}+1}{2^{m+1}}.$$

We may now add these two bounds to get the bound:

$$\frac{N-2^m+1}{2^m} - \frac{N}{2^{m+1}} \leqslant \kappa_m(N) - \kappa_{m+1}(N) \leqslant \frac{N}{2^m} - \frac{N-2^{m+1}+1}{2^{m+1}}.$$

This can then be simplified to:

$$\frac{N-2^{m+1}+2}{2^{m+1}} \leq \kappa_m(N) - \kappa_{m+1}(N) \leq \frac{N+2^{m+1}-1}{2^{m+1}}.$$

Thus, the number of all ordered pairs in which both a and b are in $[1,N]$ and both have exactly m factors of 2 is $N^2\phi_m(N)$, which has bounds:

$$\left(\frac{N-2^{m+1}+2}{2^{m+1}}\right)^2 \leqslant N^2\phi_m(N) \leqslant \left(\frac{N+2^{m+1}-1}{2^{m+1}}\right)^2.$$

Squaring all three parts of the preceding inequality is valid here if all three are nonnegative, which will be the case if $2^{m+1} \leq N+2$, or to put it differently, $m \leq \log_2(N+2) - 1$. There is one case in which this is not justified. Suppose $m = \lfloor \log_2 N \rfloor$ (i.e. the last m value for which $\phi_m(N)$ can be nonzero). Then—and only then—the value of $N - 2^{m+1} + 2$ can be negative. Because of this, we will consider $\phi_{\lfloor \log_2 N \rfloor}(N)$ separately. For smaller m values, simplifying the above inequality then gives:

$$\left(\frac{N-2^{m+1}+2}{N2^{m+1}}\right)^2 \leqslant \phi_m(N) \leqslant \left(\frac{N+2^{m+1}-1}{N2^{m+1}}\right)^2$$

or

$$\left(\frac{1}{2^{m+1}} - \frac{1}{N} + \frac{2}{N2^{m+1}}\right)^2 \leqslant \phi_m(N) \leqslant \left(\frac{1}{2^{m+1}} + \frac{1}{N} - \frac{1}{N2^{m+1}}\right)^2.$$

Now summing over m gives:

$$\phi_{\lfloor \log_2 N \rfloor}(N) + \sum_{m=1}^{\lfloor \log_2(N+2)-1 \rfloor} \left(\frac{1}{2^{m+1}} - \frac{1}{N} + \frac{2}{N2^{m+1}}\right)^2 \leq F_{EEP}(N)$$
$$\leq \phi_{\lfloor \log_2 N \rfloor}(N) + \sum_{m=1}^{\lfloor \log_2(N+2)-1 \rfloor} \left(\frac{1}{2^{m+1}} + \frac{1}{N} - \frac{1}{N2^{m+1}}\right)^2.$$

Then taking the limit as $N \to \infty$ gives:

$$\begin{aligned}
&\lim_{N\to\infty}\left(\phi_{\lfloor \log_2 N\rfloor}(N)+\sum_{m=1}^{\lfloor \log_2(N+2)-1\rfloor}\left(\frac{1}{2^{m+1}}-\frac{1}{N}+\frac{2}{N2^{m+1}}\right)^2\right)\\
&\leq \lim_{N\to\infty} F_{EEP}(N)\\
&\leq \lim_{N\to\infty}\left(\phi_{\lfloor \log_2 N\rfloor}(N)+\sum_{m=1}^{\lfloor \log_2(N+2)-1\rfloor}\left(\frac{1}{2^{m+1}}+\frac{1}{N}-\frac{1}{N2^{m+1}}\right)^2\right).
\end{aligned}$$

At this point it is tempting to simply say that since the $-\frac{1}{N}+\frac{2}{N2^{m+1}}$ terms approach zero, and the upper limit of the summation approaches infinity, the left-hand side reduces to $\sum_{m=1}^{\infty}\left(\frac{1}{2^{m+1}}\right)^2$. This is a little too fast, since we would need a lemma along the lines of $\lim_{N\to\infty}\left(\sum_{m=1}^{f(N)}\alpha_N(m)\right)=\sum_{m=1}^{\lim_{N\to\infty}f(N)}\lim_{N\to\infty}\alpha_N(m)$ subject to some suitable conditions. Instead of attempting to prove such a lemma here, we use a different strategy. The left-hand side may be rewritten as:

$$\begin{aligned}
LHS &= \lim_{N\to\infty}\Bigg(\phi_{\lfloor \log_2 N\rfloor}(N)+\\
&\sum_{m=1}^{\lfloor \log_2(N+2)-1\rfloor}\frac{1}{2^{2m+2}}+\frac{1}{N^2}+\frac{4}{N^2 2^{2m+2}}-\frac{2}{N2^{m+1}}+\frac{4}{N2^{2m+2}}-\frac{4}{N^2 2^{m+1}}\Bigg)\\
&= \lim_{N\to\infty}\Bigg(\phi_{\lfloor \log_2 N\rfloor}(N)+\sum_{m=1}^{\lfloor \log_2(N+2)-1\rfloor}\frac{1}{2^{2m+2}}+\\
&\sum_{m=1}^{\lfloor \log_2(N+2)-1\rfloor}\left(\frac{1}{N^2}+\frac{4}{N^2}\left(\frac{1-2^{m+1}}{2^{2m+2}}\right)+\frac{2}{N}\left(\frac{2-2^{m+1}}{2^{2m+2}}\right)\right)\Bigg)\\
&= \lim_{N\to\infty}\Bigg(\phi_{\lfloor \log_2 N\rfloor}(N)+\sum_{m=1}^{\lfloor \log_2(N+2)-1\rfloor}\frac{1}{2^{2m+2}}+\sum_{m=1}^{\lfloor \log_2(N+2)-1\rfloor}\left(\frac{1}{N^2}\right)+\\
&\frac{4}{N^2}\sum_{m=1}^{\lfloor \log_2(N+2)-1\rfloor}\left(\frac{1-2^{m+1}}{2^{2m+2}}\right)+\frac{2}{N}\sum_{m=1}^{\lfloor \log_2(N+2)-1\rfloor}\left(\frac{2-2^{m+1}}{2^{2m+2}}\right)\Bigg).
\end{aligned}$$

The first summation is the one we want to keep. The third and fourth have absolute values bounded above by the absolute values of the geometric series that we would get by replacing the upper limits of summation with infinity. These geometric series converge to finite numbers, so the quantity $\frac{2}{N}$ or $\frac{4}{N^2}$ multiplied by such a series will converge to zero by the product rule for limits and the Squeeze Theorem. The second summation is simply equal to $\lfloor \log_2(N+2)-1\rfloor/N^2$, and L'Hospital's rule easily shows that this converges to zero as $N \to \infty$. Thus, all of the summations after the first converge to zero. Furthermore, the fraction $\phi_{\lfloor \log_2 N\rfloor}(N)$ can be estimated separately. It is by definition the fraction of ordered pairs of integers in $[1,N]$ which contain exactly $\lfloor \log_2 N \rfloor$ factors of 2, (i.e. the maximum possible number of factors of 2 for any integer not exceeding N). There is in fact only one such integer in $[1,N]$, and it is $2^{\lfloor \log_2 N \rfloor}$. To see why, suppose there was another. It would still need to have $2^{\lfloor \log_2 N \rfloor}$ in its prime factorization, but it would also need at least one more factor, which must be at least 3. The integer would then exceed N. Therefore, $\phi_{\lfloor \log_2 N \rfloor}(N) = \frac{1}{N^2}$ which again approaches zero as $N \to \infty$. Thus, we are left with

$$\text{LHS}=\lim_{N\to\infty} \left(\sum_{m=1}^{\lfloor \log_2 (N+2)-1 \rfloor} \frac{1}{2^{2m+2}} \right) = \sum_{m=1}^{\infty} \frac{1}{2^{2m+2}} = \frac{1}{12}$$

A very similar calculation applies to the right-hand side. Then by the Squeeze Theorem, $\lim_{N\to\infty} F_{EEP}(N) = \frac{1}{12}$. This concludes the proof.

■

The Generalized Collatz Conjecture stated above applies to one-third of all possible Collatz rules. The parity expansion theorem applies to the other two thirds. Thus, the “interesting” Collatz rules—those for which the Collatz Conjecture or its generalization are not obviously false—are a minority, but they are not rare.

Conclusion

We have–unsurprisingly–not offered a proof of the original Collatz conjecture, but this analysis has contextualized it by asking in what sense the choice of $a = 3$ and $b = 1$ in the original rule is special. We find that the values of a and b do need to satisfy some constraints to avoid producing much simpler and less interesting sequences. However, within these constraints, many generalized Collatz rules appear to produce similar behavior to the original, and sometimes with more complicated terminal patterns. Many additional lines of inquiry are open here. One can for example construct a directed graph from any Collatz rule in which the vertices are all positive integers and an edge is directed from k to m iff $f(k) = m$. This graph then gives a visual representation of the paths that various Collatz sequences take through the integers. Comparison of such graphs might show, for example, that the graph G_1 corresponding to one Collatz rule is homeomorphic to the graph G_2 corresponding to a different Collatz rule. The conditions under which this occurs could give interesting insights into the comparison between various rules.

References

Aberkane, I. J. (2017, August 15). *On the Syracuse conjecture over the binary tree*. Accueil - Archive ouverte HAL. Retrieved March 7, 2023, from `https://hal.science/hal-01574521`

Carnielli, W. (2015, July 11). *Some natural generalizations of the Collatz problem*. Applied Mathematics e-Notes. Retrieved March 7, 2023, from https://www.emis.de/journals/AMEN/2015/AMEN(150711).pdf

EMN. (2020, September 8). *Collatz conjecture in all its variants*. MathOverflow. Retrieved March 7, 2023, from https://mathoverflow.net/questions/371155/collatz-conjecture-in-all-its-variants

Hartnett, K. (2019, December 11). *Mathematician proves huge result on 'dangerous' problem*. Quanta Magazine. Retrieved March 7, 2023, from https://www.quantamagazine.org/mathematician-proves-huge-result-on-dangerous-problem-20191211

Kleinnijenhuis, J., Kleinnijenhuis, A. M., & Aydogan, M. G. (2022, June 14). *The Collatz tree as an automorphism graph: A Cotree density proof of the 3x + 1 conjecture*. arXiv.org. Retrieved March 7, 2023, from https://arxiv.org/abs/2008.13643

Lagarias, J. C. (1985). *The 3x + 1 Problem and Its Generalizations*. The American Mathematical Monthly, 92(1), 3–23. https://doi.org/10.2307/2322189

Editor's note: The following note presents some novel results. While the results may be of interest in and of themselves, we include this to possibly spur student-led investigation into the questions posed at the end of the paper.

Consecutive Integers Having Power Greater Than One

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Abstract

We consider consecutive integers all with prime decompositions involving powers greater than one.

Introduction

Consider the following sequences:

1. (8,9), (24, 25), (27, 28)
2. (48, 49, 50), (98, 99, 100), (124, 125, 126)
3. (242, 243, 244, 245), (3174, 3175, 3176, 3177)

If we consider the prime decomposition of the numbers in these sequences, we see that they involve a multiple with a power greater than 1:

1. $\left(2^3, 3^2\right), \left(3 \times 2^3, 5^2\right), \left(3^3, 7 \times 2^2\right)$
2. $\left(3 \times 2^4, 7^2, 2 \times 5^2\right), \left(2 \times 7^2, 11 \times 3^2, 10^2\right), \left(31 \times 2^2, 5^3, 2 \times 7 \times 3^2\right)$
3. $\left(2 \times 11^2, 7^3, 61 \times 2^2, 5 \times 7^2\right), \left(2 \times 3 \times 23^2, 127 \times 5^2, 397 \times 2^3, 353 \times 3^2\right)$

For $k \in \mathbb{N}$ let us denote the set of k consecutive numbers with prime decompositions involving multiples with a power greater than 1 by c_k and elements of this set beginning with the number n by $c_k(n)$. Some examples:

$\underline{c_2}$

$$\begin{aligned} c_2(24) &= (24,25) = \left(3\times 2^3, 5^2\right) \\ c_2(27) &= (27,28) = \left(3^3, 7\times 2^2\right) \\ c_2(44) &= (44,45) = \left(11\times 2^2, 5\times 3^2\right) \\ c_2(63) &= (63,64) = \left(7\times 3^2, 2^6\right) \\ c_2(152) &= (152,153) = \left(19\times 2^3, 17\times 3^2\right) \\ c_2(171) &= (171,172) = \left(19\times 3^2, 43\times 2^2\right) \end{aligned}$$

$\underline{c_3}$

$$\begin{aligned} c_3(124) &= (124,125,126) = \left(31\times 2^2, 5^3, 2\times 7\times 3^2\right) \\ c_3(350) &= (350,351,352) = \left(2\times 7\times 5^2, 13\times 3^3, 11\times 2^5\right) \\ c_3(475) &= (475,476,477) = \left(19\times 5^2, 7\times 17\times 2^2, 53\times 3^2\right) \\ c_3(423) &= (423,424,425) = \left(47\times 3^2, 53\times 2^3, 17\times 5^2\right) \end{aligned}$$

$\underline{c_4}$

$$\begin{aligned} c_4(3174) &= (3174,3175,3176,3177) \\ &= \left(2\times 3\times 23^2, 127\times 5^2, 397\times 2^3, 353\times 3^2\right) \\ c_4(8523) &= (8523,8524,8525,8526) \\ &= \left(947\times 3^2, 2131\times 2^2, 11\times 31\times 5^2, 2\times 3\times 29\times 7^2\right) \\ c_4(10050) &= (10050,10051,10052,10053) \\ &= \left(2\times 3\times 67\times 5^2, 19\times 23^2, 7\times 359\times 2^2, 1117\times 3^2\right) \end{aligned}$$

Given $c_k(n)$ with $k = 1,2$, or 3, let L be the least common multiple of the primes in the decompositions of the numbers in $c_k(n)$ with powers greater than 1. For example, consider $c_3(124)$: here

$$L = lcm\left(2^2, 5^2, 3^2\right) = 900.$$

Let us denote by $c_k(n) + Lm$ the result of adding $Lm, m \in \mathbb{N}$ to each number in $c_k(n)$. Thus

$$c_3(124) + 900m = (124 + 900m, 125 + 900m, 126 + 900m)$$

for any natural number m.

Theorem 1. *If $c_k, k \in \mathbb{N}$ is non-empty, then it is an infinite set.*

Proof. Suppose

$$c_k(n) = (n, n+1, n+2, \ldots, n+k-1) \in c_k$$

and let L be the least common multiple of the primes in the decompositions of the numbers in $c_k(n)$ with powers greater than 1. Then for any $m \in \mathbb{N}$

$$\begin{aligned} c_k(n) &= (n + Lm, n+1+Lm, n+2+LM, \ldots, n+k-1+Lm) \\ &= ((n+Lm), (n+Lm)+1, (n+Lm)+2, \ldots, (n+Lm)+k-1), \end{aligned}$$

so $c_k(n) + Lm$ is a set of consecutive integers. Each $n+i, 0 \leq i \leq k-1$, and Lm have a common prime factor with a power greater than 1, so each entry in $c_k(n) + Lm$ has a prime decomposition with powers greater than 1. It follows that if c_k is non-empty, then it is infinite. ■

Theorem 2. *Suppose $c_2(n) \in c_2$. Then $c_2(n(n+1) + 3(n+1)) \in c_2$.*

Proof. Note that $n(n+1) + 3(n+1) = (n+1)(n+3)$ has a prime decomposition with a power greater than 1 since $(n+1)$ has such a factor. We also have has a prime decomposition with a power greater than 1 since has such a factor. We also have

$$n(n+1) + 3(n+1) + 1 = n^2 + 4n + 4 = (n+2)^2,$$

which then has a prime decomposition with a power greater than 1. It follows that $c_2(n(n+1) + 3(n+1)) \in c_2$. ■

Some interesting patterns:

1. Note that $c_3(124) \in c_3$ and:

$$\begin{aligned} 124 \times 3 - 22 = 350 &\text{ and } c_3(350) \in c_3 \\ 350 \times 3 - 26 = 1024 &\text{ and } c_3(1024) \in c_3 \\ 1024 \times 3 - 22 = 3050 &\text{ and } c_3(3050) \in c_3 \\ 3050 \times 3 - 26 = 9124 &\text{ and } c_3(9124) \in c_3 \end{aligned}$$

2. Note that $c_3(98) \in c_3$ and:

$$\begin{aligned} 98 \times 8 - 18 = 6174 &\text{ and } c_3(6174) \in c_3 \\ 6174 \times 8 - 18 = 49374 &\text{ and } c_3(49374) \in c_3 \\ 49374 \times 8 - 18 = 394974 &\text{ and } c_3(394974) \in c_3 \end{aligned}$$

Some interesting questions:

1. Are $c_5, c_6, c_7, \ldots$ nonempty?
2. Is it the case that c_j is empty for all j greater than equal to some k?

The Problem Corner

Edited by Pat Costello

The Problem Corner invites questions of interest to undergraduate students. As a rule, the solution should not demand any tools beyond calculus and linear algebra. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following new problems should be submitted on separate sheets before May 31, 2026. Solutions received after this will be considered up to the time when copy is prepared for publication. The solutions received will be published in the Spring 2025 issue of *The Pentagon*. Preference will be given to correct student solutions. Affirmation of student status and school should be included with solutions. New problems and solutions to problems in this issue should be sent to Pat Costello, Department of Mathematics and Statistics, Eastern Kentucky University, 521 Lancaster Avenue, Richmond, KY 40475-3102 (e-mail: pat.costello@eku.edu, fax: (859) 622-3051)

NEW PROBLEMS 937 - 944

Problem 937. *Proposed by José Luis Diaz-Barrero, Barcelona Mathematical Circle, Barcelona, Spain.*
Three roots of the equation $x^4 - px^3 + qx^2 - rx + s = 0$ are $\tan A, \tan B, \tan C$ where A, B, C are the angles of a triangle ABC. Determine the fourth root as a function of only p, q, r and s.

Problem 938. *Proposed by José Luis Diaz-Barrero, Barcelona Mathematical Circle, Barcelona, Spain.*
If a, b, c are positive reals no larger than one, prove that

$$\frac{2a - \sqrt[3]{abc}}{1+a} + \frac{2b - \sqrt[3]{abc}}{1+b} + \frac{2b - \sqrt[3]{abc}}{1+b} \geq \frac{3\sqrt[3]{abc}}{1+\sqrt[3]{abc}}.$$

Problem 939. *Proposed by Toyesh Prakash Sharma and Etisha Sharma, Agra College, Agra, India.*
Find the highest power of 5 which is contained in 777!

Problem 940. *Proposed by John Zerger, Catawba College, Salisbury, NC.*
Show that if p and q are two consecutive odd prime numbers then $p+q$ is the product of at least three prime numbers (not necessarily distinct).

Problem 941. *Proposed by Guillermo Garcia (student) and Angel Plaza, Universidad de Las Palmas de Gran Canaria, Spain.*
Evaluate the sum $\int \arctan x \left(e^x + \frac{1}{e^x}\right) dx + \int \frac{1}{1+x^2}\left(e^x - \frac{1}{e^x}\right) dx$.

Problem 942. *Proposed by D.M. Bătinetu-Giurgiu, "Matei Basarab" National College, Bucharest, Romania and Neculai Stanciu, "George Emil Palade" School, Buzău, Romania.*
If (a_n) with $n \geq 1$ is a positive real sequence such that $\lim\limits_{n\to\infty} \frac{a_{n+1}}{na_n \sqrt[n]{n!}} = a$ which is a positive real, then compute $\lim\limits_{n\to\infty} \frac{1}{n}\left(\sqrt[n+1]{a_{n+1}} - \sqrt[n]{a_n}\right)$.

Problem 943. *Proposed by D.M. Bătinetu-Giurgiu, "Matei Basarab" National College, Bucharest, Romania and Neculai Stanciu, "George Emil Palade" School, Buzău, Romania.*
If (a_n) with $n \geq 1$ is a positive real sequence such that $\lim\limits_{n\to\infty} \frac{a_{n+1}}{na_n \sqrt[n]{n!}} = \pi$, then compute $\lim\limits_{n\to\infty} \left(\sqrt[n+1]{a_{n+1}} - \sqrt[n]{a_n}\right)$.

Problem 944. *Proposed by the editor.*
Find a positive integer x which is divisible by a fourth power and $x+1$ is divisible by a cube and $x+2$ is divisible by a square.

SOLUTIONS TO PROBLEMS 920 - 927

Problem 920. *Proposed by Mohammad K. Azarian, University of Evansville, Evansville, Indiana.*
Let α be the golden ratio. Show that

$$\sum_{i=0}^{\infty} \frac{i}{\alpha^i} \left(\sum_{j=0}^{\infty} \left[\lim_{n\to\infty} \frac{F_n^2 + F_{n+2}^2 - F_{n+1}F_{n+3}}{F_{n-1}F_{n+2}}\right]^j\right)^{-1} = \alpha.$$

Solution *by Angel Plaza, Universidad de Las Palmas de Gran Canaria, Spain.*

Since $F_n = (\frac{\alpha^n - \beta^n)}{\sqrt{5}}$, then $\lim\limits_{n\to\infty} \frac{F_n}{F_{(n+p)}} = \alpha^{(-p)}$. Therefore,

$$\lim_{n\to\infty} \frac{F_n}{F_{(n-1)}} \frac{F_n}{F_{(n+2)}} = \alpha^{-1},$$
$$\lim_{n\to\infty} \frac{F_{(n+2)}}{F_{(n-1)}} \frac{F_{(n+2)}}{F_{(n+2)}} = \alpha^3,$$
$$\lim_{n\to\infty} \frac{F_{(n+1)}}{F_{(n-1)}} \frac{F_{(n+3)}}{F_{(n+2)}} = \alpha^2\alpha = \alpha^3.$$

So $\lim\limits_{n\to\infty} \frac{F_n^2+F_{(n+2)}^2 F_{(n+1)} F_{(n+3)}}{F_{(n-1)}F_{(n+2)}} = \alpha^{-1}+\alpha^3-\alpha^3 = \alpha^{-1}$. Then the proposed sum is

$$\sum_{i=0}^{\infty} \frac{i}{\alpha^i} \left(\sum_{j=0}^{\infty} \alpha^{-j}\right)^{-1} = \sum_{i=0}^{\infty} \frac{i}{\alpha^i} \left(\frac{1}{1-\alpha^{-1}}\right)^{-1} = \sum_{i=0}^{\infty} \frac{i}{\alpha^i} \left(\frac{\alpha}{\alpha-1}\right)^{-1}$$
$$= \frac{\alpha-1}{\alpha} \sum_{i=0}^{\infty} \frac{i}{\alpha^i} = \frac{\alpha-1}{\alpha} \frac{\alpha}{(1-\alpha)^2} = \frac{1}{\alpha-1} = \alpha,$$

where it has been used that $\sum\limits_{i=0}^{\infty} \frac{i}{x^i} = \frac{x}{(x-1)^2}$, for $|x|>1$ and that $\alpha^2-\alpha=1$.

Also solved by Albert Stadler, Herrliberg, Switzerland; and the proposer.

Problem 921. *Proposed by Mihaly Bencze, Braşov, Romania and Neculai Stanciu, "George Emil Palade" School, Buzău, Romania.*
Solve in real numbers the following equation:

$$\log_2(x^2+2^x) + (x^2-1)*2^{x+1} + x^4 + x^2 + 2^x = 3*4^x + x + 1.$$

Solution *by Brian Beasley, Simpsonville, South Carolina.*

We show that x is a solution of the equation if and only if $x^2 = 2^x$, which holds when $x=2, x=4$, or $x \approx -0.7666647$.
If $x^2 = 2^x$, then the left-hand side of the equation (LHS) becomes

$$\log_2(2*2^x) + \left(x^2-1\right)2x^2 + x^4 + x^2 + x^2 = 3x^4 + x + 1$$

and the right-hand side of the equation (RHS) equals

$$3\left(x^2\right)^2 + x + 1 = 3x^4 + x + 1.$$

If $x^2 > 2^x$, then

$$\begin{aligned} \text{LHS} &= \log_2\left(x^2+2^x\right) + \left(x^2-1\right)*2^{x+1} + x^4 + x^2 + 2^x \\ &> \log_2(2*2^x) + (2^x-1)2^{x+1} + 4^x + 2*2^x \\ &= x+1+2*4^x - 2^{x+1} + 4^x + 2^{x+1} \\ &= 3*4^x + x + 1 \\ &= \text{RHS}. \end{aligned}$$

If $x^2 < 2^x$, then

$$\begin{aligned}\text{LHS} &= \log_2\left(x^2+2^x\right)+\left(x^2-1\right)*2^{x+1}+x^4+x^2+2^x\\ &< \log_2(2*2^x)+(2^x-1)\,2^{x+1}+4^x+2*2^x\\ &= x+1+2*4^x-2^{x+1}+\ 4^x+\ 2^{x+1}\\ &= 3*4^x+x+1\\ &= \text{RHS}.\end{aligned}$$

Also solved by Aaron Allen, North Carolina Wesleyan University, Rocky Mount, North Carolina; Albert Stadler, Herrliberg, Switzerland; and the proposer.

Problem 922. *Proposed by Toyesh Prakash Sharma (student), Agra College, Agra, India.*
Let F_n be the n^{th} Fibonacci number defined by $F_1 = 1, F_2 = 1$ and, for all $n \geq 3, F_n = F_{n-1} + F_{n-2}$. Prove that $\sum_{n=1}^{\infty}\left(\frac{1}{9}\right)^{F_{n+2}}$ is an irrational number but not a transcendental number.

Solution *by the proposer.*

Let $x = \sum\limits_{n=1}^{\infty}\left(\frac{1}{9}\right)^{F_{n+2}} = \frac{1}{9}\sum\limits_{n=1}^{\infty}\left(\frac{1}{9}\right)^{F_{n+2}-1}$ and now using $F_{n+2}-1 = \sum\limits_{k=1}^{n} F_k$ we can rewrite x as

$$x = \frac{1}{9}\sum_{n=1}^{\infty}\left(\frac{1}{9}\right)^{\sum\limits_{n=1}^{\infty}F_n} = \frac{1}{9}\left[\frac{1}{9^{F_1}}+\frac{1}{9^{F_1}9^{F_2}}+\frac{1}{9^{F_1}9^{F_2}9^{F_3}}+\dots\right].$$

On comparing this with $\sum\limits_{k=1}^{\infty}\frac{1}{a_1a_2\dots a_k}$ we can say that $a_k = 9^{F_k}$. The series $\sum\limits_{k=1}^{\infty}\frac{1}{a_1a_2\dots a_k}$ is the Engel expansion of the positive real number x. See Wikipedia for a definition of the Engel expansion. In 1913, Engel established the following result (see Ribenboim's book ([1] page 303): Every real number x has a unique representation $c+\sum\limits_{k=1}^{\infty}\frac{1}{a_1a_2\dots a_k}$ where c is an integer and $2 \leq a_1 \leq a_2 \leq a_3 \leq \dots$ is a sequence of integers. Conversely, every such sequence is convergent and its sum is irrational if and only if $\lim\limits_{k\to\infty} a_k = \infty$ Therefore, by Engel's result x is irrational since $\lim\limits_{k\to\infty} 9^{F_k} = \infty$.
Now consider the following from Ribenboim's book ([1] page 315): Let $(f(n)), n \geq 1$ be a sequence of positive integers such that $\lim\limits_{n\to\infty}\frac{f(n+1)}{f(n)} = \mu \geq 2$. Then for every integer $d \geq 2$ the number $x = \sum\limits_{n=1}^{\infty}\frac{1}{d^{f(n)}}$ is transcendental. For the given problem $d = 9$ and $f(n) = F^{n+2}-1$ and $\lim\limits_{n\to\infty}\frac{f(n+1)}{f(n)} = \frac{1+\sqrt{5}}{2} < 2$.

[1] Paulo Ribenboim, My Numbers, My Friends: Popular Lectures on Number Theory, Springer 2000.

Editor's Note: At this point, the proposer wants to conclude that$x$ is not transcendental, but denying the antecedent does not deny the conclusion. This leaves the question of whether x is transcendental or not unknown. In fact, Albert Stadler wrote that similar numbers like the Kempner number have been proven transcendental.

Problem 923. *Proposed by José Luis Díaz-Barrero, Barcelona Tech, Barcelona, Spain.*
Let $n \geq 1$ be an integer. Compute

$$\lim_{n\to\infty} \frac{\binom{n+1}{2}}{2^{n-1}} \sum_{k=0}^{\infty} \frac{k+4}{(k+1)(k+2)(k+3)} \binom{n}{k}.$$

Solution *by Albert Stadler, Herrliberg, Switzerland.*

We use the identity $\frac{n+1}{k+1}\binom{n}{k} = \binom{n+1}{k+1}$ to deduce

$\sum_{k=0}^{\infty} \frac{k+4}{(k+1)(k+2)(k+3)} \binom{n}{k} = \sum_{k=0}^{\infty} \frac{1}{(k+1)(k+2)} \binom{n}{k} + \sum_{k=0}^{\infty} \frac{1}{(k+1)(k+2)(k+3)} \binom{n}{k}$

$= \frac{1}{(n+1)(n+2)} \sum_{k=0}^{\infty} \frac{(n+1)(n+2)}{(k+1)(k+2)} \binom{n}{k} + \frac{1}{(n+1)(n+2)(n+3)} \sum_{0}^{\infty} \frac{(n+1)(n+2)(n+3)}{(k+1)(k+2)(k+3)} \binom{n}{k}$

$= \frac{1}{(n+1)(n+2)} \sum_{k=0}^{\infty} \binom{n+2}{k+2} + \frac{1}{(n+1)(n+2)(n+3)} \sum_{k=0}^{\infty} \binom{n+3}{k+3}$

$= \frac{1}{(n+1)(n+2)} \left(2^{n+2} - \binom{n+2}{0} - \binom{n+2}{1}\right)$

$\quad + \frac{1}{(n+1)(n+2)(n+3)} \left(2^{n+3} - \binom{n+3}{0} - \binom{n+3}{1} - \binom{n+3}{2}\right).$

Thus

$\lim_{n\to\infty} \frac{\binom{n+1}{2}}{2^{n-1}} \sum_{k=0}^{\infty} \frac{k+4}{(k+1)(k+2)(k+3)} \binom{n}{k}$

$\quad = \lim_{n\to\infty} \frac{\binom{n+1}{2}}{2^{n*1}} \left(\frac{2^{n+2}}{(n+1)(n+2)}\right) = 4.$

Also solved by Devis Alvarado, UNAH y UPNFM, Tegucigalpa, Honduras; Henry Ricardo, Westchester Area Math Circle, Purchase, New York; and the proposer.

Problem 924. *Proposed by José Luis Díaz-Barrero, Barcelona Tech, Barcelona, Spain.*
Let a,b,c be three positive real numbers. Prove that

$$\frac{a}{4b+7\sqrt{ab}} + \frac{b}{4c+7\sqrt{bc}} + \frac{c}{4a+7\sqrt{ca}} \geq \frac{3}{11}.$$

Solution *by Sarah Seales, Arizona State University, Tempe, Arizona.*

First we rewrite the inequality as

$$\frac{a^2}{4ab+7a\sqrt{ab}}+\frac{b^2}{4bc+7b\sqrt{bc}}+\frac{c^2}{4ac+7c\sqrt{ca}}\geq\frac{3}{11}$$

Using Titu's Lemma and then the AM-GM we have the following inequalities

$$\begin{aligned}\frac{a^2}{4ab+7a\sqrt{ab}}&+\frac{b^2}{4bc+7b\sqrt{bc}}+\frac{c^2}{4ac+7c\sqrt{ca}}\\ &\geq\frac{(a+b+c)^2}{4ab+4bc+4ac+7a\sqrt{ab}+7b\sqrt{bc}+7c\sqrt{ca}}\\ &\geq\frac{2(a+b+c)^2}{15\,(ab+bc+ca)+7(a^2+b^2+c^2)}.\end{aligned}$$

So it is enough to show $\frac{2(a+b+c)^2}{15(ab+bc+ca)+7(a^2+b^2+c^2)}\geq\frac{3}{11}$.
Set $a+b+c=p, ab+bc+ca=q, a^2+b^2+c^2=r$ and the inequality becomes

$$\frac{2p^2}{15q+7r}\geq\frac{3}{11}.$$

Using $p^2=r+2q$, the inequality simplifies to $r\geq q$ a well-known true inequality.

Also solved by Ioan Viorel Codreanu, Satulung, Maramures, Romania; Brent Dozier, North Carolina Wesleyan University, Rocky Mount, North Carolina; Kee-Wai Lau, Hong Kong, China; Cao Minh Quang, Nguyen Binh Khiem High School, Finh Long, Vietnam; Henry Ricardo, Westchester Area Math Circle, Purchase, New York; Albert Stadler, Herrliberg, Switzerland; Daniel Vacaru, "Maria Teiuleanu" National College, Piteşti, Romania; and the proposer.

Corrected Problem 925. *Proposed by Neculai Stanciu, "George Emil Palade" School, Buzău, Romania.*
Prove that in any triangle ABC with usual notations (R = circumradius, r = inradius, s = semiperimeter, ma = median from vertex A) the following inequality is true:

$$2\sum m_a\leq 3\sqrt{\frac{R\,(s^2+r^2+4Rr)}{2r}}.$$

Solution *by proposer.*

Let F be the area of triangle ABC. We use:

$$\left(\sum x\right)^2 \leqslant 3\sum x^2;$$
$$\sum m_a^2 = \frac{3\left(a^2+b^2+c^2\right)}{4};$$
$$ab+bc+ca = s2+r2+4Rr.$$

It suffices to show that:

$$\frac{9\left(a^2+b^2+c^2\right)}{4} \leq \frac{9}{4} * \frac{R}{2r}(ab+bc+ca)$$
$$\Leftrightarrow a^2+b^2+c^2 \leq \frac{abc}{4F} * \frac{s}{2F}(ab+bc+ca)$$
$$\Leftrightarrow 16F^2(a^2+b^2+c^2) \leq abc(a+b+c)(ab+bc+ca)$$
$$\Leftrightarrow (2a^2b^2+2b^2c^2+2a^2c^2-a^4-b^4-c^4)(a^2+b^2+c^2)$$
$$\leq abc(a+b+c)(ab+bc+ca)b$$
$$\Leftrightarrow \sum a^6+\sum a^3b^2c+\sum a^3bc^2 \geq 3a^2b^2c^2+\sum a^4b^2+\sum a^2b^4. \quad (1)$$

By Schur's Inequality we have

$$\sum a^6+3a^2b^2c^2 \geqslant \sum a^4b^2+\sum a^2b^4,$$

and from the AM- GM inequality we obtain

$$\sum a^3b^2c \geqslant 3a^2b^2c^2, \sum a^3bc^2 \geqslant 3a^2b^2c^2$$

which by adding up yields inequality (1).

Problem 926. *Proposed by the editor.*
Prove that the sequence $a_1 = 1, a_2 = 1, a_n = a_{n-1} + 2 * a_{n-2}$ for all $n > 2$ gives the number of integers between 2^n and 2^{n+1} which are divisible by 3.

Solution *by Angel Plaza, Universidad de Las Palmas de Gran Canaria, Spain.*

The numbers given in the sequence are called Jacobsthal numbers, and an explicit form for a_n is given by the Binet-type formula $a_n = \frac{2^n-(-1)^n}{3}$. The property stated by the problem is a straightforward consequence of this formula. The number of integers between 2^n and 2^{n+1} which are divisible by 3, are $\left\lfloor \frac{2^{n+1}-2^n+1}{3} \right\rfloor = \left\lfloor \frac{2^n+1}{3} \right\rfloor = \frac{2^n-(-1)^n}{3}$.

Also solved by Brian Beasley, Simpsonville, South Carolina; Brent Dozier, North Carolina Wesleyan University, Rocky Mount, North Carolina; Henry Ricardo, Westchester Area Math Circle, Purchase, New York; Albert Stadler, Herrliberg, Switzerland; John Zerger, Catawba College, Salisbury, North Carolina; and the proposer.

Problem 927. *Proposed by the editor.*
Find the area below the two lines $8x+5y = 976$ and $6x+5y = 792$ that lies in the first quadrant.

Solution *by John Zerger, Catawba College, Salisbury, North Carolina.*

The vertices of the region are, in counterclockwise order, starting with the origin are $(0,0),(122,0),(92,48),(0,\frac{792}{5})$. Using the Shoelace Theorem, the area of the enclosed region is

$$A=\frac{1}{2}\left[\begin{vmatrix} 0 & 122 \\ 0 & 0 \end{vmatrix}+\begin{vmatrix} 122 & 92 \\ 0 & 48 \end{vmatrix}+\begin{vmatrix} 92 & 0 \\ 48 & 792/5 \end{vmatrix}+\begin{vmatrix} 0 & 0 \\ 792/5 & 0 \end{vmatrix}\right]=\frac{51072}{5}.$$

Also solved by Brian Beasley, Simpsonville, South Carolina; Henry Ricardo, Westchester Area Math Circle, Purchase, New York; Albert Stadler, Herrliberg, Switzerland; and the proposer.

Kappa Mu Epsilon News

Edited by Mark Hughes, Historian
Updated information as of June 2024

News of chapter activities and other noteworthy KME events should be sent to

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Chapter News

AL Theta – Jacksonville State University
Chapter President – Nicholas Covalsen; 353 Total Members; 14 New Members
Other Spring 2024 Officers: Adam Parton, Vice President; Jacob Skipper, Secretary; Lucas Saone, Treasurer; Dr. David Dempsey, Corresponding Secretary; and Dr. Jason Cleveland, Faculty Sponsor.
The Alabama Theta chapter met at least monthly during Spring 2024 for game nights. We held an in-person initiation ceremony on March 8, 2024, inducting 14 new members. Our last meeting of the year, April 19, was a combined Trivia Night with the other student organizations in our building (IEEE, UPE, Psi Chi, Psychology Club).
New Initiates – Layne Brown, Qays Ghazal, Sydney Heidrich, William Emory Hughes, Skyla Key, Avery Price, Madelyn Repzynski, Hayden Nikolas Robinson, Auburn Rollins, Madison Grace Sisson, Caelyn Stedham, Justin Toliver, Angelica Roxanna Vargas, and Jackson Washburn.

AR Beta – Henderson State University
Chapter President – Kimberly "Kristen" Harper; 74 Total Members
Other Spring 2024 Officers: Trenton Moore, Vice President; Valerie Grigar, Secretary; James Kegley, Treasurer; and Catherine Leach, Corresponding Secretary and Faculty Sponsor.

CT Beta – Eastern Connecticut State University
Corresponding Secretary and Faculty Sponsor – Dr. Mehdi Khorami; 560 Total Members; 7 New Members
New Initiates – Christian Deras-Rodriguez, Samuel Fagerquist, Tim Lee, Jarod Martin, Aaron Matus, Sean Mitchell, and Ryan Toomey.

CT Gamma – Central Connecticut State University
Corresponding Secretary – Gurbakhshash Singh; 78 Total Members
Other Spring 2024 Officer: Nelson Castaneda, Faculty Sponsor.

GA Zeta – Georgia Gwinnett College
Chapter President – Edgar Derricho; 69 Total Members
Other Spring 2024 Officers: Gabriel Amat, Vice President; Matt Elenteny, Secretary; Dr. Jamye Curry Savage, Corresponding Secretary and Faculty Sponsor; and Dr. Livy Uko, Faculty Sponsor.
Our chapter had 2 members to graduate: Edgar Derricho (Applied Math – IT Data Analysis) and Matt Elenteny (Pure Math). We also had 2 members to informally join our chapter this semester, and we will hold a formal initiation ceremony in the fall to recognize these students and their achievements.

GA Theta – College of Coastal Georgia
Chapter President – Casey Griffin; 34 Total Members; 3 New Members
Other Spring 2024 Officers: Jalen Shepard, Vice President; Zach Atkinson, Secretary; Jimmy Hutchinson, Treasurer; Dr. Syvillia Averett, Corresponding Secretary; and Aaron Yeager, Faculty Sponsor.
Over the spring semester, the Georgia Theta Chapter of Kappa Mu Epsilon inducted three new members and we had one meeting.
New Initiates – Sydnie Bass, Félix Arroyo Viglino, and Elizabeth Mahas.

IA Alpha – University of Northern Iowa
Chapter President – Grace Croat; 1125 Total Members; 2 New Members
Other Spring 2024 Officers: Quinn Robinson, Vice President; Krista Zimmer, Secretary; Erica Peters, Treasurer; and Dr. Mark D. Ecker, Corresponding Secretary and Faculty Sponsor.
Eight student members of KME and three faculty members met on Tuesday, April 30, 2024 in Wright Hall for our fall KME meeting/banquet. Grace Croat presented her senior seminar project entitled "An Analysis of the Determinants of Breast Cancer Recurrence and Deaths Among Cancer Patients" and two new student members were initiated at our meeting.

IA Gamma – Morningside University
Chapter President – Isaiah Hinners; 445 Total Members
Other Spring 2024 Officers: Kelsey Schieffer, Vice President and Secretary; Fred Lageschulte, Treasurer; and Dr. Eric Canning, Corresponding Secretary and Faculty Sponsor.
There were no new initiates in Spring 2024. We plan on having an initiation ceremony early in the fall of 2024 because a number of math majors will have met the requirements to join KME once the spring semester has concluded. Our KME math club met 6 different evenings during the fall semester. At these meetings, we had guest speakers 3 times, baked pies to sell for π-day (3/14/2024), dyed Easter

eggs, and had a game and pizza night.

IA Delta – Wartburg College
Chapter President – Beth Johll; 787 Total Members; 9 New Members
Other Spring 2024 Officers: Lakshmi Srujana Dandem, Vice President; Aden Stroup, Secretary; Eli Swanstrom, Treasurer; Brian Birgen, Corresponding Secretary; and Mariah Birgen, Faculty Sponsor.
Our initiation ceremony was held on March 16, 2024. Justin Peters, a 2007 alum, spoke.

IL Zeta – Dominican University
Corresponding Secretary and Faculty Sponsor – Mihaela Blanariu; 471 Total Members; 10 New Members
We have inducted ten new members in Spring 2024.

IL Theta – Benedictine University
Chapter President – Julia Sakowicz; 303 Total Members; 1 New Member
Other Spring 2024 Officers: Julia Sakowicz, Vice President, Secretary, and Treasurer; and Manmohan Kaur, Corresponding Secretary and Faculty Sponsor.
New Initiate – Jack Daniel Lyons.

IN Beta – Butler University
Chapter President – Evan Blom; 460 Total Members; 8 New Members
Other Spring 2024 Officers: Jenna Lane, Vice President; Dylan Laudenschlager, Secretary; Sarah Moore, Secretary; and Rasitha Jayesekere, Corresponding Secretary and Faculty Sponsor.

- Our KME president, Evan Blom, completed his honors thesis titled "New Algorithms for the Multiplication Table Problem" and presented results at the 2024 Undergraduate Research Conference at Butler University in April.
- Our KME Vice President Jenna Lane received the prestigious national Goldwater Scholarship this year, from The Barry Goldwater Scholarship and Excellence in Education Foundation.
- We initiated 8 new KME members during the Department of Mathematical Sciences awards ceremony in April.

Pictured above (left to right): Ellei Coleman, Cara Oser, Dankia Casa, and Aiden Johnson Not pictured: Gabriela Campbell, Will Kelleher, Elizabeth Pinel, Gillian Cutshaw

KS Beta – Emporia State University
Chapter President – Chris Brooke; 1550 Total Members; 4 New Members
Other Spring 2024 Officers: Lana Piepho, Vice President; Cahrin Pinkston, Secretary; Maliki Mosher, Treasurer; Tom Mahoney, Corresponding Secretary; and Brian Hollenbeck, Faculty Sponsor.
New Initiates – Cash Waddell, Chris Brooke, Cahrin Pinkston, and Zach Guiciardi.

KS Delta – Washburn University
Chapter President – James Gillin; 848 Total Members; 11 New Members
Other Spring 2024 Officers: Graci Postma, Vice President; Elly George, Secretary and Treasurer; and Sarah Cook, Corresponding Secretary and Faculty Sponsor.
On March 26, ten students and one faculty member were initiated into the Kansas Delta Chapter of Kappa Mu Epsilon. Students Nathan Walker and Mohammad Asfaque, along with faculty sponsors Jillian Kimzey and Sarah Cook, attended the North Central Regional Convention hosted by Missouri Alpha at Missouri State University in Springfield, Missouri.

MD Delta – Frostburg State University
Chapter President – Kaitlyn Custer; 552 Total Members; 6 New Members
Other Spring 2024 Officers: Ricky Day, Vice President; Faith James Sergent, Secretary; Dawson Hormuth, Treasurer; Mark Hughes, Corresponding Secretary and Faculty Sponsor; and Frank Barnet, Faculty Sponsor.

Maryland Delta Chapter had its first meeting in February where we made plans for the semester. We were pleased to welcome six new members at our March 3 initiation ceremony. At this ceremony, faculty sponsor Dr. Mark Hughes gave a presentation entitled "Huygens, Curvature, and the Pendulum Clock". On March 14, we had our annual Pi-Day Bake Sale which was a great success. After Spring Break, we had our March meeting where we enjoyed pizza, math videos and puzzles. We had our final meeting of the semester on May 1 where we elected new officers: Alyssa Kush as president, Te'a Thompson as vice president, Emilia Germain as secretary, and Gabe Hicks as treasurer. We offer congratulations to graduating members Kaitlyn Custer, Ricky Day, Faith James Sergent, Dawson Hormuth, and Christian Speir.

New Initiates – Emilia Germain, Gabriel Hicks, Alyssa Kush, Robert Moffett, Christian Speir, and Te'a Thompson.

MI Beta – Central Michigan University

Chapter President – Maleia Thompson; 1762 Total Members

Other Spring 2024 Officers: Julia Savage, Vice President; Elijah Hayes, Treasurer; and Dr. Dmitry Zakharov, Corresponding Secretary and Faculty Sponsor.

In the spring semester, the Michigan Beta Chapter held 6 general meetings. We hosted a talk and Q&A run by doctoral students from Central Michigan University's math department, a presentation of an honors capstone project, and a talk on a CMU PhD student's research. We held a math Kahoot meeting and math scavenger hunt. We also participated in a multi-club social with other STEM based clubs.

MI Epsilon – Kettering University

Corresponding Secretary and Faculty Sponsor – Valerie Sosnowskii; 1196 Total Members, 11 New Members

We are hosting math colloquiums each term (four per year).

New Initiates – Jasmine Adamson, Thomas Borst, Madison Clapp, Daniel Hadden, Lance Lipasek, Robert McCollum, Grace Pulley, Blake Ronne, Ella Ryan, Cole Thomas, and Dexter Woods.

MO Theta – Evangel University

Chapter President – Tori Risner; 313 Total Members; 8 New Members

Other Spring 2024 Officers: Abby Harrison, Vice President; and Dianne Twigger, Corresponding Secretary and Faculty Sponsor.

MO Theta initiated eight new members this term. Meetings were held monthly and many students attended the regional conference hosted by the KME chapter at Missouri State University. Two students from our chapter (Micah Herron and Tori Risner) presented at the conference.

New Initiates – Benjamin Brewster, Cole Cooper, Erin Hancock, Dayton Hansen, Abby Harrison, Isaiah Holgerson, Theodora W. Osborne, and Selah Whisman.

MO Iota – Missouri Southern State University

Chapter President – Lauren Emanuel; 445 Total Members; 3 New Members
Other Spring 2024 Officers: Weston Allen, Secretary; and Dr. Amila Appuhamy, Corresponding Secretary and Faculty Sponsor.

The 49^{th} KME Initiation Ceremony for the Missouri Iota Chapter took place on Thursday, April 18th, 2024. During the ceremony, three new members were initiated. Additionally, thirteen guests and three faculty members attended both the ceremony and the banquet.

New Initiates – Joseph Courter, Edward Emanuel III, and Jerrel Smith.

MO Kappa – Drury University

Chapter President – Samuel Fullbright; 361 Total Members, 16 New Members
Other Spring 2024 Officers: Nicolette Gaston, Vice President; Hanna Ritter, Secretary; Kylie Warden, Treasurer; and Colin T. Barker, Corresponding Secretary and Faculty Sponsor.

This year our chapter held meetings twice weekly to explore problems in mathematics. One research project came out of these meetings, *writing a piece of music using group theory*. This was presented at the MAKO regional conference in Springfield, MO. A second semester-long KME exclusive discussion revolved around palindromic numbers in multiple bases. We anticipate a presentation and short article to be written about this and would explore publishing in the KME journal. Future aspirations are to host KME alumnae to speak on interesting utilizations of mathematics within their careers.

New Initiates – Allyse (AJ) Agers, Lane Boswell, Aly Boyd, Emma Dole, Ivan Dubinin, Leila Ehrichs, Allysa Freimanis, Sean Greeley, Layden Halcomb, Joseph O'Neill, Jali Purcell, Cam Slade, Abby Stunja, Claire Conover, Preston Dotson, and Landry Parnell.

MS Alpha – Mississippi University for Women

Chapter President – Joshua White; 842 Total Members, 2 New Members
Other Spring 2024 Officers: Audrey Mitchell, Vice President; and Dr. Joshua Hanes, Secretary, Treasurer, Corresponding Secretary and Faculty Sponsor.

After not having any active members on campus for the fall of 2023, I was able to induct two new members in the spring. We are looking forward to doing activities in the fall with our new members and inducting several new members who are soon to meet the requirements for membership in the fall of 2024.

NC Epsilon – North Carolina Wesleyan University

Chapter President – Alexis Whitfield; 121 Total Members; 4 New Members
Other Spring 2024 Officers: Chance Savage, Vice President; Victoria Seggiaro Parma, Secretary; Niana Gunter, Treasurer; Bill Yankosky, Corresponding Secretary; and Brent Dozier, Faculty Sponsor.

New Initiates – Niana Gunter, Ashley Renee Riley, Chance Matoskah Savage, and Victoria Seggiaro Parma.

NC Zeta – Catawba College
Chapter President – Morgan Childress; 99 Total Members; 2 New Members
Other Spring 2024 Officers: Marit Reckmann, Vice President; Ian Shue, Secretary; and Dr. Katherine Baker, Corresponding Secretary and Faculty Sponsor.

NY Nu – Hartwick College
Chapter President – Runyararo Chaora; 357 Total Members
Other Spring 2024 Officers: Dereck Cupernall, Vice President; Jake Thorry, Secretary and Treasurer; and Min Chung, Corresponding Secretary and Faculty Sponsor.

NY Omicron – St. Joseph's University
Chapter President – Robert Kohlmann; 357 Total Members; 5 New Members
Other Spring 2024 Officers: Maddie Frascogna, Vice President; Anthony Randall, Secretary; Elana Reiser, Corresponding Secretary; and Dr. Donna Pirich, Faculty Sponsor.
This is the first year we have had an induction since 2018. We have a small, but dedicated group. Our members volunteered at The Learning Connection to tutor immigrant women in math (`https://brentwoodcsj.org/ministry-areas/education/the-learning-connection/`).
New Initiates – Jenna Rose Ansaldi, Olivia Mary Cassone, Maddy Frascogna, Robert Kohlmann III, and Anthony Randall.

OH Gamma – Baldwin Wallace University
Chapter President – Julia Gersey; 1071 Total Members; 18 New Members
Other Spring 2024 Officers: Kathryn Raubolt, Vice President; Malini Gaddamanugu, Secretary; and David Calvis, Corresponding Secretary and Faculty Sponsor.
Our Spring 2024 initiation ceremony was held on April 21 in our beautiful new Knowlton Center, in conjunction with computer science honorary Upsilon Pi Epsilon.
New Initiates – Violet Beach, Joshua P. Buxton, Braddon Dennison, Maura Devine, Lauren Dunlap, Mario Escobar, Mack Fisher, Benjamin Hughes, Kaley Lanza, Kendall Mamich, Zachary Mihok, Zella Miller, Ella Quick, Owen Rogonjic, Jenna Schifano, Corey Schwarz, Luke Vucenovic, and Isaac Zabarsky.

OH Theta – Capital University
Chapter President – Anna Kasunick; 76 Total Members; 5 New Members
Other Spring 2024 Officers: Ally Davis, Vice President; Peter Brand, Secretary; Hannah Grissom, Treasurer; Paula Federico, Corresponding Secretary; and Jon Stadler, Faculty Sponsor.

During the spring semester our chapter held two meetings and welcomed five new members. Our membership is slowly increasing after very low years during and after the Covid Pandemic. During the spring, the chapter hosted a Pi-Day event handing out mini-pies and a solar eclipse watch party handing out glasses and math inspired candy. Our initiation ceremony was on Wednesday May 1, 2024. Our guest speaker was graduating student and our chapter president, Anna Kasunick. Her interactive presentation was entitled "The Mathematics of Spot it!" All attendees had fun creating a set of cards to play the game using stickers. After his presentation, five new members were inducted into our chapter of KME. The chapter will select new officers for the 2024-2025 academic year at the beginning of the 2024 Fall Semester.

OH Theta – from left to right: Mya Swearengin, Olivia Schneider, Megan Mayse

PA Iota – Shippensburg University

Chapter President – Becca Halvorson; 770 Total Members; 4 New Members

Other Spring 2024 Officers: Autumn Chandler, Vice President; Ryan Moore, Secretary; Caniah Mayo, Treasurer; and Dr. Paul Taylor, Corresponding Secretary and Faculty Sponsor.

We are recovering from a big drop in initiates during covid.

PA Mu – Saint Francis University

Chapter President – McKenzie Watt; 533 Total Members; 14 New Members

Other Spring 2024 Officers: Sarah Evans, Vice President; Alexandra Ochs, Sec-

retary; Caleb Stivanelli, Treasurer; and Dr. Brendon LaBuz, Corresponding Secretary and Faculty Sponsor.
The Pennsylvania Mu Chapter of Kappa Mu Epsilon National Mathematics Honor Society held its initiation ceremony on Tuesday March 26, 2024 at 5:30 pm in the John N. Wozniak Atrium in the Science Center. Dr. Brendon LaBuz, faculty sponsor, began the evening with some opening remarks. An opening prayer from Brother Marius Strom followed. After dinner Dr. LaBuz gave a short talk on the history of Euclidean geometry and the discovery of hyperbolic geometry. Officers McKenzie Watt, Sarah Evans, Alexandra Ochs, and Caleb Stivanelli led the initiation ritual. New society members Sara Blanco, Connor Clarke, Toby Cree, Victoria DiRenno, Priya Gupta, Mason Hogue, Kennedy Kokoski, Madeline Menta, Dallan Schoenberger, Abigail Seese, John Shuster, Sydney Sowers, Thomas Urbain, Austin Wheeler were welcomed to the Pennsylvania Mu Chapter. Congratulations to our new members!

PA Rho – Thiel College
Chapter President – Steven Wright; 156 Total Members; 8 New Members
Other Spring 2024 Officers: Emmalee Sheeler, Vice President; Juliana Peace, Secretary; Bailey Stilts, Treasurer; Dr. Russell Richins, Corresponding Secretary; and Dr. Jie Wu, Faculty Sponsor.
During the spring 2024 semester we held several meetings, organized a successful Challenge 24 competition which benefitted the local food bank, and held our initiation ceremony at which eight new members were added.

PA Sigma – Lycoming College
Chapter President – Haley Seebold; 176 Total Members; 7 New Members
Other Spring 2024 Officers: Kaitlyn Haefner, Vice President; Allison Kelly, Secretary; Zoe Stauffer, Treasurer; Dr. Andrew Brandon, Corresponding Secretary and Faculty Sponsor.

RI Beta – Bryant University
Corresponding Secretary – Prof. John Quinn; 224 Total Members; 7 New Members
Other Spring 2024 Officer: Prof. Gao Niu, Faculty Sponsor.
We held our KME initiation ceremony on April 30, 2024 during which we welcomed 7 new student members into our Rhode Island Beta chapter.

SD Beta – Black Hills State University
Chapter President – Timothy Brown; 12 Total Members; 2 New Members
Other Spring 2024 Officers: Steven William, Vice President; Jake Siewert, Secretary and Treasurer; and Dr. Parthasarathi Nag, Corresponding Secretary and Faculty Sponsor.
The chapter members were actively involved in participating and assisting in the MAA Rocky Mountain Section meeting in the spring of 2023 which was held

at BHSU. Two of the chapter members went to the Rocky Mountain Section of the MAA meeting and one of the chapter members presented an academic undergraduate research paper at the MAA Rocky Mountain Section 2024 Meeting. The meeting was held at Colorado College on April 19-20, 2024.

TN Gamma – Union University
Chapter President – Joy Lewis; 537 Total Members; 3 New Members
Other Spring 2024 Officers: Jacob Carbonell, Vice President; Georgia Morgan, Secretary and Treasurer; Ian Banderchuck, Webmaster and Historian; Bryan Dawson, Corresponding Secretary; and Matt Lunsford, Faculty Sponsor.
Our annual initiation banquet was held at Brooksie's Barn. A student-led activity was to guess which response to various questions was provided by which mathematics professor. Fun was had by all.
New Initiates – William Brady Forester, Liliana Pettigrew, and Stacia Talbott.

TX Lambda – Trinity University
Corresponding Secretary and Faculty Sponsor – Dr. Hoa Nguyen; 330 Total Members; 4 New Members
New Initiates – Janet Jiang, Marco Botello, Burgess Rugeley, and Ashley Breu.

TX Mu – Schreiner University
Chapter President – Jacob Plummer; 217 Total Members; 8 New Members
Other Spring 2024 Officers: Dominic Civello, Vice President; Christopher Jones, Secretary; and Rachel Lynn, Corresponding Secretary and Faculty Sponsor.
In Spring 2024, the Texas Mu KME chapter at Schreiner University inducted 8 new members in an induction ceremony on April 17, 2024.
New Initiates – Ryan Bohls, Kris Ersch, Seth Joseph, Aleksander Kowalczyk, Rachel Lynn, Ryder Michael, Bradley Russell, and Kaloeb Son.

WI Alpha – Mount Mary University
Chapter President – Mary Parlier; 312 Total Members; 2 New Members
Other Spring 2024 Officers: Marissa Heraly, Vice President and secretary; Megan Schmitz, Treasurer; Jeremy Edson, Corresponding Secretary and Faculty Sponsor.
New Initiates – Magdalene Akpan and Caitlyn Booms.

WV Alpha – Bethany College
Chapter President – Karleigh Clegg; 202 Total Members; 4 New Members
Other Spring 2024 Officers: Joseph Hubert, Vice President; Alexus Reid, Secretary and Treasurer; and Dr. Adam C. Fletcher, Corresponding Secretary and Faculty Sponsor.
West Virginia Alpha is getting back on its feet! The chapter and our local Mathematics and Computer Science Club hosted small chess and gaming tournaments on campus this year. They successfully executed the return of the annual Math/Science Day Competition for local high school students and grew participation in that

event. The chapter held a joint induction ceremony (in which four new members were inducted) with the Upsilon Pi Epsilon international honor society in the computing sciences (which inducted two), engaging and networking with some of our alumni.

New Initiates – Karleigh Danielle Clegge, Kyle Eugenio D'Angela, Joseph T Hubert, and Alexus Anastacia Reid.

Active Chapters of Kappa Mu Epsilon

Listed by date of installation

Chapter	Location	Installation Date
OK Alpha	Northeastern State University, Tahlequah	18 Apr 1931
IA Alpha	University of Northern Iowa, Cedar Falls	27 May 1931
KS Alpha	Pittsburg State University, Pittsburg	30 Jan 1932
MO Alpha	Missouri State University, Springfield	20 May 1932
MS Alpha	Mississippi University for Women, Columbus	30 May 1932
NE Alpha	Wayne State College, Wayne	17 Jan 1933
KS Beta	Emporia State University, Emporia	12 May 1934
AL Alpha	Athens State University, Athens	5 Mar 1935
NM Alpha	University of New Mexico, Albuquerque	28 Mar 1935
IL Beta	Eastern Illinois University, Charleston	11 Apr 1935
AL Beta	University of North Alabama, Florence	20 May 1935
AL Gamma	University of Montevallo, Montevallo	24 Apr 1937
OH Alpha	Bowling Green State University, Bowling Green	24 Apr 1937
MI Alpha	Albion College, Albion	29 May 1937
MO Beta	University of Central Missouri, Warrensburg	10 Jun 1938
TX Alpha	Texas Tech University, Lubbock	10 May 1940
KS Gamma	Benedictine College, Atchison	26 May 1940
IA Beta	Drake University, Des Moines	27 May 1940
TN Alpha	Tennessee Technological University, Cookeville	5 Jun 1941
MI Beta	Central Michigan University, Mount Pleasant	25 Apr 1942
NJ Beta	Montclair State University, Upper Montclair	21 Apr 1944
IL Delta	University of St. Francis, Joliet	21 May 1945
KS Delta	Washburn University, Topeka	29 Mar 1947
MO Gamma	William Jewell College, Liberty	7 May 1947
TX Gamma	Texas Woman’s University, Denton	7 May 1947
WI Alpha	Mount Mary College, Milwaukee	11 May 1947
OH Gamma	Baldwin-Wallace College, Berea	6 Jun 1947
MO Epsilon	Central Methodist College, Fayette	18 May 1949
MS Gamma	University of Southern Mississippi, Hattiesburg	21 May 1949
IN Alpha	Manchester College, North Manchester	16 May 1950
PA Alpha	Westminster College, New Wilmington	17 May 1950
IN Beta	Butler University, Indianapolis	16 May 1952
KS Epsilon	Fort Hays State University, Hays	6 Dec 1952
PA Beta	LaSalle University, Philadelphia	19 May 1953
VA Alpha	Virginia State University, Petersburg	29 Jan 1955
IN Gamma	Anderson University, Anderson	5 Apr 1957
CA Gamma	California Polytechnic State University, San Luis Obispo	23 May 1958
TN Beta	East Tennessee State University, Johnson City	22 May 1959
PA Gamma	Waynesburg College, Waynesburg	23 May 1959
VA Beta	Radford University, Radford	12 Nov 1959
NE Beta	University of Nebraska—Kearney, Kearney	11 Dec 1959
IN Delta	University of Evansville, Evansville	27 May 1960
OH Epsilon	Marietta College, Marietta	29 Oct 1960
MO Zeta	University of Missouri—Rolla, Rolla	19 May 1961
NE Gamma	Chadron State College, Chadron	19 May 1962
MD Alpha	College of Notre Dame of Maryland, Baltimore	22 May 1963
CA Delta	California State Polytechnic University, Pomona	5 Nov 1964
PA Delta	Marywood University, Scranton	8 Nov 1964
PA Epsilon	Kutztown University of Pennsylvania, Kutztown	3 Apr 1965
AL Epsilon	Huntingdon College, Montgomery	15 Apr 1965
PA Zeta	Indiana University of Pennsylvania, Indiana	6 May 1965
TN Gamma	Union University, Jackson	24 May 1965
IA Gamma	Morningside College, Sioux City	25 May 1965
MD Beta	McDaniel College, Westminster	30 May 1965
IL Zeta	Dominican University, River Forest	26 Feb 1967
SC Beta	South Carolina State College, Orangeburg	6 May 1967
PA Eta	Grove City College, Grove City	13 May 1967
NY Eta	Niagara University, Niagara University	18 May 1968
MA Alpha	Assumption College, Worcester	19 Nov 1968
MO Eta	Truman State University, Kirksville	7 Dec 1968
IL Eta	Western Illinois University, Macomb	9 May 1969
OH Zeta	Muskingum College, New Concord	17 May 1969
PA Theta	Susquehanna University, Selinsgrove	26 May 1969
PA Iota	Shippensburg University of Pennsylvania, Shippensburg	1 Nov 1969
MS Delta	William Carey College, Hattiesburg	17 Dec 1970
MO Theta	Evangel University, Springfield	12 Jan 1971
PA Kappa	Holy Family College, Philadelphia	23 Jan 1971
CO Beta	Colorado School of Mines, Golden	4 Mar 1971
KY Alpha	Eastern Kentucky University, Richmond	27 Mar 1971
TN Delta	Carson-Newman College, Jefferson City	15 May 1971
NY Iota	Wagner College, Staten Island	19 May 1971
SC Gamma	Winthrop University, Rock Hill	3 Nov 1972
IA Delta	Wartburg College, Waverly	6 Apr 1973
PA Lambda	Bloomsburg University of Pennsylvania, Bloomsburg	17 Oct 1973
OK Gamma	Southwestern Oklahoma State University, Weatherford	1 May 1973

NY Kappa	Pace University, New York	24 Apr 1974
TX Eta	Hardin-Simmons University, Abilene	3 May 1975
MO Iota	Missouri Southern State University, Joplin	8 May 1975
GA Alpha	State University of West Georgia, Carrollton	21 May 1975
WV Alpha	Bethany College, Bethany	21 May 1975
FL Beta	Florida Southern College, Lakeland	31 Oct 1976
WI Gamma	University of Wisconsin—Eau Claire, Eau Claire	4 Feb 1978
MD Delta	Frostburg State University, Frostburg	17 Sep 1978
IL Theta	Benedictine University, Lisle	18 May 1979
PA Mu	St. Francis University, Loretto	14 Sep 1979
AL Zeta	Birmingham-Southern College, Birmingham	18 Feb 1981
CT Beta	Eastern Connecticut State University, Willimantic	2 May 1981
NY Lambda	C.W. Post Campus of Long Island University, Brookville	2 May 1983
MO Kappa	Drury University, Springfield	30 Nov 1984
CO Gamma	Fort Lewis College, Durango	29 Mar 1985
NE Delta	Nebraska Wesleyan University, Lincoln	18 Apr 1986
TX Iota	McMurry University, Abilene	25 Apr 1987
PA Nu	Ursinus College, Collegeville	28 Apr 1987
VA Gamma	Liberty University, Lynchburg	30 Apr 1987
NY Mu	St. Thomas Aquinas College, Sparkill	14 May 1987
OH Eta	Ohio Northern University, Ada	15 Dec 1987
OK Delta	Oral Roberts University, Tulsa	10 Apr 1990
CO Delta	Mesa State College, Grand Junction	27 Apr 1990
PA Xi	Cedar Crest College, Allentown	30 Oct 1990
MO Lambda	Missouri Western State College, St. Joseph	10 Feb 1991
TX Kappa	University of Mary Hardin-Baylor, Belton	21 Feb 1991
SC Delta	Erskine College, Due West	28 Apr 1991
NY Nu	Hartwick College, Oneonta	14 May 1992
NH Alpha	Keene State College, Keene	16 Feb 1993
LA Gamma	Northwestern State University, Natchitoches	24 Mar 1993
KY Beta	Cumberland College, Williamsburg	3 May 1993
MS Epsilon	Delta State University, Cleveland	19 Nov 1994
PA Omicron	University of Pittsburgh at Johnstown, Johnstown	10 Apr 1997
MI Delta	Hillsdale College, Hillsdale	30 Apr 1997
MI Epsilon	Kettering University, Flint	28 Mar 1998
MO Mu	Harris-Stowe College, St. Louis	25 Apr 1998
GA Beta	Georgia College and State University, Milledgeville	25 Apr 1998
AL Eta	University of West Alabama, Livingston	4 May 1998
PA Pi	Slippery Rock University, Slippery Rock	19 Apr 1999
TX Lambda	Trinity University, San Antonio	22 Nov 1999
GA Gamma	Piedmont College, Demorest	7 Apr 2000
LA Delta	University of Louisiana, Monroe	11 Feb 2001
GA Delta	Berry College, Mount Berry	21 Apr 2001
TX Mu	Schreiner University, Kerrville	28 Apr 2001
CA Epsilon	California Baptist University, Riverside	21 Apr 2003
PA Rho	Thiel College, Greenville	13 Feb 2004
VA Delta	Marymount University, Arlington	26 Mar 2004
NY Omicron	St. Joseph's College, Patchogue	1 May 2004
IL Iota	Lewis University, Romeoville	26 Feb 2005
WV Beta	Wheeling Jesuit University, Wheeling	11 Mar 2005
SC Epsilon	Francis Marion University, Florence	18 Mar 2005
PA Sigma	Lycoming College, Williamsport	1 Apr 2005
MO Nu	Columbia College, Columbia	29 Apr 2005
MD Epsilon	Stevenson University, Stevenson	3 Dec 2005
NJ Delta	Centenary College, Hackettstown	1 Dec 2006
NY Pi	Mount Saint Mary College, Newburgh	20 Mar 2007
OK Epsilon	Oklahoma Christian University, Oklahoma City	20 Apr 2007
HA Alpha	Hawaii Pacific University, Waipahu	22 Oct 2007
NC Epsilon	North Carolina Wesleyan College, Rocky Mount	24 Mar 2008
NY Rho	Molloy College, Rockville Center	21 Apr 2009
NC Zeta	Catawba College, Salisbury	17 Sep 2009
RI Alpha	Roger Williams University, Bristol	13 Nov 2009
NJ Epsilon	New Jersey City University, Jersey City	22 Feb 2010
NC Eta	Johnson C. Smith University, Charlotte	18 Mar 2010
AL Theta	Jacksonville State University, Jacksonville	29 Mar 2010
GA Epsilon	Wesleyan College, Macon	30 Mar 2010
FL Gamma	Southeastern University, Lakeland	31 Mar 2010
MA Beta	Stonehill College, Easton	8 Apr 2011
AR Beta	Henderson State University, Arkadelphia	10 Oct 2011
PA Tau	DeSales University, Center Valley	29 Apr 2012
TN Zeta	Lee University, Cleveland	5 Nov 2012
RI Beta	Bryant University, Smithfield	3 Apr 2013
SD Beta	Black Hills State University, Spearfish	20 Sept 2013
FL Delta	Embry-Riddle Aeronautical University, Daytona Beach	22 Apr 2014
IA Epsilon	Central College, Pella	30 Apr 2014
CA Eta	Fresno Pacific University, Fresno	24 Mar 2015
OH Theta	Capital University, Bexley	24 Apr 2015
GA Zeta	Georgia Gwinnett College, Lawrenceville	28 Apr 2015
MO Xi	William Woods University, Fulton	17 Feb 2016
IL Kappa	Aurora University, Aurora	3 May 2016
GA Eta	Atlanta Metropolitan University, Atlanta	1 Jan 2017
CT Gamma	Central Connecticut University, New Britan	24 Mar 2017
KS Eta	Sterling College, Sterling	30 Nov 2017
NY Sigma	College of Mount Saint Vincent, The Bronx	4 Apr 2018
PA Upsilon	Seton Hill University, Greensburg	5 May 2018

KY Gamma	Bellarmine University, Louisville	23 Apr 2019
MO Omicron	Rockhurst University, Kansas City	13 Nov 2020
AK Gamma	Harding University, Searcy	27 Apr 2021
GA Theta	College of Coastal Georgia, Brunswick	22 Oct 2021
CA Theta	William Jessup University, Rocklin	17 Oct 2022