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Mathematical Breakthroughs Driven by Wartime in the Twentieth Century

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Abstract

Significant mathematical breakthroughs have occurred during times of war. Given the historical and geopolitical relevance of wars, a sample from World War I, World War II, and the Cold War are explored to highlight their contributions to mathematics. This was achieved by examining published work from each war, while secondary sources are used to discuss the post-war impact. The twentieth century was chosen for its recency and modern applications that significantly influenced the present. The research highlighted key innovative trends during and in the aftermath of each war. World War I led to aeronautical development that affected all aviation. The research started with three equations that describe a force accelerating in the x, y, and z directions. From there, the research followed in the footsteps of George Bryan to derive the two equations that describe aeronautical stabilization. World War II spurred the development of more powerful methods of encryption and decryption to ensure secure data transmission. The research followed the original questions posed and answered by Claude E. Shannon regarding secrecy, encompassing both theoretical and practical considerations. Lastly, the Cold War fostered fast data analysis by condensing vast information. This is illustrated in the calculation of complex Fourier series by comparing the Discrete Fourier Transform (DFT) to the Fast Fourier Transform (FFT). Together, these examples illustrate that powerful advancements in mathematics have played an essential role in shaping history.

Contents

1 Introduction

While the history of mathematics is complex, attempting to dismantle it can further our understanding of how history and mathematics influence each other. Therefore, this paper focuses on innovative breakthroughs from World War I, World War II, and the Cold War to illustrate some of the most impactful discoveries of the twentieth century.

"Most impactful" is, of course, a contentious statement. This paper aims to sample one finding from each war to summarize and analyze its contributions. This demonstrates the paper's lack of dialogue on other findings in the twentieth century and during wartime. Furthermore, this paper is naturally limited, and the reader would benefit from further research on the full effects of each of these mathematical contributions, as well as additional breakthroughs not discussed here.

The paper analyzes the same three aspects from each war: historical context, mathematical development, and modern implications. The historical content will give the reader a necessary understanding of why the mathematical breakthrough was needed. This is followed by the original research that was involved, including how the theory was derived. Lastly, the modern implications of the new theory will demonstrate the useful nature of the developments for the present day.

2 WorldWar I: Aeronautical Stabilization

2.1 The Need For Stability

World War I fostered incredible growth in aviation. Aerodynamic research in Germany was very lively, with *Zeitschrift der Flugzeugmeisterei* on atmospheric boundary layers published during the war (Aubin and Goldstein, 2013, p. 31). A critical way mathematics can be applied in war is through the collaboration of mathematicians with non-mathematicians. This is particularly evident with the Royal Flying Corps, which also needed to innovate, "driven apace by the necessities and incentives of conflict" (Royle, 2017). People like Keith Lucas, a captain in the Royal Flying Corps, paved the way for the expansion of our aerial knowledge. As Royle (2017) illustrates, Lucas flew frequently and was "instrumental in designing and testing a reliable aviation compass" and in developing "more accurate bomb-aiming equipment". Unfortunately, air travel at this time was a very dangerous pursuit that ended the lives of many. This was no different for Lucas, who passed away due to a biplane crash.

George Bryan, a mathematician rather than a pilot, published the paper *Stability In Aviation; An Introduction to Dynamical Stability As Applied to the Motions of Aeroplanes*. Bryan described the conditions to keep an aircraft steady using two equations (Royle, 2017). One for longitudinal stability and one for lateral stability. One of the many intentions of this paper was to mitigate biplane crashes.

There was just one issue; the equations were unsolvable "without knowing certain parameters" of how forces "acting on the aircraft's surfaces altered its motion about its three axes" (Royle, 2017).

However, Edward Busk, a graduate of King's College, Cambridge, "was handpicked to join the Royal Aircraft Factory in 1912" to help fill gaps in the understanding of flight performance (Royle, 2017), e.g., why an aircraft may return to its original flight path after a disturbance. Busk attempted to solve Bryan's equations and to develop an aircraft that would naturally dispel oscillations caused by wind and other disruptions without the need for corrections from the pilot. In other words, how do you make a plane inherently stable so that, theoretically, it can glide so long as fuel does not run out? The movement of objects through fluids (e.g, water and air) was understood. However, for an aircraft, "how the lift created by its aerofoil–shaped wings" (Royle, 2017) affected its motion was still unknown.

It was ultimately Busk who was able to provide the necessary flight information to solve Bryan's equations through his test flights. Because of Busk's data, along with Bryan's equations, "the first inherently stable aircraft, the RE1" (Royle, 2017), was produced in 1913. This, unfortunately, did not stop Busk from also passing away in a biplane crash on November 5th, 1914.

2.2 Bryan's Stability Equations

As mentioned above, in Bryan's 1911 paper, he stated the conditions needed to keep an airplane stable. These two equations were needed in the war effort because, according to Bryan, "the development of the aeroplane has opened up a large number of interesting problems in both theoretical and experimental mechanics" (Bryan, 1911, p. 1). Thus, Bryan's pursuit of stability in airplanes was also an intellectual endeavor.

2.2.1 Longitudinal Stability

The following analysis is based on the steps in Bryan's book. Although most of the derivations are not shown, the general concepts are illustrated similarly to Bryan's work. He first considers oscillations on an airplane when rotated about each axis through some angle. Therefore, Bryan pictured the flight path as shown in Figure 1.

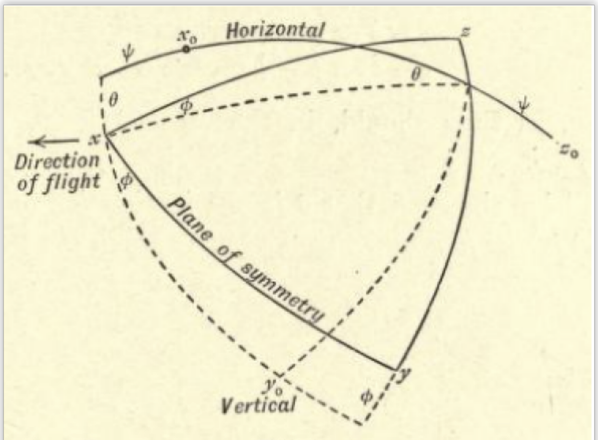

FIG. 2.

Figure 1: Direction of flight as described by Bryan (1911, p. 21).

Before deriving the equations, Bryan defines the variables as such:

- W is the weight of the airplane.
- H is the propeller thrust.
- g is the acceleration due to gravity.
- X, Y, Z are the force components of air resistance in the x, y, and z directions respectively.
- u, v, w are components of translational velocity in the x, y, and z directions respectively.
- p, q, r are components of angular velocity about the x-axis, y-axis, and z-axis respectively.
- ϕ, ψ, θ represents the amount of rotation about the x-axis, y-axis, and z-axis respectively.
- θ_0 is the constant angle of descent relative to the horizon.

To derive Bryan's two equations, we have to start with the descriptions of motion for an accelerating force in the x, y, and z directions given by the following three equations in the same order:

$$W\left(\frac{du}{gdt}+\frac{qw}{g}-\frac{rv}{g}\right)=W\sin\theta+H-X;$$
$$W\left(\frac{dv}{gdt}+\frac{ru}{g}-\frac{pq}{g}\right)=W\cos\theta\cos\phi-Y;$$
$$W\left(\frac{dw}{gdt}+\frac{pv}{g}-\frac{qu}{g}\right)=-W\cos\theta\sin\phi-Z.$$

Bryan (1911, p. 26) states that in "In small oscillations," the difference between θ and θ_0 will be equal to ε, which is a constant referred to as "Antoinette" for lateral stability. Furthermore, ϕ will be a small angle, meaning $\sin\phi \approx \phi$ andit also follows $\cos\phi \approx 1$. This gives us the following identities:

$$\begin{aligned}
\sin\theta &= \sin\theta_0 + \varepsilon\cos\theta_0;\\
\cos\theta &= \cos\theta_0 - \varepsilon\sin\theta_0;\\
\sin\phi &= \phi;\\
\cos\phi &= 1.
\end{aligned}$$

If we "let the thrust be H, initial thrust be H_0, and let thrust act at a perpendicular distance, h, below the origin" (Bryan, 1911, p. 23) then we can now state "the new propeller thrust to be $H_0 + \delta H, h$ remaining constant" (Bryan, 1911, p. 26) where δH is the change in thrust. If U is the uniform velocity of descent in the direction of the x-axis, then we can write u, the "added velocity ... in a small oscillation" (Bryan, 1911, p. 190), as $U + u$. After substituting, we get the following three equations:

$$\begin{aligned}
&W\frac{du}{gdt} = W\left(\sin\theta_0 + \varepsilon\cos\theta_0\right) = H_0 + \delta H - X_0 - uX_u - vX_v - rX_r;\\
&W\left(\frac{dv}{gdt} + \frac{rU}{g}\right) = W\left(\cos\theta_0 - \varepsilon\sin\theta_0\right) - Y_0 - uY_u - vY_v - rY_r;\\
&W\left(\frac{dw}{gdt} - \frac{qU}{g}\right) = -W\phi\cos\theta_0 - Z_0 - wZ_w - pZ_p - qZ_q.
\end{aligned}$$

Bryan (1911, p. 27) then assumes that u, v, r and ε are proportional to $e^{\lambda t}$, where λ is a "coefficient in a small oscillation or disturbance" and must be strictly greater than zero. Furthermore, λ is proportional to $e^{\lambda t}$ such that

$$\frac{du}{dt} = \lambda u, \frac{dv}{dt} = \lambda v, \frac{dv}{dt} = \lambda r, \frac{d\varepsilon}{dt} = \lambda\varepsilon.$$

From $\frac{d\varepsilon}{dt} = \lambda\varepsilon$ Bryan obtains $r = \lambda\varepsilon$. However, the equations are in a more symmetrical form when $\frac{r}{\lambda}$ is substituted for ε. Now, transposing the three equations gives:

$$\begin{aligned}
&\left(W\frac{\lambda}{g} + X_u\right)u + X_v v + \left(-\frac{W}{\lambda}\cos\theta_0 + X_r\right)r = \delta H;\\
&Y_u u + \left(W\frac{\lambda}{g} + Y_v\right)v + \left(\frac{W}{\lambda}\sin\theta_0 + W\frac{U}{g} + Y_r\right)r = 0;\\
&N_u u + N_v v + \left(C\frac{\lambda}{g} + N_r\right)r = -h\delta H,
\end{aligned}$$

where C is a moment of inertia and N is a coupling about an axis due to resulting resistance, opposed to rotation (Bryan, 1911, p.188-189). The next step is assuming the simplest case of no change in thrust "so that $\delta H = 0$" then "the right hand sides ... vanish, and on eliminating u, v, r, the result assumes the form of the determinant" (Bryan, 1911, p. 28)

$$\begin{vmatrix} W\frac{\lambda}{g}+X_u & X_v & -\frac{W}{\lambda}\cos\theta_0+X_r \\ Y_u & W\frac{\lambda}{g}+Y_v & \frac{W}{\lambda}\sin\theta_0+W\frac{U}{g}+Y_r \\ N_u & N_v & C\frac{\lambda}{g}+N_r \end{vmatrix}$$

Then Bryan multiplies by λ to simplify the expression and develops the determinant in powers of λ such that we get an equation with degree four:

$$\mathfrak{A}_0\lambda^4+\mathfrak{B}_0\lambda^3+\mathfrak{C}_0\lambda^2+\mathfrak{D}_0\lambda+\mathfrak{E}_0=0$$

where the subscript of "0" denotes the coefficients for symmetrical oscillations. The exact simplifications and steps are left as an exercise for the reader. These coefficients are given as:

$$\begin{aligned}
&\mathfrak{A}_0 = CW^2;\\
&\frac{\mathfrak{B}_0}{g} = CW(X_u+Y_v)+W^2N_rl;\\
&\frac{\mathfrak{C}_0}{g^2} = C(X_uY_v-X_vY_u)+W\{(Y_vN_r-Y_rN_v)+(X_uN_r-X_rN_u)\}-W^2\frac{U}{g}N_v;\\
&\frac{\mathfrak{D}_0}{g^3} = X_u(Y_vN_r-Y_rN_v)+X_v(Y_rN_u-Y_uN_r)+X_r(Y_uN_v-Y_vN_u)\\
&\qquad\qquad +W\frac{U}{g}(X_vN_u-X_uN_v)+\frac{W^2}{g}(N_u\cos\theta_0-N_v\sin\theta_0);\\
&\frac{\mathfrak{E}_0}{g^4} = \frac{W}{g}\{-\cos\theta_0(Y_uN_v-Y_vN_u)-\sin\theta_0(X_uN_v-X_vN_u)\},
\end{aligned}$$

where the determinant steps are not shown. To simplify the coefficients, Bryan (1911, p. 29) takes advantage of the following notation, to "write Δ_0 for the determinant," which gives a slightly condensed form of the coefficients where

$$\Delta_0 = \begin{vmatrix} X_u & X_v & X_r \\ Y_u & Y_v & Y_r \\ N_u & N_v & N_r \end{vmatrix}.$$

For every entry in Δ_0, we can write the different minors with the notation u_X for X_u, r_N for N_r, etc. This gives

$$\mathfrak{A}_0 = CW^2$$
$$\frac{\mathfrak{B}_0}{g} = CW(X_u + Y_v) + W^2 N_r$$
$$\frac{\mathfrak{C}_0}{g^2} = Cr_N + W(u_X + v_Y) - W^2 \frac{U}{g} N_v$$
$$\frac{\mathfrak{D}_0}{g^3} = \Delta_0 + W \frac{U}{g} r_Y + \frac{W^2}{g}(N_u \cos\theta_0 - N_v \sin\theta_0)$$
$$\frac{\mathfrak{E}_0}{g^4} = -\frac{W}{g}(r_X \cos\theta_0 - r_Y \sin\theta_0).$$

For the biquadratic to be stable, the roots must have all of their real parts negative. (Bryan, 1911, p.30) This means that $\mathfrak{A}_0, \mathfrak{B}_0, \mathfrak{C}_0, \mathfrak{D}_0, \mathfrak{E}_0$, and $\mathfrak{F}_0$, where

$$\mathfrak{F}_0 \equiv \mathfrak{B}_0\mathfrak{C}_0\mathfrak{D}_0 - \mathfrak{A}_0\mathfrak{D}_0^2 - \mathfrak{E}_0\mathfrak{B}_0^2,$$

shall be positive. This equation is almost the final form; however, Bryan (1911, p.30) examines what happens to the biquadratic "if the real part of a pair of roots … from being negative becomes zero." This implies that $\pm\beta_t$ are roots of $\lambda^2 + \beta^2 = 0$ where $\lambda^2 = -\beta^2$ is a solution of the two following equations:

$$\mathfrak{A}_0\lambda^4 + \mathfrak{C}_0\lambda^2 + \mathfrak{F}_0 = 0$$
$$\mathfrak{B}_0\lambda^3 + \mathfrak{D}_0\lambda = 0 \Rightarrow \lambda^2 \text{or} - \beta^2 = -\frac{\mathfrak{D}_0}{\mathfrak{B}_0}.$$

Substituting the second equation into the first gives

$$\mathfrak{A}_0\frac{\mathfrak{D}_0^2}{\mathfrak{B}_0^2} - \mathfrak{C}_0\frac{\mathfrak{D}_0}{\mathfrak{B}_0} + \mathfrak{F}_0 = 0$$

and multiplying by $-\mathfrak{B}_0^2$, we get

$$\mathfrak{B}_0\mathfrak{C}_0\mathfrak{D}_0 - \mathfrak{F}_0\mathfrak{B}_0^2 - \mathfrak{A}_0\mathfrak{D}_0^2 = 0$$

"which therefore represents the limiting case when the real part of a pair of roots changes sign." (Bryan, 1911, p. 31) From the equation of the form,

$$\mathfrak{F}_0 \equiv \mathfrak{B}_0\mathfrak{C}_0\mathfrak{D}_0 - \mathfrak{A}_0\mathfrak{D}_0^2 - \mathfrak{C}_0\mathfrak{B}_0^2$$

we can at last write the final form of the condition for stability as an inequality of the form

$$\mathfrak{C}_0 - \mathfrak{F}_0\frac{\mathfrak{B}_0}{\mathfrak{D}_0} - \mathfrak{A}_0\frac{\mathfrak{D}_0}{\mathfrak{B}_0} > 0 \tag{1}$$

since we derived that $\mathfrak{F}_0$ must be positive, and we divide by $\mathfrak{B}_0$ and $\mathfrak{D}_0$. This equation "necessarily implies the condition $\mathfrak{C}_0$ positive if all the other four coefficients are positive" (Bryan, 1911, p. 31).

2.2.2 Lateral Stability

Similar steps are used to derive the equation for lateral stability. However, Bryan defines the following new variables:

- A, B, C are the moments of inertia, and F is the product of inertia "relating to a rigid body and a pair of given perpendicular axes" (Oxford University Press, 2025).
- L, M "couples about axes due to resistance, in directions opposed to rotation." (Bryan, 1911, p. 189) Furthermore, with a subscript, it indicates "the corresponding resistance derivatives; $L_p = \frac{dL}{dp}$" (Bryan, 1911, p. 189).

The primary differences between longitudinal and lateral stability come from the forms of the equations concerning the motion and forces acting on the airplane. For w, p, q we note

$$W\left(\frac{dw}{gdt} - \frac{qU}{g}\right) = -W\phi\ \cos\theta_0 - wZ_w - pZ_p - qZ_q$$
$$A\frac{dp}{gdt} - F\frac{dq}{gdt} = -wL_w - pL_p - qL_q$$
$$B\frac{dq}{gdt} - F\frac{dq}{gdt} = -wM_w - pM_p - qM_q$$

For asymmetric oscillations, we can let w, p, q, and ϕ be proportional to $e^{\lambda t}$ such that

$$\lambda\phi\cos\theta_0 = \frac{d\phi}{dt}\cos\theta_0 = p\cos\theta_0 - q\sin\theta_0$$

which, Bryan (1911, p. 31) notes, "we use to substitute for ϕ in terms of p and q." This will transpose the three equations of equilibrium to be the following:

$$\left(W\frac{\lambda}{g} + Z_w\right)w + \left(\frac{W}{\lambda}\cos\theta_0 + Z_p\right)p + \left(-W\frac{U}{g} - \frac{W}{\lambda}\sin\theta_0 + Z_q\right)q = 0$$
$$L_w w + \left(A\frac{\lambda}{g} + L_p\right)p + \left(-F\frac{\lambda}{g} + L_q\right)q = 0$$
$$M_w w + \left(-F\frac{\lambda}{g} + M_p\right)p + \left(B\frac{\lambda}{g} + M_q\right)q = 0$$

and it follows that the determinate form becomes

$$\begin{vmatrix} W\frac{\lambda}{g} + Z_w & \frac{W}{\lambda}\cos\theta_0 + Z_P & -W\frac{U}{g} - \frac{W}{\lambda}\sin\theta_0 + Z_q \\ L_w & A\frac{\lambda}{g} + L_p & -F\frac{\lambda}{g} + L_q \\ M_w & -F\frac{\lambda}{g} + M_p & B\frac{\lambda}{g} + M_q \end{vmatrix} = 0.$$

Bryan (1911, p. 32) then simplifies through "Multiplying by λ and expanding the determinant in powers of λ" repeating the steps for the longitudinal equation. This gives

$$\mathfrak{A}_1\lambda^4+\mathfrak{B}_1\lambda^3+\mathfrak{C}_1\lambda^2+\mathfrak{D}_1\lambda+\mathfrak{E}_1=0$$

where the subscript, "1", indicates the coefficients for asymmetric oscillations. The exact simplifications and steps are, again, left as an exercise for the reader. From this, we can note what each coefficient is.

$$\begin{aligned}
&\mathfrak{A}_1 = W(AB-F^2)\\
&\frac{\mathfrak{B}_1}{g} = Z_w(AB-F^2)+W\{AM_q+BL_p+F(L_q+M_q)\\
&\frac{\mathfrak{C}_1}{g^2} = Z_w\{AM_q+BL_p+F(L_q+M_p)\}+W(L_pM_q-L_qM_p)\\
&\qquad\qquad -Z_p(FM_w+BL_w)-\left(Z_q-W\frac{U}{g}\right)(FL_w+AM_w)\\
&\frac{\mathfrak{D}_1}{g^3} = Z_w(L_pM_q-L_qM_p)+Z_p(L_qM_w-M_qL_w))+\left(Z_q-W\frac{U}{g}\right)(L_wM_p-L_pM_w)\\
&\qquad\qquad +\frac{W}{g}\{(FL_w+AM_w)\sin\theta_0-(BL_w+FM_w)\cos\theta_0\}\\
&\frac{\mathfrak{E}_1}{g^4} = \frac{W}{g}\{(L_qM_w-M_qL_w)\cos\theta_0-(L_wM_p-M_wL_p)\sin\theta_0\}
\end{aligned}$$

Now, as was the case for Δ_0, we will write Δ_1 such that

$$\Delta_1=\begin{vmatrix} Z_w & Z_p & Z_q\\ L_w & L_p & L_q\\ M_w & M_p & M_q \end{vmatrix}$$

where we can then use the same notation from Bryan (1911, p. 32) as seen for the equation on longitudinal stability "e.g., w_Z to denote the minor of Z_w, we get"

$$\begin{aligned}
&\mathfrak{A}_1 = W(AB-F^2)\\
&\frac{\mathfrak{B}_1}{g} = Z_w(AB-F^2)+W\{AM_q+BL_p+F(L_q+M_q)\}\\
&\frac{\mathfrak{C}_1}{g^2} = Ap_L+Bq_M-F(q_L+p_M)+Ww_Z+W\frac{U}{g}(AM_w+FL_w)\\
&\frac{\mathfrak{D}_1}{g^3} = \Delta_1-W\frac{U}{g}q_Z+\frac{W}{g}\{(A\sin\theta_0-F\cos\theta_0)M_w+(F\sin\theta_0-B\cos\theta_0)L_w\}\\
&\frac{\mathfrak{E}_1}{g^4} = \frac{W}{g}(p_Z\cos\theta_0-q_Z\sin\theta_0).
\end{aligned}$$

Next, we can write

$$\mathfrak{F}_1\equiv\mathfrak{B}_1\mathfrak{C}_1\mathfrak{D}_1-\mathfrak{A}_1\mathfrak{D}_1^2-\mathfrak{F}_1\mathfrak{B}_1^2$$

and it can thus be stated, "the conditions of asymmetric stability require that $\mathfrak{A}_1, \mathfrak{B}_1, \mathfrak{C}_1, \mathfrak{D}_1, \mathfrak{E}_1$, and $\mathfrak{F}_1$" (Bryan, 1911, p. 33) are positive. As we saw for the derivation of longitudinal stability, we can then divide by $\mathfrak{B}_1$ and $\mathfrak{D}_1$ with the expression being greater than zero, such that

$$\mathfrak{C}_1 - \mathfrak{F}_1 \frac{\mathfrak{B}_1}{\mathfrak{D}_1} - \mathfrak{A}_1 \frac{\mathfrak{D}_1}{\mathfrak{B}_1} > 0. \tag{2}$$

It will be noted that the forms of the longitudinal equation and lateral equation are the same; however, the coefficients are different. From these two equations, we can now state that we have derived Bryan's equations, as he had in 1911. This represents an airplane with inherent longitudinal and lateral stability "when propelled horizontally, and also when the inclination of the flightpath to the horizon does not exceed certain determinate limits" (Bryan, 1911, p. 165). Consequently, the final representations of longitudinal stability (1) and lateral stability (2) are displayed similarly, with the subscript denoting the oscillation type. The primary assumptions, such as small-angle approximations and no change in the thrust of the airplane, make the theoretical accuracy of these equations lower. However, this does not take away from the ingenuity Bryan displayed and its impact on aeronautical flight.

2.3 Contemporary Utilization

Bryan's contributions to mathematics and aviation are unfathomable. His equations for an airplane in flight are essentially the ones used today, although with some notation change (Abzug and Larrabee, 2002, p. 9). Modern aircraft, even the most advanced, use the theory of motion set by Bryan for analysis and simulations. However, a somewhat interesting result from his assumptions is "that the force on an airfoil is perpendicular to the airfoil chord," according to W. Hewitt Phillips, such that the equations are most accurate for supersonic aircraft "particularly those with nearly unswept wings, such as the Lockheed F-104" (Abzug and Larrabee, 2002, p. 10) pictured in Figure 2. This indicates that Bryan was decades ahead of his time, regardless of the limitations in computational power and assumptions to simplify the problems.

Figure 2: Lockheed F-104 (Anonymous, 2005)

There are some limiting factors to consider, as Bryan conceded. His theory "rested on fundamental theories of Sir Isaac Newton (1642–1727) and Leonhard Euler (1707–1783)" (Abzug and Larrabee, 2002, p. 9).. This is an amazing revelation; however, Bryan's theory does not contain control derivatives, only stability. This follows the original logic set forth earlier, as the goal was to ensure a plane could glide along, and not maneuver optimally (e.g., for a dogfight). Furthermore, disturbances from, for example, wind "gusts is ... not addressed" (Abzug and Larrabee, 2002, p. 10), which compounds the observation from Phillips regarding subsonic aircraft not adhering to Bryan's theory as accurately as supersonic ones.

Although his contributions were groundbreaking and critical to World War I efforts, they remain relevant over a century later. His stability equations excel at describing modern aircraft stability. This is especially seen with modern analysis and simulations where "using mathematics to predict the success or failure of an aircraft structure mostly involves tapping keys on a computer" (Royle, 2017). This development warrants thanks to Keith Lucas, Edward Busk, George Bryan, and many more who, through a need to innovate World War I aviation, substantially accelerated the world's understanding of aircraft behavior. However, many of them lost their lives in the process.

Innovation was not limited to aviation. Aubin and Goldstein (2013) highlight how "the war indeed redistributed" mathematics such that "many mathematicians of the war generation extended the limits of their profession beyond academic research" (p. 45). Therefore, Bryan is a prime example. Ultimately, modern mathematicians should be the ones to bridge pure theory and contemporary applications by "knitting all these aspects, from the most theoretical and general theorems" (Aubin and Goldstein, 2013, p. 45) and a combination of close contact with industries.

3 WorldWar II: Cryptography

3.1 Breakthrough That Shortened the War

Mathematical applications are rich in science, and cryptography is an example of the bridge between pure theory and contemporary applications. Encrypting and decrypting messages long predate World War II. The University of Tulsa (2024) contends "cryptography has existed for thousands of years" and "many experts believe ancient Egyptian hieroglyphics were created as a secret code." Julius Caesar did the same in 50 B.C., where "he developed a cipher...that allowed him to communicate" with his generals because he "grew wary of his messengers" (University of Tulsa, 2024). The cipher worked by shifting each letter down by the same constant amount. An amount of zero would leave the encryption unchanged. This is true for the amount of twenty-six, as it would cycle around the alphabet. Therefore, the cipher only encrypts for an amount between one and twenty-five. Figure 3 pictures an example of Caesar's encryption method.

A	B	C	D	...	W	X	Y	Z
↓	↓	↓	↓	↓	↓	↓	↓	↓
D	E	F	G	...	Z	A	B	C

Figure 3: Caesar Shift moving the alphabet three places.

This led to a new branch of mathematics: Cryptography. There were many variations used in coding before the twentieth century. However, “the mathematical nature of cryptography developed through the centuries such that cyphers become entirely mathematical by World War II” (Fries,2022). The Enigma machine highlights this new branch of mathematics. The contributions from World War II were those of pure mathematical theory, but still allowed for applications. The Enigma machine, created by Nazi Germany, encrypted data in a manner so that the resulting messages were extremely difficult to decode. Therefore, the Enigma machine had strategic value for the war, and consequently, the Allied powers worked relentlessly to break the Enigma codes. The National Museum of theUnited States Air Force (2007) extends this point, stating, “they used early computers and raw mathematical talent to work through millions of scrambled possibilities ... until they found the right solution.” As a result, Fries (2022) argues that in World War II “there were significant advancements in cryptography.”

The cryptographic developments in World War II were especially critical because “wireless radio communication was very important for directing military forces spread all over the world” (National Museum of the United States Air Force, 2007). However, radio communications can be intercepted, and thus, the encryption and decryption of communications were a vast improvement in keeping messages secret. Therefore, a secure means of communication became very important not just in World War II but for essentially all later conflicts.

Figure 4: Enigma Machine (CIA, 2011)

The Enigma machine, pictured in Figure 4, was incredibly important to both the Allied and Axis powers, not only because "the Enigma machine [was] used extensively by the Nazi Armed forces," but also because "three Polish mathematicians, after a few weeks of listening to German radio communications, were able to" break the Enigma machine's encryption (Fries, 2022). Moreover, these Polish mathematicians could, without knowing how Nazi Germany developed it, recreate a machine similar to it. Alan Turing then used the information that the Polish mathematicians presented at a conference to develop a device to break the Enigma codes. It was these code breakers that "gave the Allies an important advantage ... and shortened the war, by some estimates as much as two years" (National Museum of the United States Air Force, 2007).

3.2 Theoretical Versus Practical Secrecy

In 1945, Claude Elwood Shannon, the "father of the information age" (Viterbi, 2002, p. 10), wrote "A Mathematical Theory of Cryptography," laying the groundwork for modern understanding. In this paper, Shannon (1945) explores two types of secrecy: theoretical and practical. Theoretical secrecy is connected to "how immune is a system to cryptanalysis when the cryptanalyst has unlimited time and manpower available for the analysis of cryptograms?" (Shannon, 1945, p. 55). On the other hand, practical secrecy is the analysis of ensuring "a system which is not ideal and therefore *has* a unique solution ... will require a large amount of work" (Shannon, 1945, p. 87). In a now declassified document (previously withheld due to the sensitive nature of cryptography, especially during World War II, and its threat to national security), Shannon (1945) analyzes these questions in such a manner that a secure system can be established and a strategy on how to crack these secure systems is formalized.

3.2.1 Theoretical Secrecy

Shannon (1945, p. 55) starts by letting the possible messages be finite and represented by $M_1,\dots,M_n$ and having respective probabilities of $P(M_1),\dots,P(M_n)$ which "are enciphered into the possible cryptograms $E_1 \dots E_m$ by"

$$E = T_i M$$

where T is the number of possible keys. If a cryptanalyst intercepts E, then they can "calculate the *a posteriori* probabilities for the various messages, $P_E(M)$" (Shannon, 1945, p. 55). The notation for $P_E(M)$ is read as the 11conditional probability of M if cryptogram E is intercepted" (Shannon, 1945, p. 31). *Perfect secrecy* is then defined as the condition that the *a posteriori* probabilities equal the *a priori* probabilities for all E. Shannon (1945) illustrates how, if the previous condition is true, "intercepting the message has given the cryptanalyst no information" (p. 55). This implies that even if a cryptanalyst can decipher a message, that information should not aid in trying to decipher another message. Bayes' theorem states

$$P_E(M) = \frac{P(M)P_M(E)}{P(E)} \tag{3}$$

implying that $P_E(M)$ "must equal $P(M)$ for perfect secrecy" (Shannon, 1945, p. 56). Two cases must be true for the equality to be independent of $P(M)$. Either $P(M) = 0$, which we exclude, or

$$P_M(E) = P(E)$$

for all M and E. If

$$P_M(E) = P(E)$$

then

$$P_E(M) = \frac{P(M)P_M(E)}{P(E)} \implies P_E(M) = P(M).$$

This describes Shannon's ninth theorem for perfect secrecy, which is as follows:

"Theorem 9: A necessary and sufficient condition for [perfect] secrecy is that

$$P_M(E) = P(E)$$

for all M and E. That is $P_M(E)$ must be independent of M" (Shannon, 1945, p. 56). Moreover, "the probability of all keys that transform M_i" into the "cryptogram E is equal to that of all keys transforming M_j into the same E" (Shannon, 1945, p. 56). Shannon follows this by stating that the number of E and M must be equal since with i fixed, "T_i gives a one-to-one correspondence between all [of] the M's and some of the E's." Nevertheless,

$$P_M(E) = P(E) \neq 0$$

implying there must exist some key to transform some M into one of the Es. This demonstrates the significant mathematical logic underpinning modern cryptography.

3.2.2 Practical secrecy

Shannon (1945) also discusses practical secrecy, which is indicative of the more national security concerns. In practical secrecy, a solution will always exist. Therefore, the system's goal is to maximize the time it takes for the enemy to decrypt a message, either to delay them days, weeks, etc., or to make decryption infeasible, especially during times of war. In contrast, cryptanalysts' goal is "isolating this single solution" (Shannon, 1945, p. 86) as quickly as possible. Let the measurement in terms of time to solve a key be given by $W(N)$ where N is the number of letters, so $W(N)$ is "the work characteristic of the system" (Shannon, 1945, p. 86). In other words, $W(N)$ is the measure, in man-hours, of how long it takes to decrypt a system. Therefore, a practical system seeks to maximize $W(N)$ for someone decrypting the message.

On the other hand, for someone trying to decrypt a message, they want to minimize $W(N)$. The most straightforward approach to decrypting a key is a brute-force method. Shannon (1945, p. 88) demonstrates the impossibility of this with a key of 26! possible solutions. Assuming the best case (based on computational speeds from World War II), where a cryptanalyst can test a solution every microsecond and can find the correct one halfway through, then the time to do so is

$$\frac{2 \times 10^{26}}{2 \times 60^2 \times 24 \times 365 \times 10^6} = 3 \times 10^{12} \text{ years}$$

where Shannon approximates 26! as 2×10^{26} (Shannon, 1945, p. 89). This is not an effective method of decrypting the key. Essentially, it maximizes $W(N)$, which is the opposite of what an individual trying to decode the message wants. Given the key length of only twenty-six characters, the system takes an unrealistic amount of time to solve. Consequently, this method is only used when the key is small, such as the Caesar Shift, where the possible alterations to the key are a shift of 1-25, so only 26 possibilities must be checked. If we look at the computing speed defined above, even assuming the worst case where all possibilities must be checked, the time to solve is only 26 microseconds. Even without a computer, Caesar's enemies should not have taken too long to decipher a message; however, the solution also comes from knowing which encryption message was used in the first place.

3.3 New Eras of Security

The impact of cryptography can be seen throughout our daily lives (e.g., banking, phone networks, etc.). Fries (2022) contends "a new era of cryptography began … at the start of the Cold War." This was because of the necessity, brought upon by the United States and the Union of Soviet Socialist Republics, for information security. At this time, "cryptography entered the academic research mainstream" while both countries also "set up agencies dedicated to information security and warfare" (Fries, 2022).

"The Data Encryption Standard (DES)" (Fries, 2022), from the United States, used 56-bit keys, suggesting the "DES is considered insecure by modern standards because the encryption is easily broken." The real advancement came in 2000 from the system that replaced DES; the Advanced Encryption Standard (AES), which has keys up to 256 bits. This is currently the standard not just for the United States, but in most other countries, since it is incredibly fast and secure. However, AI, quantum computers, etc., may uproot this system, demanding an even more secure one.

	DES	AES
Year	1979-2005	2000-Present
Bit Size	56	128, 192, 256
$W(N)$	22 Hours	2.29×10^{32} years

Figute 5: DES versus AES (Fries, 2022; Wood, 2022).

However, the AES is still needed because "cybercriminals continuously evolve their tactics to find new ways to break into systems with sensitive data" (University of Tulsa, 2024). The (University of Tulsa, 2024) points out that this is why cryptography is needed in military and defense, health care, entertainment, supply chains, and government communications. All of this is to ensure confidentiality, data integrity, authentication of users, and non-repudiation. Therefore, the advancement of cryptography, as was the case during World War II and the Cold War, is a necessity for data security in the modern era.

This is why, for example, the National Institute of Standards and Technology (2016) "provides Federal agencies with a security metric to use" to ensure "strong, trusted cryptographic standards and guidelines." The Institute highlights how networks are becoming more open and interconnected, which is a major reason why these standards and guidelines are critical, especially for the United States Federal Government, whose data is incredibly sensitive. The government works to "[secure] top-secret federal data" so "public collaborations for developing modern cryptography" (National Museum of the United States Air Force, 2007) has become a focus, which originated from World War II, and only grew with the Cold War.

4 The Cold War: Fast Data Analysis

4.1 Nuclear Accountability

The Cold War is characterized by a war of information security (Fries, 2022), which logically and chronologically follows from the ideas laid out in the previous section. Brand (2006) demonstrates this clearly: "In an attempt to curb rising tensions, a ban on nuclear testing for both sides was proposed" (p. 1). This was never implemented since the United States would not commit unless it could detect "nuclear testing without having to physically visit the Soviet facilities" (Brand, 2006, p. 1). Perhaps it would have de-escalated tensions had the United States committed immediately.

Regardless, two mathematicians, James W. Cooley and John W. Tukey, were able to develop a method to determine if the Soviet Union was indeed performing nuclear tests by using offshore seismic detectors. Brand (2006) states this method became known as "the Cooley-Tukey fast Fourier transform, or FFT" (p. 1), although it was not developed in time for the nuclear ban on testing. If they had been successful, it might have led to significant changes in the events of the Cold War. Nevertheless, the FFT is an improvement to the algorithm by "Baron Fourier (1768-1830), a French mathematician" who "was able to show that conduction of heats in solids can be analyzed in terms of infinite ... series now called the Fourier Series" (Gasmi, 2022, p. 1).

Brand (2006, p. 1) and Burrus (1984, p. 14) contend that Cooley and Tukey rediscovered the FFT because earlier Gauss had developed a similar method in 1805 relating to a finite Fourier series. Gauss' results "appeared only in his collected works ... as an unpublished manuscript" (Burrus, 1984, p. 14), 153 years before Cooley and Tukey's publication. Such a case of independently rediscovering a concept, proof, etc., is especially common throughout the history of mathematics. In fact, according to Burrus (1984), "various FFT-type algorithms were used in Great Britain and elsewhere in the nineteenth century, but were unrelated to the work of Gauss" (p. 19). Nevertheless, the impact Cooley and Tukey had is significant because, even though they did not discover the FFT first, they revolutionized its applications.

4.2 Computational Optimization

The algorithm Cooley and Tukey (1965) developed used mathematician Irving John Good's methods "in which one must multiply an N-vector by an $N \times N$ matrix" (p. 297). Therefore, "these methods are applied...to the calculation of complex Fourier series" (Cooley and Tukey, 1965, p. 297), because the Discrete Fourier Transform (DFT) has a computation time of $O(N^2)$ compared to FFT's faster run-time of $O(N \log_2 N)$, allowing "real-time digital signal processing" (Meher, 2019, p. 1).

4.2.1 DFT and FFT

Cooley and Tukey (1965, p. 297) begin with "the problem of calculating the complex Fourier series"

$$X(j) = \sum_{k=0}^{N-1} A(k) \cdot W^{jk}, \quad j = 0, 1, \ldots, N-1.$$

As mentioned earlier, if we apply the DFT to the problem, then the time complexity grows very quickly. Cooley and Tukey (1965) define $A(k)$ as the complex Fourier coefficients "and W is the principal Nth root of unity" (p. 297) such that

$$W = e^{\frac{2\pi i}{N}}.$$

To derive the FFT algorithm, Cooley and Tukey "suppose N is a composite, i.e., $N = r_1 \cdot r_2$" (1965, p. 297); then the indices, j and k, can be expressed as

$$j = j_1 r_1 + j_0, \quad j_0 = 0, 1, \ldots, r_1 - 1, \quad j_1 = 0, 1, \ldots, r_2 - 1,$$

$$k = k_1 r_2 + k_0, \quad k_0 = 0, 1, \ldots, r_2 - 1, \quad k_1 = 0, 1, \ldots, r_1 - 1$$

such that

$$X(j_1, j_0) = \sum_{k_0} \sum_{k_1} A(k_1, k_0) \cdot W^{jk_1 r_2} W^{jk_0}.$$

Essentially, what Cooley and Tukey are doing is breaking up the array in the problem into two smaller arrays to make the computation faster and less redundant. Cooley and Tukey observe

$$W^{jk_1 r_2} = W^{j_0 k_1 r_2},$$

implying "the inner sum, k_1, depends only on j_0 and k_0 and can be defined as a new array," (1965, p. 298)

$$A_1(j_0, k_0) = \sum_{k_1} A(k_1, k_0) \cdot W^{j_0 k_1 r_2}.$$

This gives the final result of

$$X(j_1, j_0) = \sum_{k_0} A_1(j_0, k_0) \cdot W^{(j_1 r_1 + j_0) k_0} \tag{4}$$

for "N elements in the array A_1" (Cooley and Tukey, 1965, p. 298) requiring r_1 operations. Before continuing, we need to define $F(j)$. If a, b, c represent the size for a three-dimensional array, $2^a \times 2^b \times 2^c$ (Cooley and Tukey, 1965, p. 300), then

$$F(j) = aX(j+1) + bX(j) + cX(j-1).$$

A few pages later, they analyze the Fourier amplitude and solutions. For example, "the present method could be first applied to calculate the Fourier amplitudes of $F(j)$ from the formula"

$$B(k) = \frac{1}{N} \sum_j F(j) W^{-jk}.$$

Therefore, "the Fourier amplitudes of the solutions are, then," (Cooley and Tukey, 1965, p. 300)

$$A(k) = \frac{B(k)}{aW^k + b + cW^{-k}}. \tag{5}$$

"The $B(k)$ and $A(k)$ arrays are in bit-inverted order" (Cooley and Tukey, 1965, p. 300) to extrapolate the solution for the Fourier amplitudes. However, this is for the case that the "Fourier sums are to be evaluated twice," and that "no bit-inversion is necessary" (Cooley and Tukey, 1965, p. 300).

4.2.2 Time Analysis

For every N in the array A_1, it requires r_1 operations to calculate the Fourier solutions, implying the total array takes Nr_1 operations and X takes Nr_2 operations from A_1. To compute both of these, it takes

$$T = N(r_1 + r_2)$$

operations, as Cooley and Tukey (1965, p. 298) explain. This can be generalized to "give an m-step algorithm" requiring

$$T = N(r_1 + r_2 + \cdots + r_m)$$

operations where

$$N = r_1 \cdot r_2 \ldots r_m.$$

If r_j is a composite equal to the product of s_j and t_j, which are greater than one, then "$s_j + t_j < r_j$ unless $s_j = t_j = 2$ when $s_j + t_j = r_j$" (Cooley and Tukey, 1965, p. 298). From this, if we assume r is equal to all r_j, then,

$$N = r_1 \cdot r_2 \ldots r_m \implies N = r^m \implies m = \log_r N$$

and so “the total number of operations is” (Cooley and Tukey, 1965, p. 298)

$$T(r) = rN \log_r N.$$

To conclude this section, for the FFT, Cooley and Tukey (1965) wrote a program for an IBM 7094 computer to run the method discussed for $A(k)$ (5) and recorded “the computing time taken for... three-dimensional $2^a \times 2^b \times 2^c$ arrays of data points” (p. 300) giving the following table:

a	b	c	No. Pts.	Time (minutes)
4	4	3	2^{11}	.02
11	0	0	2^{11}	.02
4	4	4	2^{12}	.04
12	0	0	2^{12}	.07
5	4	4	2^{13}	.10
5	5	3	2^{13}	.12
13	0	0	2^{13}	.13

Table 1: IBM 7094 computing time for $2^a \times 2^b \times 2^c$ arrays using the FFT (Cooley and Tukey, 1965, p. 301).

Although Cooley and Tukey did not compute the run-time for the DFT, we can approximate how long the IBM 7094 computer would theoretically take. Since the DFT takes $O(n^2)$ operations and the FFT takes $O(n \log_2 n)$, then we can estimate the DFT runtime from the experimental data Cooley and Tukey presented in Table 1. If T is time, then

$$\frac{T_{DFT}}{T_{FFT}} \approx \frac{n^2}{n \log_2 n}$$

and so solving for T_{DFT}, we get

$$T_{DFT} \approx \frac{T_{FFT} \cdot n}{\log_2 n}.$$

Using Table 1, we can create a new table to demonstrate the difference between the FFT’s speed and the DFT’s speed. Table 2 shows the DFT run-time equivalences of the FFT.

a	b	c	No. Pts.	Time (minutes)
4	4	3	2^{11}	3.72
11	0	0	2^{11}	3.72
4	4	4	2^{12}	13.65
12	0	0	2^{12}	23.89
5	4	4	2^{13}	63.02
5	5	3	2^{13}	75.62
13	0	0	2^{13}	81.92

Table 2: Estimated DFT times for the IBM 7094 derived from the data shown in Table 1.

To demonstrate the magnitude of the performance difference, consider the array $2^{13} \times 2^0 \times 2^0 = 2^{13}$ number of points. The DFT would have taken approximately 81.92 minutes, whereas the FFT took .13 minutes. A simple division shows

$$\frac{81.92 \text{ minutes}}{.13 \text{ minutes}} \approx 630.15.$$

In other words, this example shows the FFT was approximately 630.15 times faster than the DFT.

4.3 Implementations Today

The FFT is one of the most important developments in modern society. The primary reason why the FFT became so mainstream is because "it was not patented and thus placed in the public domain immediately" (Brand, 2006, p. 2). This is why the FFT is considered "a landmark development in the field of digital signal processing (DSP), since it could expedite the DSP algorithms" to allow "real time digital signal processing" (Meher, 2019, p. 1). However, the impact of the FFT is not felt only by DSP, but rather in almost all fields, especially where optimization is concerned.

Ever since the academic introduction of the FFT algorithm, researchers have studied and expanded its application base. This has permitted the FFT to expand in application and efficiency (Meher, 2019, p. 1). This often means very specialized FFT algorithms. This includes applications in "speech, audio and image processing, signal analysis, communication systems, and many others" (Meher, 2019, p. 1). The universality of the FFT as an advancement in technology demanded the coding of the FFT into software since "it requires smaller memory and storage compared to. . . Discrete Fourier Transform" Transform" (Gasmi, 2022, p. 10).

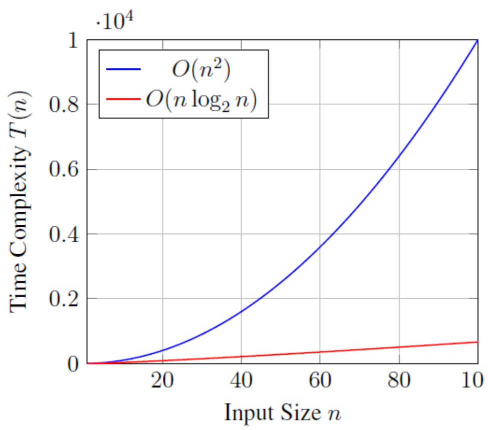

Figure 6: Time Complexity of the Discrete Fourier Transform and the Fast Fourier Transform.

The computational significance all comes down to the difference between $O\left(n^2\right)$ and $O\left(n \log_2 n\right)$. The first is the computational run-time of the DFT algorithm, which has quadratic time complexity. The second, however, represents the quasi-linear time complexity of the FFT algorithm. This slight difference alters the computational run-time significantly as n increases. Figure 6 pictures this difference. For example, if a field requires some computation such that $n = 10$, then the time complexity for the DFT algorithm is $10^2 = 100$. This is compared to the time complexity of the FFT, which is illustrated by $10 \log_2 10 \approx 33.219$. The difference between $O\left(n^2\right)$ and $O\left(n \log_2 n\right)$ grows as n increases. Consequently, the vast decrease in computational speed "had a revolutionary effect on many digital processing methods, and remains the most widely used method of computing Fourier transforms" (Burrus, 1984, p. 14).

5 Summary

World War I, World War II, and the Cold War significantly reshaped geopolitics and mathematics. Although the breakthroughs discussed in this paper were not always unique to each war, their impact was. Due to a need to innovate in each war, many groups developed groundbreaking inventions, saw significant breakthroughs, etc.

Lucas oversaw the Royal Flying Corps. In this group was Busk, an experimentalist by heart, who collected the necessary data to solve Bryan's stability equations. Lastly, Bryan provided significant insight into how an aircraft can inherently be stable. This insight is still the modern foundation of aviation theory.

Caesar's cipher is a notable example of cryptography being used in an ancient conflict. However, this easy-to-decrypt system differs significantly from the complexity of the Enigma machine. Thanks to Turing and many others, World War II ended two years sooner than expected. Mathematical interest in cryptography surged, as seen with Shannon, who analyzed theoretical and practical secrecy in systems. These mathematical explorations and techniques for decrypting a coded message have led to a race to update security, as seen with the Advanced Encryption System.

Unknowingly, Cooley and Tukey rediscovered the work of Gauss to calculate the Fourier series. Specifically, they found an optimization to the Discrete Fourier Transform by breaking down the array into two separate arrays. This led to the number of operations performed decreasing substantially, such that, instead of $O(n^2)$, the time complexity is now $O(n \log_2 n)$. Since the FFT was not patented, mathematicians led a thorough study of the FFT and many variations of it, which further led to applications in numerous fields.

Due to the inherent limitations of this paper, further research would reveal additional impacts on mathematics. This is two-fold. First is a description of wars outside of the twentieth century (e.g., the Civil War), and the second is a description of smaller-scale, but still incredibly significant conflicts (e.g., the Korean War). The literature is inherently limited in its coverage of both the depth and breadth of mathematics needed in conflicts; however, the research in this paper serves as a stepping stone for further analysis and different perspectives.

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The Problem Corner

Edited by Pat Costello

The Problem Corner invites questions of interest to undergraduate students. As a rule, the solution should not demand any tools beyond calculus and linear algebra. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following new problems should be submitted on separate sheets before July 31, 2026. Solutions received after this will be considered up to the time when copy is prepared for publication. The solutions received will be published in the Fall 2025 issue of *The Pentagon*. Preference will be given to correct student solutions. Affirmation of student status and school should be included with solutions. New problems and solutions to problems in this issue should be sent to Pat Costello, Department of Mathematics and Statistics, Eastern Kentucky University, 521 Lancaster Avenue, Richmond, KY 40475-3102 (e-mail: pat.costello@eku.edu, fax: (859) 622-3051)

NEW PROBLEMS 945 - 953

Problem 945. *Proposed by José Luis Dĭaz-Barrero, Barcelona Mathematical Circle, Barcelona, Spain.*
Let F_n be the n^{th} Fibonacci number defined by $F_0 = 0, F_1 = 1$ and for $n \geq 2, F_n = F_{n-1} + F_{n-2}$. Prove that the following holds:

$$\sum_{k=1}^{n} \sqrt{\frac{F_k^5}{F_n F_{k+1} F_{n+1}}} \leq \sqrt{\sum_{k=1}^{n} \frac{F_k^3}{F_{k+1}}}$$

(Here the subscripts are taken mod n.)

Problem 946. *Proposed by José Luis Dĭaz-Barrero, Barcelona Mathematical Circle, Barcelona, Spain.*
Find all triples of real numbers a, b, c such that each of the equations

$$\begin{aligned} x^3 + (a-1)x^2 + (b-3)x + (c-2) &= 0, \\ x^3 + (a-2)x^2 + (b-1)x + (c-3) &= 0, \\ x^3 + (a-3)x^2 + (b-2)x + (c-1) &= 0, \end{aligned}$$

has three distinct roots in the set of real numbers, but in total there are only five distinct numbers among them all.

Problem 947. *Proposed by Neculai Stanciu, "George Emil Palade School", Buzău, Romania.*
If $a, b, c > 0$ with $a+b+c=1$, prove the following inequality

$$\frac{1}{2abc} - \frac{1+3a}{b+c} - \frac{1+3b}{c+a} - \frac{1+3c}{a+b} - \frac{a^3+b^3+c^3}{2abc} \geq 3.$$

Problem 948. *Proposed by Toyesh Prakash Sharma, Agra College, Agra, India.*
Evaluate $\int e^{\arcsin t} * \left(\frac{5t^2+1}{\sqrt{t^3-t^5}}\right) dt$.

Problem 949. *Proposed by Daniel Sitaru, "Theodor Costescu" National Economic College, Drobeta Turnu – Severin, Mehedinti, Romania.*
Solve for real numbers $-1 \leq a, b, c, d, e \leq 1$, where $3a+3b+3c+3d+3e+1=0$ and $a^3 + b^3 + c^3 + d^3 + e^3 = 1$.

Problem 950. . *Proposed by Albert Natian, Los Angeles Valley College, Valley Glen, California.*
Given that $n \geq 4$, what are the coefficients of x^2 and x^3 in the polynomial

$$p(x) = p_n(x) = \prod_{k=2}^{n} \left(\frac{1}{k} + \sum_{l=2}^{k} l x^{2^{l-2}} \right).$$

Problem 951. *Proposed by Vasile Mircea Popa, Lucian Blaga University, Sibiu, Romania.*
Calculate the following integral

$$\int_0^1 \frac{ln^2 x}{x^3 + x\sqrt{x} + 1}\, dx.$$

Problem 952. *Proposed by the editor.*
Find all real solutions of $2^x - \sqrt{2}^{x+6} + 15 = 0$.

Problem 953. *Proposed by the editor.*
Find the area of the 5-sided region with vertices $(0,0), (0,6), (3,3), (6,6), (6,0)$. Show your method.

SOLUTIONS TO PROBLEMS 928 - 936

928. *Proposed by José Luis Díaz-Barrero, School of Civil Engineering, Barcelona Tech - UPC, Barcelona, Spain.*
Solve in the set of positive integers the following equation: $x^2+y^2=137(x-y)$.

Solution *by Brian Beasley, Simpsonville, SC.*

In general, let p be a prime with $p\equiv 1\ (\ mod\ 4)$. Then we seek all positive integers x and y such that $x^2+y^2=p\,(x-y)$. Since $x^2-px+\left(y^2+py\right)=0$,

$$x=\frac{p\pm\sqrt{2p^2-(2y+p)^2}}{2}.$$

This implies that $2p^2-(2y+p)^2=q^2$ for some odd integer q, so $2p^2=(2y+p)^2+q^2$. Since p is a prime with $p\equiv 1\ (\ mod\ 4)$, we write p uniquely as $p=m^2+n^2$ for positive integers $m<n$. Using $2=1^2+1^2$ and the identity

$$\left(a^2+b^2\right)\left(c^2+d^2\right)=(ac+bd)^2+(ad-bc)^2.$$

We obtain $2p^2=p^2+p^2=\left(n^2+2mn-m^2\right)^2+\left(n^2-2mn-m^2\right)^2$ as the only representation of $2p^2$ as the sum of two squares. This implies that $2y+p$ is either p or $n^2+2mn-m^2$ or $|n^2-2mn-m^2|$. But $2y+p=p$ means $y=0$ while $2y+p=|n^2-2mn-m^2|$ means $y<0$. Hence we conclude that $2y+p=n^2+2mn-m^2\to y=mn-\ m^2$, which in turn yields

$$x=\frac{m^2+n^2\pm(n^2-2mn-m^2)}{2}.$$

Thus $(x,y)=\left(n^2-mn,mn-m^2\right)$ or $(m^2+mn,mn-m^2)$. For the original equation, $p=137=4^2+11^2$ and we obtain $(x,y)=(77,28)$ or $(x,y)=(60,28)$.

Also solved by Brian Bradie, Christopher Newport University, Newport News, VA; Kee-Wai Lau, Hong Kong, China; Carl Libis, University of Maryland Global Campus; Missouri State University Problem Solving Group, Springfield, MO; and the proposer.

929. *Proposed by José Luis Díaz-Barrero, School of Civil Engineering, Barcelona Tech - UPC, Barcelona, Spain.*
In how many ways can the rational $\frac{2025}{2024}$ be written as the product of two rational numbers of the form $\frac{(n+1)}{n}$, where n is a positive integer?

Solution *by Cao Minh Quang, Nguyen Binh Khiem high school for the gifted, Vinh Long, Vietnam.*

Assume that $\frac{2025}{2024} = \frac{n+1}{n} * \frac{m+1}{m}$, where m, n are positive integers, or

$$2025nm = 2024 * (n+1) * (m+1) \Leftrightarrow mn - 2024m - 2024n - 2024 = 0.$$

Let $k = 2024$, we can rewrite $(m-k)*(n-k) = k^2 + k$. Since $k^2 + k = 2024 * 2025 = 2^3 * 3^4 * 5^2 * 11 * 23$, hence it has $(3+1)*(4+1)*(2+1)*(1+1)*(1+1) = 240$ factors. From this we know the fraction $\frac{2025}{2024}$ can be written as the product of two rational numbers of the form $(n+1)/n$ in $\frac{240}{2} = 120$ distinct ways.

Also solved by Aaron Allen, North Carolina Wesleyan University, Rocky Mount, NC; Missouri State University Problem Solving Group, Springfield, MO; Angel Plaza, Universidad de Las Palmas de Gran Canaria, Spain; Albert Stadler, Herrliberg, Switzerland; Daniel Văcaru, National Economic College, Pitesti, Romania; and the proposer.

930. *Proposed by Mathew Cropper, Eastern Kentucky University, Richmond, KY.* The Mycielski construction is done to a finite simple graph G producing a graph $M(G)$ as follows: set the vertex set of G to be $\{v_1, v_2, \ldots, v_k\}$, then add a set of vertices $\{u_1, u_2, \ldots, u_k\}$ and one more vertex w. Set u_i to be adjacent to every vertex in G to which v_i is adjacent and make w adjacent to every vertex in $\{u_1, u_2, \ldots, u_k\}$. Note that the set of vertices $\{u_1, u_2, \ldots, u_k\}$ is an independent set [Introduction to Graph Theory, West, pg. 205]. Let $M^n(G)$ denote the graph obtained from a given finite simple graph G by applying the Mycielski construction n times. Determine a formula for the number of edges in the graph $M^n(G)$.

Solution *by Missouri State University Problem Solving Group, Springfield, MO.*

Let v denote the number of vertices and e the number of edges in G and let v_n denote the number of vertices and e_n the number of edges in $M_n(G)$. Note that $v_0 = v$ and $e_0 = e$. We claim that

$$v_n = \ 2^n v + (2^n - 1) \text{ and } e_n = \ 3^n e + (3^n - 2^n)\, v + \frac{1 - 2^{n+1} + 3^n}{2}.$$

Clearly, $v_{n+1} = 2v_n + 1$ and one readily verifies the claimed formula for v_n. Each edge in $M_n(G)$ gives rise to three edges in $M_{n+1}(G)$ (the original edge v_i, v_j and edges u_i, v_j and v_i, u_j). Each vertex of $M_n(G)$ also gives rise to an edge (u_i, w). Therefore, $e_{n+1} = 3e_n + v_n$. We now proceed by induction. If $n = 0, e_0 = 3^0 e + \left(3^0 - 2^0\right) v + \frac{1-2^1+3^0}{2} = e$. We also have

$$
\begin{aligned}
e_{n+1} &= 3e_n + v_n \\
&= 3\left(3^n e + (3^n - 2^n)v + \frac{1-2^{n+1}+3^n}{2}\right) + 2^n v + (2^n - 1) \\
&= 3^{n+1}e + \left(3^{n+1} - 3*2^n + 2^n\right)v + \frac{3 - 3*2^{n+1} + 3^{n+1} + 2^{n+1} - 2}{2} \\
&= 3^{n+1}e + \left(3^{n+1} - 2^{n+1}\right)v + \frac{1-2^{n+2}+3^{n+1}}{2}
\end{aligned}
$$

and the formula holds.

Also solved by Kee-Wai Lau, Hong Kong, China; and the proposer.

931. *Proposed by Richard Hasenauer, Eastern Kentucky University, Richmond, KY.*
Prove that 7! divides $n^7 - 14n^5 + 49n^3 - 36n$ for all positive integers n.

Solution *by Aaron Allen, North Carolina Wesleyan University, Rocky Mount, NC.*

We factor $n^7 - 14n^5 + 49n^3 - 36n$ as follows:

$$
\begin{aligned}
n\left(n^6 - 14n^4 + 49n^2 - 36\right) &= n[\left(n^3 - 7n\right)^2 - 6^2] \qquad (1) \\
&= n[\left(n^3 - 7n - 6\right) * \left(n^3 - 7n + 6\right)] \\
&= n[(n+1)\left(n^2 - n - 6\right)(n-1)\left(n^2 + n - 6\right)] \\
&= n[(n+1)(n+2)(n-3)(n-1)(n-2)(n+3)].
\end{aligned}
$$

Case I: If $n < 4$, it follows from (1) that $n^7 - 14n^5 + 49n^3 - 36n = 0$. Hence, 7! divides $n^7 - 14n^5 + 49n^3 - 36n$ for $n < 4$.
Case II: If $n \geq 4$, set $m = n - 3$ which is an integer. Then (1) implies the following

$$
\begin{aligned}
n^7 - 14n^5 + 49n^3 - 36n &= (m+6)(m+5)(m+4)(m+3)(m+2)(m+1)m \\
&= \frac{(m+6)!}{(m-1)!} = \frac{[(m-1)+7]!}{(m-1)7!} * 7! \\
&= \binom{m+6}{7} * 7! = \binom{n+3}{7} * 7!
\end{aligned}
$$

Since $\binom{n+3}{7}$ is a natural number, for all $n \geq 4$, $7!$ divides $n^7 - 14n^5 + 49n^3 - 36n$.

Also solved by Brian Bradie, Christopher Newport University, Newport News, VA; Kee-Wai Lau, Hong Kong, China; Missouri State University Problem Solving

Group, Springfield, MO; Albert Stadler, Herrliberg, Switzerland; and the proposer.

932. *Proposed by Tom Richmond, Western Kentucky University, Bowling Green, KY.*
If a and b are distinct square-free natural numbers and c and d are nonzero rational numbers, find necessary and sufficient conditions for $c\sqrt{a}+d\sqrt{b}$ to be a nonzero rational number.

Solution *by Missouri State University Problem Solving Group, Springfield, MO.*

The condition that a and b be square-free forces there to be no situation where $c\sqrt{a}+d\sqrt{b}$ can be a rational number. Assume that $c\sqrt{a}+d\sqrt{b}$ is rational. Then $(c\sqrt{a}+d\sqrt{b})^2 = ac^2+2cd\sqrt{ab}+bd^2$ is rational which means that $2cd\sqrt{ab}$ is rational. Since c and d are nonzero rationals, we have $\sqrt{ab}$ is rational. The only integers with rational square roots are perfect squares, so $ab = n^2$ for some positive integer n. Since a and b are square-free, we must have $a = b$. This violates the condition they are distinct.

Also solved by Kee-Wai Lau, Hong Kong, China; and the proposer.

933. *Proposed by Tom Richmond, Western Kentucky University, Bowling Green, KY.*
For cube-free integers $a,b > 1$ and nonzero rationals c,d, show that $c\sqrt[3]{a}+d\sqrt[3]{b}$ is rational if and only if $a = b$ and $c = -d$.

Solution *by the proposer.*

Let $r = c\sqrt[3]{a}+d\sqrt[3]{b}$. If $a = b$ and $c = -d$, then $r = 0$ is rational.
Now assume that r is rational. Consider the case that $r = 0$. With $c = \frac{c_1}{c_2}, d = \frac{d_1}{d_2}$ where c_i, d_i are nonzero integers, we have $c_1 d_2 \sqrt[3]{a} = c_2 d_1 \sqrt[3]{b}$. Cubing gives $c_1^3 d_2^3 a = c_2^3 d_1^3 b$. If two integers are equal, then their cube-free parts are equal, so $a = b$. Then $c\sqrt[3]{a}+d\sqrt[3]{a} = 0$ implies $c = -d$. Now suppose $r \neq 0$. Cubing gives

$$\begin{aligned} c^3 a + 3c^2 d\sqrt[3]{a^2 b} + 3cd^2\sqrt[3]{ab^2} + d^3 b = & \; r^3 c^3 a + d^3 b - r^3 \\ &= -3(c^2 d\sqrt[3]{a^2 b} + cd^2\sqrt[3]{ab^2} \\ &= -3cd(c\sqrt[3]{a} + d\sqrt[3]{b})\sqrt[3]{ab} \\ &= -3cdr\sqrt[3]{ab}. \end{aligned}$$

It follows that $\sqrt[3]{ab}$ is rational. Write $a = a_1 a_2^2, b = b_1 b_2^2$ where a_i, b_i are square-free natural numbers and $\gcd(a_1.a_2) = 1 = \gcd(b_1, b_2)$. $\sqrt[3]{ab}$ is rational implies $\sqrt[3]{a_1 a_2^2 b_1 b_2^2}$ is rational

$\Rightarrow a_1a_2^2b_1b_2^2$ is a perfect cube

$$\Rightarrow \begin{cases} \text{every prime factor of } a\_1 \text{ appears in } b\_2 \\ \text{every prime factor of } a\_2 \text{ appears in } b\_1 \\ \text{every prime factor of } b\_1 \text{ appears in } a\_2 \\ \text{every prime factor of } b\_2 \text{ appears in } a\_1 \end{cases}$$

$\Rightarrow a_1a_2^2 = b_1b_2^2$

$\Rightarrow a = b.$

Now $c\sqrt[3]{a} + d\sqrt[3]{b} = (c+d)\sqrt[3]{a} = r$ is rational. If $c = -d$, we have the desired conclusion. If $c \neq -d$, it follows that $\sqrt[3]{a} = \frac{r}{c+d}$ is rational so $\sqrt[3]{a} = \sqrt[3]{b}$ is a natural number contrary to a, b cube-free and greater than 1.

Also solved by Kee-Wai Lau, Hong Kong, China; and the Missouri State University Problem Solving Group, Springfield, MO.

934. *Proposed by John Wilson, recently retired from Centre College, Danville, KY.* A Squarely puzzle is a logic puzzle played on a 5×5 grid. The solution requires that the digits 1 through 9 be placed in the grid with two rules:

1. No digit appears more than once in any row, column or diagonal;
2. the 25 cells must contain exactly 3 copies of 8 of the digits and one copy of the ninth digit.

You are given the five digits of each row, column and diagonal.

Row	**Column**	**Diagonal**
13458	12459	
12369	34689	\12589
24569	35678	
24568	12569	\35689
13789	12348	

Get 10 free puzzles at squarelypuzzle.com.

Solution *by Kee-Wai Lau, Hong, China.*

1	4	8	5	3
2	9	3	6	1
4	6	5	9	2
5	8	6	2	4
9	3	7	1	8

Also solved by the proposer.

935. *Proposed by the editor.*
The binomial transform of sequence $a_0, a_1, a_2, \ldots, a_n$ is sequence $b_0, b_1, b_2, \ldots, b_n$ where

$$b_k = \sum_{i=0}^{k} (-1)^i \binom{k}{i} a_i$$

Starting with sequence $a_0 = 1, a_1 = -2, a_2 = 4, a_3 = -8, a_4 = 16$, find b_4.

Solution *by Brian Bradie, Christopher Newport University, Newport News, VA.*

Recognize that $a_k = (-2)^k$ for each k. Then, by the binomial theorem,

$$\begin{aligned} b_k &= \sum_{i=0}^{k} (-1)^i \binom{k}{i} a_i = \sum_{i=0}^{k} (-1)^i \binom{k}{i} (-2)^i \\ &= \sum_{i=0}^{k} \binom{k}{i} 2^i = (1+2)^k = 3^k. \end{aligned}$$

Thus $b_4 = 3^4 = 81$.

Also solved by Kee-Wai Lau, Hong Kong, China; Albert Stadler, Herrliberg, Switzerland; and the proposer.

936. *Proposed by the editor.*
A pair of numbers (A, M) is called *amicable* when $\sigma(A) - A = M$ and $\sigma(M) - M = A$ where $\sigma(n)$ is the sum of all positive divisors of n. The smallest amicable pair is (220,284) which was known to Pythagoras.
4230338853909611459680566139419405 3 is one member of a previously-unknown amicable pair. Find the other member.

Solution *by Albert Stadler, Herrliberg, Switzerland.*

We find, with the help of *Mathematica*, that

42303388539096114596805661394194053
$= 3^4 * 7^4 * 11^2 41 * 53 * 2801 * 55601 * 1609079 * 3333637019.$
Hence
$\sigma(42303388539096114596805661394194053)$
$- 42303388539096114596805661394194053$
$= 42304183130750004852115222943687547.$
Mathematica finds the prime factorization
42304183130750004852115222943687547
$= 3^4 * 7^4 * 11^2 41 * 53 * 2801 * 3572398961 * 82677621599$
so that
$\sigma(42304183130750004852115222943687547)$
$- 42304183130750004852115222943687547$
$= 42303388539096114596805661394194053.$
So the other member is 42304183130750004852115222943687547.

Also solved by Aaron Allen, North Carolina Wesleyan University, Rocky Mount, NC; Brian Beasley, Simpsonville, SC; Kee-Wai Lau, Hong Kong, China; and the proposer.

Kappa Mu Epsilon News

Edited by Mark Hughes, Historian
Updated information as of March 2025

News of chapter activities and other noteworthy KME events should be sent to

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Chapter News

AL Theta – Jacksonville State University
Chapter President – Lucas Saone; 353 Total Members
Other Fall 2024 Officers: Susie Landis, Vice President; Qays Ghazal, Secretary; Angelica Vargas, Treasurer; Dr. David Dempsey, Corresponding Secretary; and Dr. Jason Cleveland, Faculty Sponsor.
The Alabama Theta chapter met monthly during Fall 2024 to enjoy games and fellowship. We look forward to welcoming new members during our spring initiation ceremony and travelling to the national convention.

AR Beta – Henderson State University
Corresponding Secretary and Faculty Sponsor – Catherine Leach; 74 Total Members

CA Theta – William Jessup University
Chapter President – John Guerrero; 16 Total Members; 8 New Members
Other Fall 2024 Officers: Hannah-Jeanne Bethards, Vice President; Rebekah Jacobs, Secretary; Bradley Wagner, Corresponding Secretary; and Michelle Clark, Faculty Sponsor.
This past semester, our chapter, California Theta, initiated 8 new members. Additionally, our president, John Guerrero, graduated. Our entire chapter wishes him well as he moves on to greater things! Our chapter also hosted two events. First, we hosted the initiation ceremony for new members and then a Christmas Movie Night in which we all had fun riffing on a B Christmas movie.

CT Gamma – Central Connecticut State University
Corresponding Secretary – Gurbakhshash Singh; 78 Total Members
Other Fall 2024 Officer: Nelson Castaneda, Faculty Sponsor.

GA Zeta – Georgia Gwinnett College
Chapter President – Tamara Crawford; 71 Total Members; 2 New Members
Other Fall 2024 Officers: William Day, Vice President; Dr. Jamye Curry Savage, Corresponding Secretary and Faculty Sponsor; and Dr. Livy Uko, Faculty Sponsor.

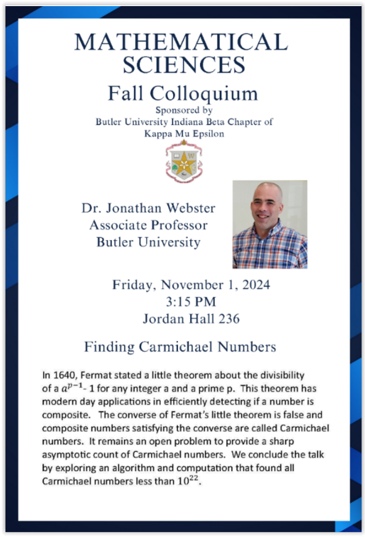
IA Alpha – University of Northern Iowa
Chapter President – Erica Peters; 1128 Total Members; 3 New Members
Other Fall 2024 Officers: Quinn Robinson, Vice President; Mackenzie Halbur, Secretary; Alyssa McDowell, Treasurer; and Dr. Mark D. Ecker, Corresponding Secretary and Faculty Sponsor.
Seven student members of KME and two faculty members met on Tuesday, December 10, 2024 in Wright Hall for our Fall KME meeting/banquet. Erica Peters presented her senior seminar project entitled "Collision Consequences: Analyzing Crash Factors on Total Property Damage" and three new KME student members were initiated at our meeting.

IA Gamma – Morningside University
Chapter President – Kelsey Schieffer; 453 Total Members; 8 New Members
Other Fall 2024 Officers: Piper Ross, Vice President; Kiara Howard, Secretary; Eein McKinley, Treasurer; and Dr. Eric Canning, Corresponding Secretary and Faculty Sponsor.
We initiated 8 new members into KME at our first meeting of the fall semester. Our KME math club met a total of 4 times during the fall semester. At the other 3 meetings, we twice had guest speakers and had a pizza/games night.

IL Zeta – Dominican University
Corresponding Secretary and Faculty Sponsor – Mihaela Blanariu; 471 Total Members
No new members were initiated in Fall 2024.

IN Beta – Butler University
Chapter President – Jenna Lane; 460 Total Members
Other Fall 2024 Officers: Danika Casa, Vice President; Will Kelleher, Secretary; Dr. Rasitha Jayesekere, Corresponding Secretary and Faculty Sponsor.

KME Indiana Beta Chapter sponsored one of their departmental colloquia in the fall. The speaker was our very own faculty member, an algorithmic and computational number theorist, Dr. Jonathan Webster. Dr. Webster spoke on “Finding Carmichael Numbers”. A big shout out to our KME Indiana Beta Chapter President Jenna Lane, Vice President Danika Casa, and Secretary Will Kelleher for their work in organizing and hosting the event. The colloquium poster is presented here.



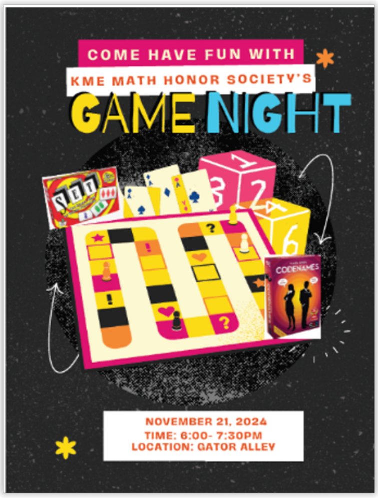
KS Beta – Emporia State University

Chapter President – Chris Brooke; 1552 Total Members; 2 New Members
Other Fall 2024 Officers: Lana Piepho, Vice President; Cahrin Pinkston, Secretary; Maliki Mosher, Treasurer; Tom Mahoney, Corresponding Secretary; and Brian Hollenbeck, Faculty Sponsor.

For ESU Math Day this past fall, KME Kansas Beta Chapter held a virtual “Escape Room” competition for teams of high school students. The escape room included three sections, a geometry section that involved using trigonometry to find a side length of a triangle, an algebra portion that required solving systems of equations and finding polynomial roots, and a math scramble that highlighted Caesar Ciphers. We had 19 schools from surrounding area districts participate, with the fastest completion being 28 minutes and 29 seconds. Additionally, the Kansas Beta Chapter held a biannual initiation where we welcomed new members to the chapter. We also had a group from our chapter visit the Linda Hall Library in Kansas City, where we visited their rare book collection. The books included several copies of Euclid’s *Elements*, some dating back to just after the invention of the printing press, and early editions of Newton’s *Principia*.

New Initiates – Alex Stewart and Isabel Ayala.

MD Alpha – Notre Dame of Maryland University
Chapter President – Kaitlyn Stephenson; 415 Total Members
Other Fall 2024 Officers: Kritika KC, Vice President; Keira Humpreys, Secretary; Dr. Charlie Buehrle, Corresponding Secretary and Faculty Sponsor.
MD Alpha chapter hosted a Game Night in November.



MD Delta – Frostburg State University
Chapter President – Alyssa Kush; 552 Total Members
Other Fall 2024 Officers: Te'a Thompson, Vice President; Emilia Germain, Secretary; Gabe Hicks, Treasurer; Mark Hughes, Corresponding Secretary and Faculty Sponsor; and Frank Barnet, Faculty Sponsor.
We had three meetings during the semester where we enjoyed puzzles, math videos, and pizza. We also represented the Mathematics Department at our university's annual Majors Fair.

MI Epsilon – Kettering University
Corresponding Secretary and Faculty Sponsor – Valerie Sosnowski; 1207 Total Members; 11 New Members
New Initiates – Nicholas Buchholz, Victoria Brender, Daniel Budd, Michael Cercone, Jack Krugh, Augusta Hayek, Phu Nguyen, Bernardo Lozano Porter, Christa Spencer, Jamie Robbins, Emma Arntson.

MO Alpha – Missouri State University
Chapter President – Yahasvi Moon; 1870 Total Members; 1 New Member
Other Fall 2024 Officers: Layna Mangiapanello, Vice President; Rebecca Jolivette, Secretary; Mia Jones, Treasurer; Ngoc Do, Corresponding Secretary and Faculty Sponsor.

MO Epsilon – Central Methodist University
Chapter President – Dillan Lembke; 474 Total Members; 6 New Members
Other Fall 2024 Officers: Abigail Michael, Vice President; Luke Green, Secretary; Noble Yiga, Treasurer; Tyler Kenefake, Corresponding Secretary and Faculty Sponsor.

MO Theta – Evangel University
Chapter President – Victoria Risner; 313 Total Members
Other Fall 2024 Officers: Abby Harrison, Vice President; Dianne Twigger, Corresponding Secretary; and Jeremy Osborne, Faculty Sponsor.
The MO Theta Chapter of KME met 3 times during the fall term. Students heard from a current graduate student and KME member, Micah Herron, on graduate work and graduate assistantships. Many students also attended the Fall 2024 MAKO conference sponsored by Missouri State University. Two of our students presented talks at the conference as well. Plans are underway for the chapter to attend the 2025 biennial convention in March.

MO Kappa – Drury University
Chapter President – Samuel Fullbright; 361 Total Members
Other Fall 2024 Officers: Nicolette Gaston, Vice President; Hannah Ritter, Secretary; Aly Boyd, Co-Secretary; Kylie Warden, Treasurer; and Colin T. Barker, Corresponding Secretary and Faculty Sponsor.
This fall our chapter became officially recognized as a student organization on campus and held meetings twice weekly to explore problems in mathematics. We also held our first campus event—a call for problems on campus. We had a lot of participation and have begun analyzing parking on campus by examining Queue Theory models. Another research project which came out of the weekly meetings, *Super Basic Palindromic Numbers*, was presented at the MAKO regional conference in Springfield, MO. We anticipate a presentation and short article to be written about this and will explore publishing in the Pentagon. Future aspirations are to host KME alumnae to speak on interesting utilizations of mathematics within their careers, and to host a spring event that illustrates some of the problems we attempted to solve from the fall event.

MS Alpha – Mississippi University for Women
Chapter President – Audrey Mitchell; 844 Total Members; 2 New Members
Other Fall 2024 Officer: Dr. Joshua Hanes, Secretary, Treasurer, Corresponding Secretary and Faculty Sponsor.
After having only one active member on campus for the fall of 2024, we are looking forward to doing activities this spring by recruiting new associate members.

NH Alpha – Keene State College
Chapter Co-Presidents – Madeleine Forhan and Genevieve Steenhoek; 331 Total Members

Other Fall 2024 Officers: Rachel Rainey, Secretary; Sarah Gerardi, Treasurer; and Caitlyn Parmalee, Corresponding Secretary and Faculty Sponsor.
The NH Alpha Chapter initiated three new members in 2024. Our chapter meets several times a semester and though we were unable to attend any conferences this fall, we look forward to sending several students to the Nebraska Conference for Undergraduate Women in Mathematics and the Hudson River Undergraduate Mathematics Conference this spring!

NY Omicron – St. Joseph's University
Chapter President – Robert Kohlmann; 352 Total Members
Other Fall 2024 Officers: Madison Frascogna, Vice President; Katelyn Raby, Secretary; Vanessa Stabile, Treasurer; Elana Reiser, Corresponding Secretary; and Donna Pirich, Faculty Sponsor.
Our chapter is planning an Easter basket drive to collect materials and then we will make baskets to distribute to kids of victims of domestic abuse. Our members have also volunteered to help run a family math night for local elementary students. We will also be attending the play "Proof" together.

OK Alpha – Northeastern State University (Spring Semester)
Chapter President – Ryan McAbee; 1880 Total Members
Other Spring 2024 Officers: Allen Ortiz, Vice-President; Mark Buckles, Secretary, Treasurer, Corresponding Secretary, and Faculty Sponsor.
During the spring semester, we held an ice cream social.

OK Alpha – Northeastern State University
Chapter President – Ryan McAbee; 1884 Total Members; 4 New Members
Other Fall 2024 Officer: Mark Buckles, Secretary, Treasurer, Corresponding Secretary, and Faculty Sponsor.
We had a Kappa Mu Epsilon Initiation Meeting on October 25, 2024. We invited parents, alumni, and retired faculty. At the initiation, we had pizza and towards the end of the event, faculty sponsor Mark Buckles gave a 15 minute talk entitled, "A Little Bit of Naïve Set Theory (Part I)."
New Initiates – Taylor Singleton, Joseph Dobson, Aviel Spurgeon, and Sarder Sadique.

OK Delta – Oral Roberts University
Chapter President – C. Scott Alons; 219 Total Members; 4 New Members
Other Fall 2024 Officers: Prem Thannicka, Vice President; Victor Gomes, Secretary; Nathan Anderson, Treasurer; and Enriques Valderrama Araya, Corresponding Secretary and Faculty Sponsor.
Topics for this fall semester: Grad School Prep, How to Read Graduate Mathematics Texts, Higher Proof Studies, Lie Algebras (and Necessary Prerequisite Concepts), Mathematical Theology
New Initiates – Nathan Anderson, Queena Lee Chin, Gwenevere Rose Ashton, and William Charles Moses George.

PA Rho – Thiel College
Chapter President – Bailey Stilts; 156 Total Members
Other Fall 2024 Officers: Kaitlyn Schmidt, Vice President; Jessica Wagner, Secretary; Alexa Beck, Treasurer; and Dr. Jie Wu, Corresponding Secretary and Faculty Sponsor.

RI Beta – Bryant University
Corresponding Secretary – Professor John Quinn; 224 Total Members
Other Fall 2024 Officer: Professor Gao Niu, Faculty Sponsor.
We have our KME nominations and initiation ceremony every spring semester and we are planning to do the same for spring 2025.

TN Gamma – Union University
Chapter President – Jacob Carbonell; 537 Total Members
Other Fall 2024 Officers: Georgia Morgan, Vice President; Stacia Talbott, Secretary and Treasurer; Ian Banderchuk, Webmaster and Historian; Bryan Dawson, Corresponding Secretary; and Matt Lunsford, Faculty Sponsor.

TX Mu – Schreiner University
Chapter President – Jacob Plummer; 217 Total Members
Other Fall 2024 Officers: Dominic Civello, Vice President; Rachel Lynn, Corresponding Secretary and Faculty Sponsor.

WV Alpha – Bethany College
Chapter President – Karleigh D. Clegg; 202 Total Members (since 2001 reorganization)
Other Fall 2024 Officers: Joseph T. Hubert, Vice President; Alexus A. Reid, Secretary and Treasurer; and Dr. Adam C. Fletcher, Corresponding Secretary and Faculty Sponsor.
West Virginia Alpha has had a rather quiet fall semester. While small, the chapter continues to join forces with the local Mathematics and Computer Science Club to offer board game nights, chess tournaments, support for research presentations, and the like. They are busily planning for the annual Math/Science Day Competition for local high school students on campus in March and providing service to local high school mathematics competitions in the community. We anticipate a larger (for us!) induction ceremony in the next few months and look forward to traveling to Missouri for the National Biennial Convention this spring.

Active Chapters of Kappa Mu Epsilon

Listed by date of installation

Chapter	Location	Installation Date
OK Alpha	Northeastern State University, Tahlequah	18 Apr 1931
IA Alpha	University of Northern Iowa, Cedar Falls	27 May 1931
KS Alpha	Pittsburg State University, Pittsburg	30 Jan 1932
MO Alpha	Missouri State University, Springfield	20 May 1932
MS Alpha	Mississippi University for Women, Columbus	30 May 1932
NE Alpha	Wayne State College, Wayne	17 Jan 1933
KS Beta	Emporia State University, Emporia	12 May 1934
AL Alpha	Athens State University, Athens	5 Mar 1935
NM Alpha	University of New Mexico, Albuquerque	28 Mar 1935
IL Beta	Eastern Illinois University, Charleston	11 Apr 1935
AL Beta	University of North Alabama, Florence	20 May 1935
AL Gamma	University of Montevallo, Montevallo	24 Apr 1937
OH Alpha	Bowling Green State University, Bowling Green	24 Apr 1937
MI Alpha	Albion College, Albion	29 May 1937
MO Beta	University of Central Missouri, Warrensburg	10 Jun 1938
TX Alpha	Texas Tech University, Lubbock	10 May 1940
KS Gamma	Benedictine College, Atchison	26 May 1940
IA Beta	Drake University, Des Moines	27 May 1940
TN Alpha	Tennessee Technological University, Cookeville	5 Jun 1941
MI Beta	Central Michigan University, Mount Pleasant	25 Apr 1942
NJ Beta	Montclair State University, Upper Montclair	21 Apr 1944
IL Delta	University of St. Francis, Joliet	21 May 1945
KS Delta	Washburn University, Topeka	29 Mar 1947
MO Gamma	William Jewell College, Liberty	7 May 1947
TX Gamma	Texas Woman's University, Denton	7 May 1947
WI Alpha	Mount Mary College, Milwaukee	11 May 1947
OH Gamma	Baldwin-Wallace College, Berea	6 Jun 1947
MO Epsilon	Central Methodist College, Fayette	18 May 1949
MS Gamma	University of Southern Mississippi, Hattiesburg	21 May 1949
IN Alpha	Manchester College, North Manchester	16 May 1950
PA Alpha	Westminster College, New Wilmington	17 May 1950
IN Beta	Butler University, Indianapolis	16 May 1952
KS Epsilon	Fort Hays State University, Hays	6 Dec 1952
PA Beta	LaSalle University, Philadelphia	19 May 1953
VA Alpha	Virginia State University, Petersburg	29 Jan 1955
IN Gamma	Anderson University, Anderson	5 Apr 1957
CA Gamma	California Polytechnic State University, San Luis Obispo	23 May 1958
TN Beta	East Tennessee State University, Johnson City	22 May 1959
PA Gamma	Waynesburg College, Waynesburg	23 May 1959
VA Beta	Radford University, Radford	12 Nov 1959
NE Beta	University of Nebraska—Kearney, Kearney	11 Dec 1959
IN Delta	University of Evansville, Evansville	27 May 1960
OH Epsilon	Marietta College, Marietta	29 Oct 1960
MO Zeta	University of Missouri—Rolla, Rolla	19 May 1961
NE Gamma	Chadron State College, Chadron	19 May 1962
MD Alpha	College of Notre Dame of Maryland, Baltimore	22 May 1963
CA Delta	California State Polytechnic University, Pomona	5 Nov 1964
PA Delta	Marywood University, Scranton	8 Nov 1964
PA Epsilon	Kutztown University of Pennsylvania, Kutztown	3 Apr 1965
AL Epsilon	Huntingdon College, Montgomery	15 Apr 1965
PA Zeta	Indiana University of Pennsylvania, Indiana	6 May 1965
TN Gamma	Union University, Jackson	24 May 1965
IA Gamma	Morningside College, Sioux City	25 May 1965
MD Beta	McDaniel College, Westminster	30 May 1965
IL Zeta	Dominican University, River Forest	26 Feb 1967
SC Beta	South Carolina State College, Orangeburg	6 May 1967
PA Eta	Grove City College, Grove City	13 May 1967
NY Eta	Niagara University, Niagara University	18 May 1968
MA Alpha	Assumption College, Worcester	19 Nov 1968
MO Eta	Truman State University, Kirksville	7 Dec 1968
IL Eta	Western Illinois University, Macomb	9 May 1969
OH Zeta	Muskingum College, New Concord	17 May 1969
PA Theta	Susquehanna University, Selinsgrove	26 May 1969
PA Iota	Shippensburg University of Pennsylvania, Shippensburg	1 Nov 1969
MS Delta	William Carey College, Hattiesburg	17 Dec 1970
MO Theta	Evangel University, Springfield	12 Jan 1971
PA Kappa	Holy Family College, Philadelphia	23 Jan 1971
CO Beta	Colorado School of Mines, Golden	4 Mar 1971
KY Alpha	Eastern Kentucky University, Richmond	27 Mar 1971
TN Delta	Carson-Newman College, Jefferson City	15 May 1971
NY Iota	Wagner College, Staten Island	19 May 1971
SC Gamma	Winthrop University, Rock Hill	3 Nov 1972
IA Delta	Wartburg College, Waverly	6 Apr 1973
PA Lambda	Bloomsburg University of Pennsylvania, Bloomsburg	17 Oct 1973
OK Gamma	Southwestern Oklahoma State University, Weatherford	1 May 1973

NY Kappa	Pace University, New York	24 Apr 1974
TX Eta	Hardin-Simmons University, Abilene	3 May 1975
MO Iota	Missouri Southern State University, Joplin	8 May 1975
GA Alpha	State University of West Georgia, Carrollton	21 May 1975
WV Alpha	Bethany College, Bethany	21 May 1975
FL Beta	Florida Southern College, Lakeland	31 Oct 1976
WI Gamma	University of Wisconsin—Eau Claire, Eau Claire	4 Feb 1978
MD Delta	Frostburg State University, Frostburg	17 Sep 1978
IL Theta	Benedictine University, Lisle	18 May 1979
PA Mu	St. Francis University, Loretto	14 Sep 1979
AL Zeta	Birmingham-Southern College, Birmingham	18 Feb 1981
CT Beta	Eastern Connecticut State University, Willimantic	2 May 1981
NY Lambda	C.W. Post Campus of Long Island University, Brookville	2 May 1983
MO Kappa	Drury University, Springfield	30 Nov 1984
CO Gamma	Fort Lewis College, Durango	29 Mar 1985
NE Delta	Nebraska Wesleyan University, Lincoln	18 Apr 1986
TX Iota	McMurry University, Abilene	25 Apr 1987
PA Nu	Ursinus College, Collegeville	28 Apr 1987
VA Gamma	Liberty University, Lynchburg	30 Apr 1987
NY Mu	St. Thomas Aquinas College, Sparkill	14 May 1987
OH Eta	Ohio Northern University, Ada	15 Dec 1987
OK Delta	Oral Roberts University, Tulsa	10 Apr 1990
CO Delta	Mesa State College, Grand Junction	27 Apr 1990
PA Xi	Cedar Crest College, Allentown	30 Oct 1990
MO Lambda	Missouri Western State College, St. Joseph	10 Feb 1991
TX Kappa	University of Mary Hardin-Baylor, Belton	21 Feb 1991
SC Delta	Erskine College, Due West	28 Apr 1991
NY Nu	Hartwick College, Oneonta	14 May 1992
NH Alpha	Keene State College, Keene	16 Feb 1993
LA Gamma	Northwestern State University, Natchitoches	24 Mar 1993
KY Beta	Cumberland College, Williamsburg	3 May 1993
MS Epsilon	Delta State University, Cleveland	19 Nov 1994
PA Omicron	University of Pittsburgh at Johnstown, Johnstown	10 Apr 1997
MI Delta	Hillsdale College, Hillsdale	30 Apr 1997
MI Epsilon	Kettering University, Flint	28 Mar 1998
MO Mu	Harris-Stowe College, St. Louis	25 Apr 1998
GA Beta	Georgia College and State University, Milledgeville	25 Apr 1998
AL Eta	University of West Alabama, Livingston	4 May 1998
PA Pi	Slippery Rock University, Slippery Rock	19 Apr 1999
TX Lambda	Trinity University, San Antonio	22 Nov 1999
GA Gamma	Piedmont College, Demorest	7 Apr 2000
LA Delta	University of Louisiana, Monroe	11 Feb 2001
GA Delta	Berry College, Mount Berry	21 Apr 2001
TX Mu	Schreiner University, Kerrville	28 Apr 2001
CA Epsilon	California Baptist University, Riverside	21 Apr 2003
PA Rho	Thiel College, Greenville	13 Feb 2004
VA Delta	Marymount University, Arlington	26 Mar 2004
NY Omicron	St. Joseph's College, Patchogue	1 May 2004
IL Iota	Lewis University, Romeoville	26 Feb 2005
WV Beta	Wheeling Jesuit University, Wheeling	11 Mar 2005
SC Epsilon	Francis Marion University, Florence	18 Mar 2005
PA Sigma	Lycoming College, Williamsport	1 Apr 2005
MO Nu	Columbia College, Columbia	29 Apr 2005
MD Epsilon	Stevenson University, Stevenson	3 Dec 2005
NJ Delta	Centenary College, Hackettstown	1 Dec 2006
NY Pi	Mount Saint Mary College, Newburgh	20 Mar 2007
OK Epsilon	Oklahoma Christian University, Oklahoma City	20 Apr 2007
HA Alpha	Hawaii Pacific University, Waipahu	22 Oct 2007
NC Epsilon	North Carolina Wesleyan College, Rocky Mount	24 Mar 2008
NY Rho	Molloy College, Rockville Center	21 Apr 2009
NC Zeta	Catawba College, Salisbury	17 Sep 2009
RI Alpha	Roger Williams University, Bristol	13 Nov 2009
NJ Epsilon	New Jersey City University, Jersey City	22 Feb 2010
NC Eta	Johnson C. Smith University, Charlotte	18 Mar 2010
AL Theta	Jacksonville State University, Jacksonville	29 Mar 2010
GA Epsilon	Wesleyan College, Macon	30 Mar 2010
FL Gamma	Southeastern University, Lakeland	31 Mar 2010
MA Beta	Stonehill College, Easton	8 Apr 2011
AR Beta	Henderson State University, Arkadelphia	10 Oct 2011
PA Tau	DeSales University, Center Valley	29 Apr 2012
TN Zeta	Lee University, Cleveland	5 Nov 2012
RI Beta	Bryant University, Smithfield	3 Apr 2013
SD Beta	Black Hills State University, Spearfish	20 Sept 2013
FL Delta	Embry-Riddle Aeronautical University, Daytona Beach	22 Apr 2014
IA Epsilon	Central College, Pella	30 Apr 2014
CA Eta	Fresno Pacific University, Fresno	24 Mar 2015
OH Theta	Capital University, Bexley	24 Apr 2015
GA Zeta	Georgia Gwinnett College, Lawrenceville	28 Apr 2015
MO Xi	William Woods University, Fulton	17 Feb 2016
IL Kappa	Aurora University, Aurora	3 May 2016
GA Eta	Atlanta Metropolitan University, Atlanta	1 Jan 2017
CT Gamma	Central Connecticut University, New Britan	24 Mar 2017
KS Eta	Sterling College, Sterling	30 Nov 2017
NY Sigma	College of Mount Saint Vincent, The Bronx	4 Apr 2018
PA Upsilon	Seton Hill University, Greensburg	5 May 2018

KY Gamma	Bellarmine University, Louisville	23 Apr 2019
MO Omicron	Rockhurst University, Kansas City	13 Nov 2020
AK Gamma	Harding University, Searcy	27 Apr 2021
GA Theta	College of Coastal Georgia, Brunswick	22 Oct 2021
CA Theta	William Jessup University, Rocklin	17 Oct 2022